On propagation of short pulses in strong dispersion managed optical lines

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We show that the propagation of short pulses in optical lines with strong dispersion management is described by an integrable Hamiltonian system. The leading nonlinear effect is the formation of a collective dispersion which is a result of the interaction of all pulses propagating along the line.
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One of the most important practical applications of nonlinear optics is the theory of propagation optical pulses in nonlinear optical fibers. Due to dispersion, pulses of small amplitude in such a system suffer chromatic spreading, which limits the transmission capacity of a fiber. In 1973 Hasegawa and Tappert[1] proposed to use a focusing nonlinearity for compensation of chromatic spreading. The competion between spreading and nonlinear focusing in a conservative fiber leads to the formation of stationary pulses — optical solitons — which can be used as units of information. The theory of optical solitons and their interactions was developed by Zakharov and Shabat in 1971.[2] Optical solitons have now been proposed for use in dozens of planned large-scale telecommunication systems.

Real optical lines are not conservative. For compensation of the damping one must install a periodic array of amplifiers. Hasegawa and Kodama[3] showed that such ''conventional'' soliton lines inherit the basic features of conservative lines.

A more advanced (see Ref. 4) proposal is to design a line that includes a periodic system of fiber legs with opposite signs of the dispersion in addition to a periodic array of amplifiers. If such a line were linear, the designer could achieve compensation of chromatic spreading. This makes such lines the most promising systems for ultrafast communication. However, to suppress the noise and provide a low-error transmission of information, one should use optical pulses of relatively high amplitude. As a result, in the process of long-distance pulse propagation the nonlinear effects inevitably become important.

The theory of pulse propagation in long periodic structures is a new and interesting chapter of nonlinear physics. In many aspects periodic fibers differ from homogeneous ones. In the latter the dispersion is a smooth function of frequency which can be approximated inside a narrow spectral band by a low-order polynomial. As a result, a pulse envelope can be described by a partial differential equation of the second or third order, and the pulse interaction is local in time. In this situation, optical solitons propagating without chromatic distortion are the basic objects. Because of the locality of the interaction, the shape of an individual soliton does not depend on the presence of other pulses in the system. We will show in this letter that this is not necessarily true in periodic systems.

We will consider lines in which the local nonlinearity and mean dispersion are much less than the local dispersion. Such lines are characterized by a time τ_0. This is the duration of a pulse that broadens by a factor of two in passing through a fiber leg with constant dispersion. If the pulse is short ($\tau \ll \tau_0$, where τ is the pulse duration), one can speak of strong dispersion management (SDM). In this letter we show that in this limit the line is described approximately by a completely integrable Hamiltonian system.

The leading nonlinear effect is the appearance of a collective average dispersion formed by a whole ensemble of pulses propagating through the line. Since this dispersion is very nonlocal, the pulse envelopes cannot be described by any partial differential equation in coordinate space. The pulses pass through each other without interaction, but the rate of chromatic spreading of an individual pulse depends on the presence of neighboring pulses.

THE THEORY OF SHORT-PULSE PROPAGATION

The basic model for describing dispersion managed fibers is the nonlinear Schrödinger equation with periodic coefficients,

$$i\frac{\partial\Psi}{\partial x}+[d+\Phi'(x)]\Psi_{tt}+R(x)|\Psi^2|\Psi=0. \tag{1}$$

Here $\Psi(x,t)$ is the envelope of a wave pulse, and $\Phi(x)$ and $R(x)$ are periodic functions of x with the same period 2π. Insofar as $\langle\Phi'(x)\rangle=0$, d is the average dispersion of the fiber. We assume that

$$|R(x)|\ll|d+\Phi'(x)|. \tag{2}$$

In real transmission lines, this condition is usually satisfied.

Under the assumption (2), Eq. (1) can be replaced with the approximate Gabitov–Turitzyn (GT) model[5–7]

$$i\frac{\partial\chi}{\partial x}=d\omega^2\chi_\omega-\int G(\Delta)\chi^*_{\omega_1}\chi_{\omega_2}\chi_{\omega_3}\delta_{\omega+\omega_1-\omega_2-\omega_3}\,d\omega_1 d\omega_2 d\omega_3. \tag{3}$$

Here

$$\chi_\omega=\Psi_\omega e^{i\Phi(x)\omega^2},\quad \Delta=\omega^2+\omega_1^2-\omega_2^2-\omega_3^2,$$

$$G(\Delta)=\frac{1}{2\pi}\int_0^{2\pi}R(x)e^{i\Phi(x)\Delta}dx=J(\Delta\tau_0^2). \tag{4}$$

Here τ_0 is a characteristic parameter of the line, $J(\xi)$ is a function of the dimensionless variable $\xi\sim 1$, and $\Psi_\omega(x)$ is the Fourier transform of $\Psi(x,t)$. In general

$$J(-\xi)=J^*(\xi).$$

In the special case of constant nonlinearity ($R=\mathrm{const}$) and piecewise constant dispersion, one has $J(\xi)=\sin\xi/\xi$.

Taking the inverse Fourier transform of (3) leads to the equation

$$i\frac{\partial\chi}{\partial x}+d\frac{\partial^2\chi}{\partial t^2}+\frac{1}{\tau_0^2}\int F\left(\frac{pq}{\tau_0^2}\right)\chi^*(t+p+q)\,\chi(t+p)\,\chi(t+q)\,dp\,dq=0. \tag{5}$$

If $s=pq/\tau_0^2>0$, then $F(s)$ is given by the expression.

$$F(s)=\frac{1}{2\pi}\int_0^\infty\{2J^*(z)K_0(\sqrt{2sz})-\pi J(z)N_0(\sqrt{2sz})\}ds. \tag{6}$$

Here $K_0(q)$ is the Bessel function of imaginary argument, and $N_0(q)$ is the Neumann function. For negative s, $F(-s)=F^*(s)$. In the case of strong dispersion $s\ll 1$, one can use asymptotic expansions of the Bessel function at small values of the argument.

Expansion in powers of s leads to the result

$$F=F_0+F_1+F_2+\cdots, \tag{7}$$

$$F_0=\frac{2}{\pi}\int_0^\infty\left[\ln\frac{2}{|s|z}-C\right]\mathrm{Re}\,J(z)dz, \tag{8}$$

$$F_1=\frac{2is}{\pi}\int_{-\infty}^\infty z\left[\ln\frac{2}{|s|z}+1-C\right]\mathrm{Im}\,J(z)dz, \tag{9}$$

$$F_2=\frac{s^2}{4\pi}\int_0^\infty z^2\left[\ln\frac{2}{|s|z}+\frac{3}{2}-C\right]\mathrm{Re}\,J(z)dz. \tag{10}$$

Here C is the Euler constant. Let $\chi=Ae^{i\Phi}$. Plugging $F=F_0$ in (5) and performing the Fourier transform, one obtains a system of equations for A and Φ:

$$\frac{\partial A}{\partial x}=0,\quad \frac{\partial\Phi}{\partial x}=-d\omega^2+\hat{K}(A). \tag{11}$$

Here (see Ref. 8)

$$\hat{K}(A)=\frac{4}{\tau_0^2}\left[fA^2+\int_{-\infty}^\infty\frac{A^2(\omega')-A^2(\omega)}{|\omega'-\omega|}d\omega'\right], \tag{12}$$

$$f=(2\ln\tau_0+C)\,a+b,$$

$$a=\int_0^\infty J(z)dz,\quad b=\int_0^\infty\ln\frac{2}{z}J(z)dz. \tag{13}$$

In order of magnitude

$$K(A)\simeq\frac{4}{\tau_0^2}\ln\frac{\tau_0}{\tau}A^2(\omega). \tag{14}$$

In the limit $\ln \tau_0/\tau \to \infty$ formula (14) is exact. Equations (11) mean that the system (4) is integrable in the limit $\tau_0/\tau \to 0$.

The leading nonlinear effect is the formation of an additional collective dispersion $\hat{K}(A)$. Let the signal $\chi(t)$ be a superposition of a large number of identical pulses with random phases, separated by arbitrary intervals of time:

$$\chi(t) = \sum_{n=1}^{N} \chi_0(t-\tau_n), \quad N \gg 1. \tag{15}$$

Then

$$\chi(\omega) = \chi_0(\omega) \sum_{n=1}^{N} e^{-i(\omega\tau_n - \phi_n)},$$

$$A^2(\omega) = N|A_0(\omega)|^2 = |\chi(\omega)|^2. \tag{16}$$

According to (11) all the pulses in the system suffer the same chromatic spreading and remain identical. The value of spreading is determined by all the pulses present in the line. This is a result of the strong dispersional spreading of each pulse in the fibers of constant dispersion. Due to nonlinearity, this spreading cannot be completely compensated in the next fibers. As a result, each pulse generates long ''tails'' that influence the shapes of the other pulses. In the model developed here, the pulse interaction is very nonlocal — the pulses ''feel'' each other when separated by an arbitrarily long distance. This ultimate nonlocality is a weak point of the simple model developed.

Another weak point of the model is the plethora of soliton solutions. To find such a solution, one can put

$$\Phi(\omega,x) = \lambda x + \Phi_0(\omega),$$

where $\Phi_0(\omega)$ is an arbitrary function of ω and λ is an arbitrary constant.

The amplitude $A(\omega)$ is an arbitrary positive solution of the equation

$$A(\omega)[\lambda + d\omega^2 - \hat{K}(A)] = 0. \tag{17}$$

One can arbitrarily separate the axis $-\infty < \omega < \infty$ into two sets $\Omega_1 \cup \Omega_2$ and put

$$A(\omega) = 0, \quad \omega \in \Omega_1, \quad \lambda + d\omega^2 - \hat{K}(A) = 0, \quad \omega \in \Omega_2. \tag{18}$$

Here (18) is the Fredholm integral equation of the first kind. The system (18) has an infinite number of solutions. To improve the model, one should take into account higher orders of the expansion (7). This is beyond the scope of this article.

We have shown that the propagation of short optical pulses in strong dispersion managed nonlinear systems is quite different from the propagation in ''conventional'' lines with constant dispersion. The interaction of the pulses is very nonlocal — even very distant pulses interact by the formation of a substantial average dispersion. In the leading order the system of pulses is described by an integrable Hamiltonian system. In the framework of this model, solitons do not have a universal form, and their importance from the practical standpoint is unclear. One can say that in such systems the nonlinearity is purely detrimental. The program for the designer is to make the line as ''linear'' as possible.

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