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PHYSICS

On Transillumination of Wave Barriers for Electromagnetic Radiation in an Inhomogeneous Plasma

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The interaction of electromagnetic waves with inhomogeneous and nonstationary media has been studied on the basis of exactly solvable models [1–4]. Such problems are interesting for a number of applications (e.g., for elevating the efficiency of electromagnetic radiation absorption during plasma heating, for explaining the mechanism of propagation of radiation from sources located in dense plasmas in astrophysics, and also for optimizing structures to elevate the effectiveness of nonreflection and absorbing coating layers in the radio-wave range and the development of thin streamlined radio-transparent antenna coverings (radomes). This is also important in the search for the optimal permittivity distribution over the thickness of the antireflection layer, which would ensure the minimum reflectance or transmission of electromagnetic signals from antennas covered by a dense plasma layer [5]. Earlier investigations (see, for example, [2, 3]) showed that nonreflection transmission of electromagnetic waves from a vacuum to the cover can be ensured in spite of a permittivity jump at the interface. In addition, these studies make it possible to improve considerably the understanding of electromagnetic field dynamics in space–time dielectric structures.

Here, we analyze nonreflection nonlinear transmission of an electromagnetic wave through the region of an inhomogeneous plasma, which is an inhomogeneous wave barrier in the nonlinear regime; the number of model parameters can be large, which makes it possible to vary over wide limits the spatial profile of the inhomogeneous structure in which transillumination of the barrier takes place. The mathematical model of transillumination of a barrier during the interaction of a wave with an inhomogeneous medium used here is based on solving the Helmholtz equation with cubic nonlinearity,

$$F'' + k_0^2[\varepsilon_L(x) + \alpha(x)|F(x)|^2]F = 0, \tag{1}$$

where $k_0 = \omega/c$ is the vacuum wavenumber, ω is the wave frequency, $\varepsilon_L(x)$ is the linear part of the nonuniform permittivity, and $\alpha(x)|F(x)|^2 \equiv \delta\varepsilon$ is a nonlinear correction. Model (1) was used, for example, in [6, 7] in analysis of the interaction between electromagnetic waves and an inhomogeneous plasma. We assume that $\alpha(x) > 0$, which corresponds to the expulsion of the plasma from the region of the microwave field peak by a ponderomotive force. It should be noted that the linear problem with $\alpha(x) = 0$ was considered in [1–4].

It is convenient for subsequent analysis to pass to the dimensionless variable $\xi = k_0x$. Analogously to [1–3], the exact solution to Eq. (1) is taken in the semiclassical form

$$F(\xi) = A\exp[i\psi(\xi)]\left[\frac{1}{q(\xi)}\right]^{1/2}, \quad \frac{d\Psi}{d\xi} = q(\xi). \tag{2}$$

Here, A = const. Substituting relation (2) into Eq. (1), we find that the problem being solved corresponds to the quantum-mechanical problem of scattering of a zero-energy particle from the following nonuniform potential:

$$u(\xi) = -q^2 - \frac{d^2q}{d\xi^2}\frac{1}{2q} + 0.75\left(\frac{dq}{d\xi}\right)^2\frac{1}{q^2}$$

$$= -\varepsilon_L(\xi) - \frac{\alpha(\xi)|A|^2}{q(\xi)}, \tag{3}$$

where the spatial wavenumber profile $q(\xi)$, which is assumed to be set, determines the form of potential $U(\xi)$. Let us consider specific examples.

Let us suppose that $q(\xi) = 1 - \dfrac{\beta}{1+\gamma^2\xi^2}$, where $0 < \beta < 1$ and $\gamma^2 \ll 1$ in the case of smooth inhomogeneity. In the case of plasmas, this model corresponds to a nonlinear transmission of the wave through the density

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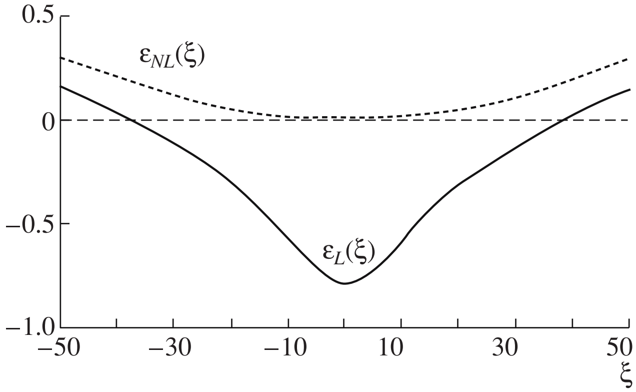

Fig. 1. Profiles of linear $\varepsilon_L(\xi)$ and nonlinear $\varepsilon_{NL}(\xi)$ permittivity of a plasma in the case of inhomogeneity of the type of a plasma density hump.

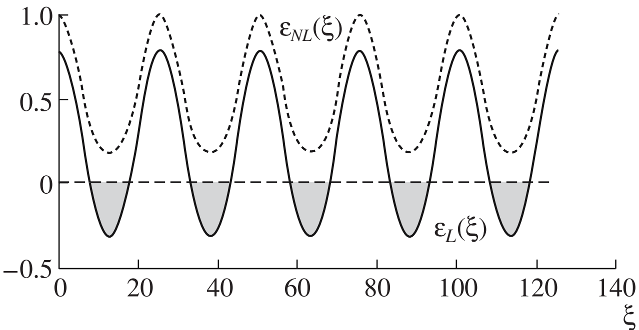

Fig. 2. Profiles of linear $\varepsilon_L(\xi)$ and nonlinear $\varepsilon_{NL}(\xi)$ permittivity of a plasma in the case of nonreflection interaction of an electromagnetic wave with a periodically modulated plasma layer.

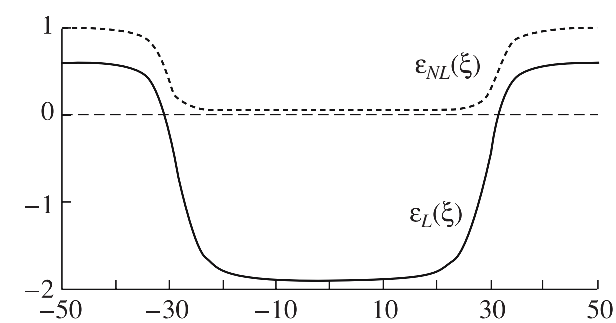

Fig. 3. Profiles of linear $\varepsilon_L(\xi)$ and nonlinear $\varepsilon_{NL}(\xi)$ permittivity of a plasma in the case of an inhomogeneity of the type of a density hump and strong nonlinearity for $\lambda = -0.8$, $\rho = 3$, and $\sigma = 0.4$.

hump. To simplify the analysis, we assume that $\sigma = \alpha|A|^2$ = const. Figure 1 shows the profiles of nonlinear $\varepsilon_{NL}(\xi)$ and linear $\varepsilon_L(\xi) = \varepsilon_{NL}(\xi) - \dfrac{\sigma|A|^2}{q(\xi)}$ permittivities of the medium, where

$$\varepsilon_{NL}(\xi) = q^2 + \frac{d^2q}{d\xi^2}\frac{1}{2q} - 0.75\left(\frac{dq}{d\xi}\right)^2\frac{1}{q^2}$$

for the following set of initial parameters: $\beta = 0.9$, $\gamma = 0.02$, and $\sigma = 0.08$. It can be seen that in the linear problem we have an opacity region (wave barrier) $|\xi| < 37.6$, in which $\varepsilon_L(\xi) < 0$. Owing to nonlinearity, transillumination of the wave barrier takes place ($\varepsilon_{NL}(\xi) > 0$) and $\min\varepsilon_{NL}(\xi) = 0.014$ in the given case. Nonreflection transillumination of an inhomogeneous wave barrier is also possible for other values of the initial parameters β, γ, and σ.

We will also consider the example of transillumination of a periodically inhomogeneous structure. Let us suppose that $q(\xi) = \mu + \chi\cos(\lambda\xi)$, where $0 < \chi < \mu < 1$, $\mu + \chi = 1$. The inhomogeneous layer occupies a region $0 \le \xi \le 2\pi n/\lambda$, where n is an integer. In the case considered here, joining with the vacuum solution, for which $q = 1$ and $dq/d\xi = 0$ takes place at its boundaries $\xi_1 = 0$ and $\xi_2 = 2\pi n/\lambda$. As a result, nonreflection transmission through a periodic structure located in layer (ξ_1, ξ_2) takes place. Figure 2 shows the profiles of nonlinear $\varepsilon_{NL}(\xi)$ and linear $\varepsilon_L(\xi)$ permittivities for the following values of the parameters: $\mu = 0.7$, $\lambda = 0.25$, and $n = 5$. According to this figure, $\varepsilon_{NL}(\xi) > 0$ everywhere in the layer; i.e., plasma is transparent due to nonlinearity. At the same time, for linear permittivity, there exist n opaque layers (hatched) in which $\varepsilon_L(\xi) < 0$. By changing the initial parameters μ and λ and number n of the layers, we can obtain a large number of other versions of nonreflection transillumination of inhomogeneous wave barriers of the type of periodic structures.

Let us consider an interesting case when a function of the well type is chosen for wavevector $q(\xi)$; for example, $q(\xi) = 1 + \lambda g_1(\xi)g_2(\xi)$, where $g_1(\xi)$ and $g_2(\xi)$ are defined as

$$g_1(\xi) = 0.5\left\{1 + \frac{\xi + b}{[\rho^2 + (\xi + b)^2]^{1/2}}\right\},$$

$$g_2(\xi) = 0.5\left\{1 + \frac{b - \xi}{[\rho^2 + (b - \xi)^2]^{1/2}}\right\}$$

with parameters ρ and λ. The characteristic width of the wave barrier is of the order of $2b$, and the blurring of well walls is characterized by parameter ρ. Figure 3 shows permittivities profiles of $\varepsilon_L(\xi)$ and $\varepsilon_{NL}(\xi)$ of the medium with minimum values $\min\varepsilon_L(\xi) = -1.919$ and $\min\varepsilon_{NL}(\xi) = 0.042$ for $\lambda = -0.8$, $\rho = 3$, and $\sigma = 0.4$. Figure 4 shows the spatial structure of the wave field $W(\xi) = \mathrm{Re}\,F(\xi)$ for $A = 1$.

Thus, other versions of nonreflection nonlinear transillumination of inhomogeneous wave barriers in the layered inhomogeneous plasma are also possible in other media. In particular, this effect is possible for quasi-periodic inhomogeneities with any (preset) num-

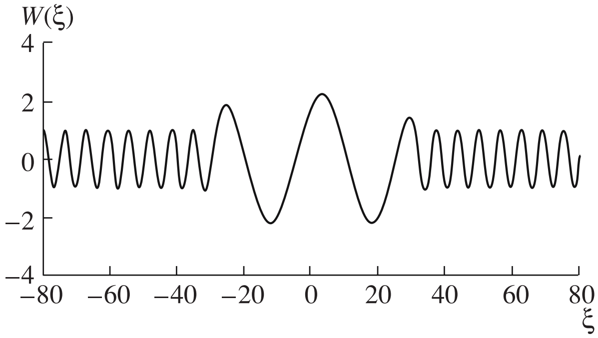

Fig. 4. Typical spatial structure of the wave field $W(\xi)$ = Re$F(\xi)$ for $A = 1$.

ber of layers; the spatial profile of inhomogeneous structures may strongly depend on any (preset) number of parameters. The following remarks of fundamental importance should be made in this connection. Having set any number of parameters and using various basis functions in the dimensionless wavenumber $q(\xi)$ for describing the lattice, the transition layer, the opacity barrier, or other localized structures, we can construct a wide set of models of nonreflection interaction of electromagnetic waves with an inhomogeneous nonlinear medium. In particular, for preset basis models of local inhomogeneity, their sum in $q(\xi)$ with a random set of model parameters will define a model of nonlinear transillumination of barriers in the case of nonreflection interaction of an electromagnetic wave with a randomly inhomogeneous medium. We can expect that, in the addition of electromagnetic modes, nonreflection transmission and transillumination of barriers is also possible for other types of waves in an inhomogeneous plasma. Since a continuum of various inhomogeneity profiles with nonreflection transillumination of wave barriers exists, their closeness to experimentally observed situations cannot be ruled out.

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