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## PLASMA OSCILLATIONS AND WAVES

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# Reflectionless Passage of an Electromagnetic Wave through an Inhomogeneous Plasma Layer

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**Abstract**—An exactly solvable model is used as a basis to study the reflectionless passage of a transverse electromagnetic wave through an inhomogeneous plasma containing large-amplitude, small-scale (subwave-length) structures (in particular, opaque regions) that cannot be correctly described by approximate methods. It is shown that, during the reflectionless passage of an electromagnetic wave, strong wave field splashes can occur in certain plasma sublayers. The nonuniform spatial plasma density profile is characterized by a number of free parameters describing the modulation depth of the dielectric function, the characteristic sizes of the structures and their number, the thickness of the inhomogeneous plasma region, and so on. Such plasma density structures are shown to be very diverse when, e.g., a wave that is incident from vacuum propagates without reflection through a plasma layer (wave barrier transillumination). With the cubic nonlinearity taken into account, a one-dimensional problem of the nonlinear transillumination of an inhomogeneous plasma can be solved exactly.

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## 1. INTRODUCTION

In recent years, the characteristic features of the interaction between electromagnetic waves and inhomogeneous media with large-amplitude, small-scale structures have been actively studied (see, e.g., [1–7]). In particular, much is being given to analysis of the possibility of reflectionless wave passage by using exactly solvable physico-mathematical models with which to study wave processes when approximate methods are inapplicable and with which to predict new effects that are introduced by strong small-scale inhomogeneities and are of great interest for various applications. Thus, reflectionless passage is important for such issues as plasma heating by electromagnetic waves, understanding of the mechanisms for the escape of radiation from sources in a high-density plasma [8], and raising the efficiency of antireflecting and absorbing coatings in radiophysics (in particular, with the aim of developing thin radiotransparent fairings) [9]. In the reflectionless passage problem, it is of interest to seek an optimum spatial profile of the dielectric function that ensures a minimum coefficient of reflection and/or an efficient transmission of electromagnetic signals from antennas with a high-density plasma layer on their surface [8]. It should be noted that the exactly solvable models in question can reveal fundamentally new features of the wave dynamics and propagation in inhomogeneous media and can also demonstrate interesting practical applications when the parameters of the medium can be varied in a con-

trolled manner. The problems mentioned above deal in fact with the resonant tunneling of electromagnetic waves through a stratified plasma containing, in particular, opaque regions. Hence, in order to ensure the reflectionless passage of an electromagnetic wave through an inhomogeneous plasma, it is necessary to investigate the possibility of making the wave parameters consistent with the parameters of an inhomogeneous plasma layer. For the problems of raising the efficiency of absorption of electromagnetic waves in resonant plasma layers, this issue was addressed earlier, e.g., in [10, 11].

In the present paper, we analyze exactly solvable models for describing the reflectionless passage of an electromagnetic wave through a thick inhomogeneous plasma layer with large-amplitude, small-scale structures. We describe analytic models for this process, investigate the generation of strong wave field splashes in certain plasma sublayers, and examine a nonlocal relationship between the spatial profiles of the wave vector and the effective plasma dielectric function. We show that, depending on the choice of the initial parameters of the problem, the wave amplitude in the splashes can increase by an order of magnitude or more. The mathematical model used here to describe the barrier transillumination in the interaction of a wave with an inhomogeneous plasma is based on solving the Helmholtz equation. In this model, the number of free parameters determining the number of structures and splashes, as well as the amplitudes and

characteristic sizes of the inhomogeneities and their spatial profiles, can be large enough for the nonuniform spatial profile of a plasma density structure in which the wave barriers are transilluminated to be substantially varied.

A fundamentally important point to note is that, in a stratified plasma, there are large-amplitude sub-wavelength inhomogeneities, so the problem cannot be solved by standard approximate methods. It is quite obvious that the analysis of exactly solvable models for the reflectionless interaction of an electromagnetic wave with an inhomogeneous plasma will considerably refine current views about the dynamics of electromagnetic fields in highly inhomogeneous and nonstationary dielectric media such as laboratory and space plasmas.

## 2. BASIC EQUATIONS AND THEIR ANALYSIS

We consider a one-dimensional problem of the reflectionless passage of an electromagnetic wave through an inhomogeneous plasma layer. For an s-polarized electromagnetic wave in a plasma in the absence of a magnetic field, or for a wave propagating in a magnetized plasma across an external uniform magnetic field, we use the standard representation for the wave field,  $E(x, t) = \text{Re}[F(x)\exp(-i\omega t)]$ , where  $\omega$  is the wave frequency and the function  $F(x)$  satisfies the Helmholtz equation

$$d^2F/dx^2 + k_0^2 \varepsilon_{\text{ef}}(x)F = 0. \quad (1)$$

Here, the  $x$  axis points in the direction in which the plasma density varies,  $k_0 = \omega/c$  is the vacuum wave-number, and  $\varepsilon_{\text{ef}}(x)$  is the effective plasma dielectric function. For the linear wave propagation in a plasma in the absence of an external magnetic field, we have  $\varepsilon_{\text{ef}}(x) = 1 - [\omega_{\text{pe}}(x)/\omega]^2$  with  $\omega_{\text{pe}}(x)$  being the electron Langmuir frequency. For an extraordinary wave propagating in a magnetized plasma across an external magnetic field, we have  $\varepsilon_{\text{ef}}(x) \equiv N^2(x) = \varepsilon_{\perp} - (\varepsilon_c^2/\varepsilon_{\perp})$ , where  $N$  is the refractive index and  $\varepsilon_{xx} = \varepsilon_{yy} \equiv \varepsilon_{\perp}$  and  $\varepsilon_{xy} = -i\varepsilon_c$  are the components of the plasma dielectric tensor [12]. For further analysis, it is convenient to switch to the dimensionless spatial variable  $\xi = k_0 x$  and dimensionless wave vector  $p(\xi) = ck_x(x)/\omega$ . By analogy with [2, 5, 12], the exact solution to Eq. (1) can be written as

$$F(\xi) = F_0 \exp[i\Psi(\xi)] [1/p(\xi)]^{1/2}, \quad (2)$$

$$d\Psi/d\xi = p(\xi), \quad F_0 = \text{const.}$$

We can see from Eq. (1) and solution (2) that the dielectric function  $\varepsilon_{\text{ef}}(x)$  is related to the dimensionless wave vector  $p(\xi)$  by the nonlinear equation

$$\varepsilon_{\text{ef}}(\xi) = [p(\xi)]^2 + (d^2p/d\xi^2)/2p - 0.75(dp/d\xi)^2/p^2, \quad (3)$$

which describes the nonlocal relationship between the functions  $\varepsilon_{\text{ef}}(\xi)$  and  $p(\xi)$ . We also introduce the normalized wave amplitude  $|F/F_0| \equiv A(\xi) = [1/p(\xi)]^{1/2}$ . Equation (3) can then be rewritten as a nonlinear equation for the wave amplitude  $A$ :

$$d^2A/d\xi^2 + \varepsilon_{\text{ef}}(\xi)A - [1/A(\xi)]^3 = 0. \quad (4)$$

For a given effective dielectric function  $\varepsilon_{\text{ef}}(\xi)$ , nonlinear equation (4) describes the spatial profile of the dimensionless amplitude of the electromagnetic wave. It should be noted at this point that, even for a homogeneous plasma ( $\varepsilon_{\text{ef}}(\xi) = \text{const}$ ), the solution to Eq. (4) with a fixed wave frequency describes a spatially modulated wave packet with a free parameter characterizing the (possibly rather large) variations in the amplitude  $A$ .

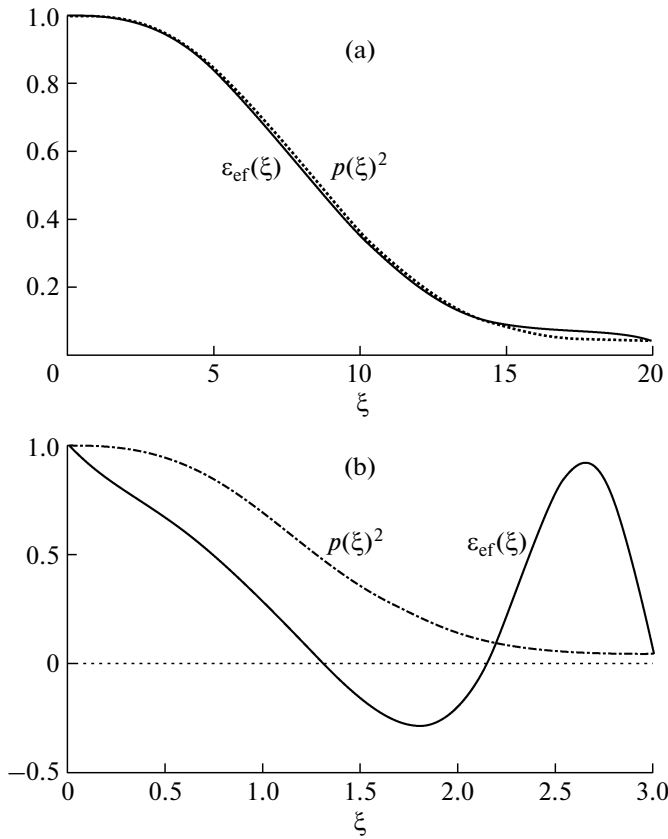
In what follows, the solutions to Eq. (4) are analyzed by using definite analytic functions  $A(\xi)$  and  $p(\xi)$  and calculating the effective dielectric function  $\varepsilon_{\text{ef}}(\xi)$  corresponding to the reflectionless interaction of an electromagnetic wave with an inhomogeneous plasma.

Let us consider reflectionless passage of a transverse electromagnetic wave through a transient plasma layer that occupies the region  $0 \leq \xi \leq b$  and is in contact with vacuum on the left side ( $\xi = 0$ ) and with a homogeneous subcritical plasma ( $\omega_{\text{pe}} < \omega$ ) on the right side. The model expression for the dimensionless wave vector that ensures reflectionless matching with the electromagnetic waves incident from vacuum and escaping into the homogeneous plasma can be taken to have the form  $p(\xi) = \alpha + \beta(1 - 1.5s + 0.6s^2)s^3$  with the free parameters  $\alpha$ ,  $\beta$ , and  $b$ . The variable  $s = \xi/b$  is introduced for convenience. Note that  $p(b) = \alpha + 0.1\beta$  and  $p(0) = \alpha$ . The choice  $\alpha = 1$  corresponds to vacuum ( $\xi < 0$ ), and the plasma density in the region  $\xi > b$  is below the critical value when  $\alpha + 0.1\beta > 0$ . The derivatives of the wave vector are given by the formulas

$$g(\xi) = dp/d\xi = 3\beta(1 - 2s + s^2)s^2/b,$$

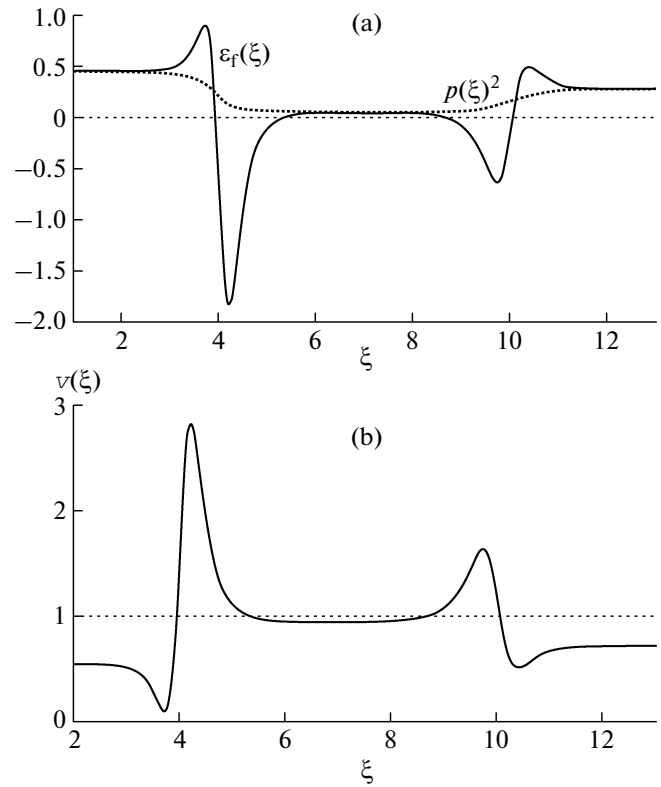
$$h(\xi) = d^2p/d\xi^2 = 6(1 - 3s + 2s^2)s/b^2.$$

It is easy to verify that the reflectionless matching conditions for the wave fields (the tangential components) are satisfied at the boundaries of the transient plasma layer:  $g(0) = g(b) = h(0) = h(b) = 0$ . The plots of the functions  $\varepsilon_{\text{ef}}(\xi)$  and  $p^2(\xi)$  for  $\beta = -0.8$  and  $b = 20$  are shown in Fig. 1a. This model version corresponds to a sufficiently smooth nonuniform plasma density profile (a wide transient layer) such that the dimensionless plasma density increases toward the right layer boundary in the range  $0 \leq v(\xi) = (\omega_{\text{pe}}/\omega)^2 \leq 0.96$ . That is why the functions  $\varepsilon_{\text{ef}}(\xi)$  and  $p^2(\xi)$  differ insignificantly from one another (by an amount of about 3% in the main part of the layer). Owing to a decrease in the wave vector  $p(\xi)$ , the wave amplitude  $A(\xi)$  gradually increases toward the right boundary, in which case the field in the layer is amplified by  $A_{\text{max}}/A_{\text{min}} \approx 2.235$ .



**Fig. 1.** (a) Profiles of the plasma dielectric function and the wavenumber squared in a smoothly inhomogeneous transient plasma layer in the model with  $p(\xi) = \alpha + \beta(1 - 1.5s + 0.6s^2)s^3$ . (b) Profiles of the plasma dielectric function and the wavenumber squared in a transient plasma layer with large-amplitude, small-scale inhomogeneities in the model with  $p(\xi) = \alpha + \beta(1 - 1.5s + 0.6s^2)s^3$ .

Let us consider a model version with a smaller parameter  $b$  ( $b = 3$ , which corresponds to a thinner transient layer), the remaining parameters being the same. For this case, the plots of the functions  $\epsilon_f(\xi)$  and  $p^2(\xi)$  are shown in Fig. 1b. We can see that, because of the strong gradient dispersion (subwavelength inhomogeneity), the profiles of the wavenumber and plasma dielectric function in the transient layer differ radically from one another. The wave vector  $p(\xi)$  again decreases gradually as  $\xi$  increases, while the function  $\epsilon_f(\xi)$  has a deep well and a large hill in the layer. In addition, in the layer, there is the opaque region  $1.3 < \xi < 2.15$ , in which  $\epsilon_f(\xi) < 0$ , such that  $\epsilon_f(\xi) = -0.28$  at  $\xi = 1.8$ . The profile  $v(\xi)$  of the dimensionless plasma density in the transient layer has a similar shape. Thus, in the hill region, we have  $\max v(\xi) \approx 1.284$  at  $\xi = 1.81$ ; and the local minimum in the plasma density,  $v(\xi) \approx 0.077$ , is at  $\xi = 2.64$ . Hence, by varying the initial parameters of the problem, we can vary the nonuniform plasma density profile.



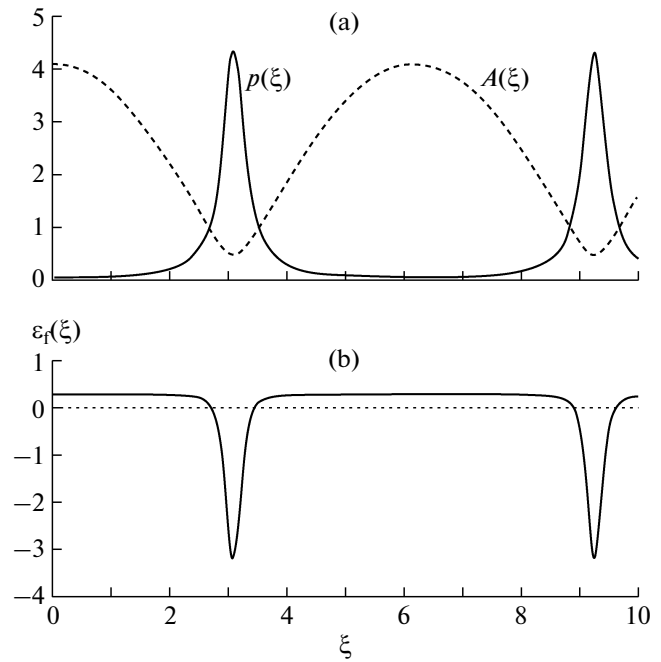
**Fig. 2.** (a) Profiles of the plasma dielectric function  $\epsilon_f(\xi)$  and function  $p^2(\xi)$  in a transient plasma layer with two steplike structures in the model with  $p(\xi) = \alpha + \beta_1 f_1(\xi) + \beta_2 f_2(\xi)$ , where the function  $f_n(\xi) = 0.5\{1 + (\xi - b_n)/[\gamma_n^2 + (\xi - b_n)^2]^{0.5}\}$  has the parameters  $\alpha = 0.67$ ,  $\beta_1 = -0.43$ ,  $\beta_2 = 0.29$ ,  $\gamma_1 = 0.45$ ,  $\gamma_2 = 0.61$ ,  $b_1 = 4$ , and  $b_2 = 10$ . (b) Plasma density profile  $v(\xi) = (\omega_{pe}/\omega)^2$  in a transient plasma layer with two steplike structures.

In another version of the model of an inhomogeneous transient plasma layer ( $1 \leq \xi \leq 13$ ), the dimensionless wave vector is given by the expression  $p(\xi) = \alpha + \sum_n \beta_n f_n(\xi)$ , where the function  $f_n(\xi) = 0.5\{1 + (\xi - b_n)/[\gamma_n^2 + (\xi - b_n)^2]^{0.5}\}$  has the structure of a step centered at the point  $\xi = b_n$  and having a characteristic thickness of  $\delta\xi_n \sim 2\gamma_n$ . The parameters  $\beta_n$ ,  $b_n$ , and  $\gamma_n$  describe the inhomogeneity of a plasma sublayer with the number  $n$ . The parameter  $\beta_n$  can be either positive or negative. A smoothly inhomogeneous layer (to which the quasiclassical approximation is applicable) refers to the case  $\gamma_n^2 \gg 1$ , in which the  $p^2(\xi)$  and  $\epsilon_f(\xi)$  profiles differ only slightly. In an inhomogeneous plasma layer such that  $\gamma_n^2 \ll 1$ , there are large-amplitude, small-scale structures and the gradient dispersion is strong. In this case, the  $p^2(\xi)$  and  $\epsilon_f(\xi)$  profiles differ both qualitatively and quantitatively. This is illustrated in Fig. 2a, which shows a model version of a transient layer in the form of two steps with the param-

eters  $\alpha = 0.67$ ,  $\beta_1 = -0.43$ ,  $\beta_2 = 0.29$ ,  $\gamma_1 = 0.45$ ,  $\gamma_2 = 0.61$ ,  $b_1 = 4$ , and  $b_2 = 10$ . The wavenumber profile has the form of a well such that  $p(1) \approx 0.688$  and  $p(13) \approx 0.53$ , with  $\min p \approx 0.247$  at the layer center. The profile of the dielectric function has three hills and two wells, with two opaque regions  $3.95 \leq \xi \leq 5.33$  and  $8.7 \leq \xi \leq 10.05$ , in which  $\varepsilon_f(\xi) < 0$ . The parameters of the effective dielectric function  $\varepsilon_f(\xi)$  are as follows:  $\max \varepsilon_f(3.71) \approx 0.896$ ,  $\max \varepsilon_f(10.41) \approx 0.486$ , and  $\varepsilon_f(7) \approx 0.06$  for the hills; and  $\min \varepsilon_f(4.22) \approx -1.82$  and  $\min \varepsilon_f(9.72) \approx -0.63$  for the opaque regions. The plasma density profile  $v(\xi) = 1 - \varepsilon_f(\xi)$  is double-humped (see Fig. 2b), with  $\max v(\xi) \approx 2.81$  at  $\xi \approx 4.23$  and  $\min v(\xi) \approx 0.115$  at  $\xi \approx 3.74$ ; that is, the maximum and minimum plasma densities differ by a factor of 24.4. We can see from Fig. 2b that, over a substantial part of the inhomogeneous region, the plasma density is overcritical,  $v(\xi) > 1$ . The above analysis shows that, by increasing the number of functions  $f_n(\xi)$  and varying their parameters, it is possible to model many various structures in an inhomogeneous plasma layer, while ensuring the reflectionless passage of an electromagnetic wave through these rather complicated wave barriers. Thus, we can speak of the 100% transillumination of wave barriers.

Let us now consider an interesting model version of an inhomogeneous plasma layer in which variations in the dielectric function  $\varepsilon_f(\xi)$  are small, of about 3.7%, while variations in the wavenumber  $p(\xi)$  and normalized wave field amplitude  $A(\xi)$  are rather large. In this model of an inhomogeneous plasma through which an electromagnetic wave can propagate without reflection, the wavenumber can be described, in particular, by the formula  $p(\xi) = \mu\sigma/[1 + B\cos(2\mu\xi)]$  with the free parameters  $\mu$ ,  $\sigma$ , and  $B$ . Let these parameters be  $\mu = 0.51$ ,  $\sigma = 0.2325$ , and  $B = 0.973$ , and let the plasma layer occupy the region  $0 \leq \xi \leq 10$ . For this case, the functions  $p(\xi)$  and  $A(\xi)$  are plotted in Fig. 3a and  $\varepsilon_f(\xi)$  is plotted in Fig. 3b. We can see that variations in the functions  $p(\xi)$  and  $A(\xi)$  are rather large. Thus, for  $p(\xi)$  and  $A(\xi)$ , we have  $p_{\max}/p_{\min} \approx 71.5$  and  $A_{\max}/A_{\min} \approx 8.45$ , respectively. Conversely, the dielectric function  $\varepsilon_f(\xi)$  in the layer varies only slightly, such that  $\max \varepsilon_f(\xi) \approx 0.26$  and  $\min \varepsilon_f(\xi) \approx 0.251$ . Moreover, if we choose the parameter  $\sigma$  to be  $\sigma_c = (1 - B^2)^{1/2}$ , then we find that, in the plasma layer, the dielectric function and plasma density,  $\varepsilon_f(\xi)$  and  $v(\xi)$ , are constant, while variations in the functions  $p(\xi)$  and  $A(\xi)$  remain large. The profile of the dimensionless plasma density,  $v(\xi) = 1 - \varepsilon_f(\xi)$ , corresponds to the  $\varepsilon_f(\xi)$  profile.

It should be noted at this point that the  $\varepsilon_f(\xi)$  profile is sensitive to a small variation in the parameter  $\sigma$ . Thus, in a model version with  $\sigma = 0.21$  and the same remaining parameters  $\mu = 0.51$  and  $B = 0.973$ , we can see from Fig. 3b that the dielectric function  $\varepsilon_f(\xi)$  is highly nonuniform:  $\max \varepsilon_f(\xi) \approx 0.259$  and  $\min \varepsilon_f(\xi) \approx$



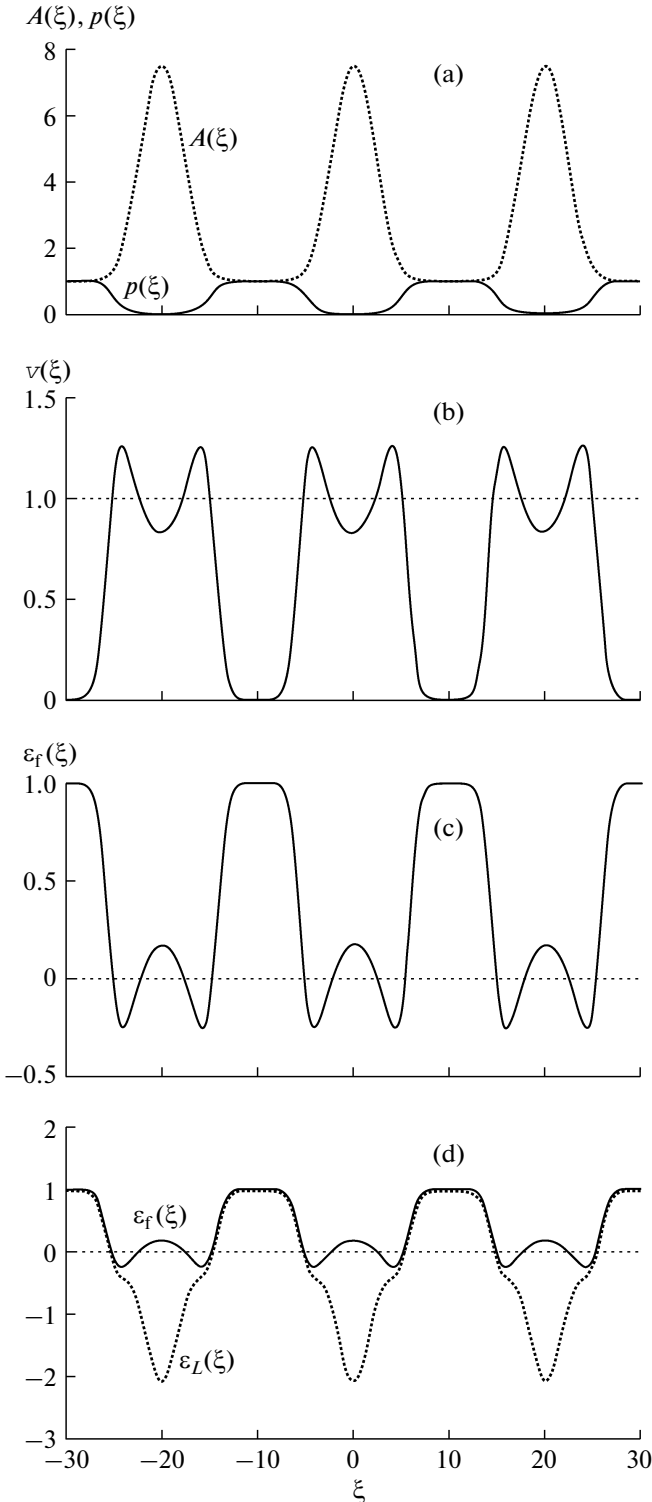
**Fig. 3.** (a) Profiles of the wavenumber  $p(\xi) = 1/A(\xi)^2$  and normalized wave amplitude  $A(\xi)$  for an inhomogeneous plasma layer with a small variation in the dielectric function  $\varepsilon_f(\xi)$  in the model with  $p(\xi) = \mu\sigma/[1 + B\cos(2\mu\xi)]$  with the parameters  $\mu = 0.51$ ,  $\sigma = 0.2325$ , and  $B = 0.973$ . The layer occupies the region  $0 \leq \xi \leq 10$ . (b) Profile of the dielectric function  $\varepsilon_f(\xi)$  in the model with  $p(\xi) = \mu\sigma/[1 + B\cos(2\mu\xi)]$  with the parameters  $\mu = 0.51$ ,  $\sigma = 0.21$ , and  $B = 0.973$ .

–3.144. In addition, in the plasma layer, there are now two opaque regions. In this case, the  $p(\xi)$  and  $A(\xi)$  profiles differ slightly from those in the previous case, while the plasma density varies to a great extent:  $\max v(\xi) \approx 4.194$  and  $\min v(\xi) \approx 0.741$ . We thus have  $\max v(\xi)/\min v(\xi) \approx 5.66$ .

In this model, the inhomogeneity is described by a periodic function, so the thickness of the plasma layer can be increased by any integer number of times. Nevertheless, we again deal with the reflectionless passage of an electromagnetic wave through a layer, i.e., with the transillumination of wave barriers for an electromagnetic wave.

Let us also consider the onset of strong splashes of the wave field amplitude in a periodically inhomogeneous plasma layer. To do this, we can use, e.g., the following model expression for the normalized amplitude:  $A(\xi) = \mu + \alpha(1 + \cos\gamma\xi)^4/16$ , where  $\mu$ ,  $\alpha$ , and  $\gamma$  are the parameters of the problem. Let the layer occupy the region  $-3b \leq \xi \leq 3b$ , the parameter values being  $\gamma = \pi/b$ ,  $\mu = 1$ , and  $\alpha = 6.5$ . The relationship of the function  $A(\xi)$  to the wave vector  $p(\xi)$  and dielectric function  $\varepsilon_f(\xi)$  has been given above. Note that, at the plasma–vacuum interfaces, we have  $p(\xi) = 1$ ,  $\varepsilon_f(\xi) = 1$ , and  $dp/d\xi = 0$ —the conditions that ensure the reflec-

tionless matching with the vacuum wave field at both layer boundaries. The profiles of the normalized amplitude  $A(\xi)$  and wave vector  $p(\xi)$  are shown in Fig. 4a. We can see that, in the plasma layer, the wave field is greatly amplified at the center of the splash,  $A_{\max} = 7.5$ , and the variation in the wave vector  $p(\xi)$  is large,  $p(10)/p(0) = 56.25$ , with  $p(0) = 0.18$ . The profile



of the plasma density  $v(\xi)$  is displayed in Fig. 4b, from which we can see that, in the layer, there are six opaque regions in which  $v(\xi) > 1$ , with  $\max v(\xi) = 1.251$ . The profile of the dielectric function in the plasma layer is given in Fig. 4c, according to which we have  $\min \epsilon_f(\xi) \approx -0.248$  in the opaque plasma regions. The smaller humps of the function  $\epsilon_f(\xi)$  correspond to  $\epsilon_f(\xi) \approx 0.171$ . Note that, in this case, the thickness of the plasma layer can also be increased by any integer number of times, while keeping the  $\epsilon_f(\xi)$ ,  $A(\xi)$ , and  $p(\xi)$  profiles unchanged and ensuring the reflectionless interaction of an electromagnetic wave incident from vacuum with an inhomogeneous plasma in the region  $-3bn \leq \xi \leq 3bn$  (with  $n = 2, 3, \dots$ ).

It is of interest to consider how the cubic nonlinearity is accounted for in the model in analyzing the possibility of reflectionless passage of an electromagnetic wave through an inhomogeneous plasma layer. Let the functions  $A(\xi)$ ,  $p(\xi)$ , and  $\epsilon_f(\xi)$  be given by the above formulas. We use the following relationship between the nonlinear,  $\epsilon_f(\xi)$ , and linear,  $\epsilon_L(\xi)$ , plasma dielectric functions in the absence of an external magnetic field:  $\epsilon_f(\xi) = \epsilon_L(\xi) + \chi|A|^2$ , where the nonlinearity parameter is set to be  $\chi = \text{const}$  for simplicity. For a weak nonlinearity ( $\chi = 0.04$ ), the functions  $\epsilon_f(\xi)$  and  $\epsilon_L(\xi)$  are plotted in Fig. 4d. A comparison between the profiles shows that, even when the nonlinearity parameter is small, the linear dielectric function  $\epsilon_L(\xi)$  has far deeper wells,  $\epsilon_L(\xi) = -2.079$ , and is characterized by far greater opaque regions. However, in certain sublayers, the  $\epsilon_f(\xi)$  and  $\epsilon_L(\xi)$  profiles are rather close to one another. For the linear dielectric function  $\epsilon_L(\xi)$  in a plasma layer with the thickness  $\delta\xi = 60$ , the total thickness of opaque regions is 31.5, a value that is more than 50% of the layer thickness  $\delta\xi$ . For the nonlinear dielectric function  $\epsilon_f(\xi)$ , the total thickness of the opaque regions is far smaller, of about 18% of the layer thickness. Hence, due to nonlinearity and resonant tunneling, an electromagnetic wave can propagate through an inhomogeneous plasma without

**Fig. 4.** (a) Profiles of the wavenumber  $p(\xi) = 1/A(\xi)^2$  and normalized wave amplitude  $A(\xi)$  in the case of a periodically inhomogeneous plasma layer with the generation of strong splashes of the wave field amplitude in the model with  $A(\xi) = \mu + \alpha(1 + \cos\gamma\xi)^4/16$  with the parameters  $\gamma = \pi/b$ ,  $\mu = 1$ ,  $\alpha = 6.5$ , and  $b = 10$ . (b) Plasma density profile in a periodically inhomogeneous plasma layer for the parameters  $\gamma = \pi/b$ ,  $\mu = 1$ ,  $\alpha = 6.5$ , and  $b = 10$ . (c) Profile of the plasma dielectric function in a periodically inhomogeneous plasma layer for the parameters  $\gamma = \pi/b$ ,  $\mu = 1$ ,  $\alpha = 6.5$ , and  $b = 10$ . (d) Profiles of the linear and nonlinear plasma dielectric functions in a periodically inhomogeneous plasma layer in the model with  $\epsilon_f(\xi) = \epsilon_L(\xi) + \chi|A|^2$  with the parameters  $\gamma = \pi/b$ ,  $\mu = 1$ ,  $\alpha = 6.5$ ,  $b = 10$ , and  $\sigma = 0.04$ .

reflection, in which case it can generate strong electromagnetic field splashes in certain plasma sublayers.

### 3. CONCLUSIONS

In the present paper, reflectionless passage of a transverse electromagnetic wave through an inhomogeneous plasma layer and the onset of fairly strong wave field splashes in certain plasma sublayers due to a sharp decrease in the wave vector have been studied based on an exactly solvable model Helmholtz equation. The relationship of the electromagnetic field amplitude  $A(\xi)$  to the effective plasma dielectric function  $\varepsilon_r(\xi)$  is described by a nonlinear equation and is nonlocal because there are large-amplitude subwavelength structures in the plasma.

An electromagnetic wave can propagate through an inhomogeneous plasma layer without reflection both in the presence and absence of an external magnetic field and independently of the thickness of the layer, in which there may be fairly thick opaque regions, where  $\varepsilon_r(\xi) < 0$ .

In the reflectionless passage of an electromagnetic wave through an inhomogeneous plasma, the spatial profiles of the wave fields and subwavelength plasma density structures are characterized by a number of free parameters. By varying them, it is possible to substantially vary such factors as the type of inhomogeneity, the number and total thickness of the opaque regions, and the extent to which the electromagnetic wave field is amplified in the splash regions. The analysis carried out here shows that, even for a comparatively small variation in the effective nonuniform dielectric function  $\varepsilon_r(\xi)$  in a plasma layer, variations in the wave vector and wave field amplitude can be rather large because of the strong gradients of  $A(\xi)$ . It is important to note that, for plasma inhomogeneity with a random component, the effect described above—transillumination of the wave barriers—can also be

expected to occur. However, this issue requires a separate study.

Note finally that the exactly solvable models under consideration can reveal new effects in the wave dynamics and in the propagation of electromagnetic waves through an inhomogeneous plasma, as well as in the development of nonlinear processes against a highly inhomogeneous background, and they can also demonstrate interesting practical applications when the parameters of inhomogeneous media can be varied in a controlled manner.

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