ISSN 0021-3640, JETP Letters, 2013, Vol. 98, No. 1, pp. 43–47. © Pleiades Publishing, Ltd., 2013.

On the Nonintegrability of the Free Surface Hydrodynamics[¶]

A. I. Dyachenko[a, b], D. I. Kachulin[b], and V. E. Zakharov[a–d]

[a] Novosibirsk State University, Novosibirsk, 630090 Russia

[b] Landau Institute for Theoretical Physics, Russian Academy of Sciences, Chernogolovka, Moscow region, 142432 Russia

[c] Department of Mathematics, University of Arizona, AZ 857201 Tucson, USA

[d] Lebedev Physical Institute, Russian Academy of Sciences, Moscow, 119991 Russia

e-mail: alexd@itp.ac.ru

Received May 21, 2013

The integrability of the compact 1D Zakharov equation has been analyzed. The numerical experiments show that the multiple collisions of breathers (which correspond to envelope solitons in the NLSE approximation) are not pure elastic. The amplitude of six-wave interactions for the compact 1D Zakharov equation has also been analyzed. It has been found that the six-wave amplitude is not canceled for this equation. Thus, the 1D Zakharov equation is not integrable.

DOI: 10.1134/S002136401314004X

1. INTRODUCTION

The work described here is motivated by remarkable fact regarding two-dimensional free surface hydrodynamics—four-wave interaction coefficient vanishes on the resonant manifold

$$k + k_1 = k_2 + k_3,$$

$$\omega_k + \omega_{k_1} = \omega_{k_2} + \omega_{k_3}.$$

This cancellation was derived in [1] and brought the hypothesis of integrability of 2D free surface hydrodynamics. Also the cancellation allows to consider surface waves moving in the same direction only. Namely, if initial state consists of such waves, evolution equation keeps this property. In this article we study the problem of integrability in more details.

So, we consider two-dimensional potential flow of an ideal incompressible fluid with a free surface in a gravity field fluid which is described by the following set of equations:

$$\phi_{xx} + \phi_{zz} = 0 \quad (\phi_z \longrightarrow 0,\, z \longrightarrow -\infty),$$

$$\eta_t + \eta_x\phi_x = \phi_z|_{z=\eta},$$

$$\phi_t + \frac{1}{2}(\phi_x^2 + \phi_z^2) + g\eta = 0|_{z=\eta};$$

here, $\eta(x, t)$ is the shape of a surface, $\phi(x, z, t)$ is a potential function of the flow, and g is a gravitational acceleration. As was shown in [2] these equations are Hamiltonian with respect to variables $\eta(x, t)$ and $\psi(x, t) = \phi(x, z, t)|_{z=\eta}$,

$$\frac{\partial\psi}{\partial t} = -\frac{\delta H}{\delta\eta}, \quad \frac{\partial\eta}{\partial t} = \frac{\delta H}{\delta\psi}.$$

Here $H = K + U$ is the total energy of the fluid with the following kinetic and potential energy terms:

$$K = \frac{1}{2}\int dx \int_{-\infty}^{\eta} v^2 dz, \quad U = \frac{g}{2}\int \eta^2 dx.$$

Hamiltonian can be expanded in infinite series of characteristic wave steepness $k\eta_k \ll 1$ and we consider this series up to the fourth order:

$$H = \frac{1}{2}\int g\eta^2 + \psi\hat{k}\psi\, dx - \frac{1}{2}\int\{(\hat{k}\psi)^2 - (\psi_x)^2\}\eta\, dx + \frac{1}{2}\int\{\psi_{xx}\eta^2\hat{k}\psi + \psi\hat{k}[\eta\hat{k}(\eta\hat{k}\psi)]\}\, dx + \dots. \tag{1}$$

Applying canonical transformation along with introducing normal complex variable $b(x, t)$ Hamiltonian (1) transforms to the equivalent compact form:

$$H = \int b^*\hat{\omega}_k b\, dx + \frac{1}{2}\int |b'|^2\left[\frac{i}{2}(bb'^* - b^*b') - \hat{k}|b|^2\right]dx. \tag{2}$$

Here, $b' = \partial b/\partial x$, $\omega_k = \sqrt{gk}$, and $\hat{k}$ is the modulus operator. All the details of this transformation can be found in [3].

Corresponding equation of motion is the following:

$$i\frac{\partial b}{\partial t} = \hat{\omega}_k b + \frac{i}{4}\hat{P}^+\left[b^*\frac{\partial}{\partial x}(b'^2) - \frac{\partial}{\partial x}\left(b^{*\prime}\frac{\partial}{\partial x}b^2\right)\right] - \frac{1}{2}\hat{P}^+\left[b\hat{k}(|b'|^2) - \frac{\partial}{\partial x}[b'\hat{k}(|b|^2)]\right]. \tag{3}$$

[¶]The article is published in the original.

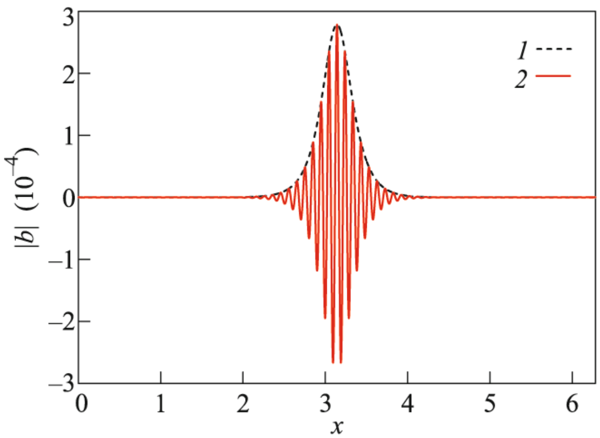

Fig. 1. Modulus of $b(x)$ and real part of $b(x)$ with $V = 1/16$ and $\Omega = 4.01$, carrier wavenumber appears to be ~64. Dashed line corresponds to modulus of $b(x)$, solid line corresponds to the real part of $b(x)$.

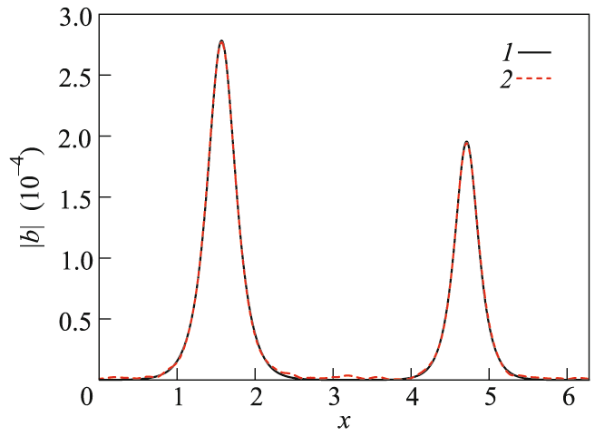

Fig. 2. Modulus of $b(x)$ for two points of time. Solid line corresponds the initial statement ($t = 0$), dashed line corresponds to the state after 100 breather collisions (t ~ 88000).

For Fourier harmonics Hamiltonian can be written as following

$$H = \int \omega_k |b_k|^2$$
$$+ \frac{1}{2}\int T_{k_1k_2}^{k_3k_4} b_{k_1}^* b_{k_2}^* b_{k_3} b_{k_4} \delta_{k_1+k_2-k_3-k_4} dk_1 dk_2 dk_3 dk_4.$$

Here

$$T_{k_2k_3}^{kk_1} = \frac{\theta(k)\theta(k_1)\theta(k_2)\theta(k_3)}{8\pi}$$
$$\times \{[kk_1(k+k_1) + k_2k_3(k_2+k_3)](kk_2|k-k_2| \quad (4)$$
$$+ kk_3|k-k_3| + k_1k_2|k_1-k_2| + k_1k_3|k_1-k_3|\},$$

where θ functions in (4) correspond to waves moving in the same direction. Corresponding evolution equation is the following:

$$i\frac{\partial b_k}{\partial t} = \omega_k b_k$$
$$+ \int T_{kk_1}^{k_2k_3} b_{k_1}^* b_{k_2} b_{k_3} \delta_{k+k_1-k_2-k_3} dk_1 dk_2 dk_3. \quad (5)$$

Below we will analyze this equation from the point of view of its integrability.

2. BREATHERS AND NUMERICAL SIMULATION OF ITS COLLISIONS

Breather is the localized solution of (3) of the following type:

$$b(x, t) = B(x - Vt)e^{i(k_0x - \omega_0 t)}, \quad (6)$$

where k_0 is the wavenumber of the carrier wave, V is the group velocity and ω_0 is the frequency close to ω_{k_0}. In the Fourier space breather can be written as follow:

$$b_k(t) = e^{-i(\Omega + Vk)t}\phi_k, \quad (7)$$

where Ω is close to $\omega_{k_0}/2$.

For ϕ_k the following equation is valid:

$$(\Omega + Vk - \omega_k)\phi_k$$
$$= \int \tilde{T}_{kk_1}^{k_2k_3} \phi_{k_1}^* \phi_{k_2} \phi_{k_3} \delta_{k+k_1-k_2-k_3} dk_1 dk_2 dk_3. \quad (8)$$

One can treat ϕ_k as pure real function of k.

To solve Eq. (8) one can use Petviashvili iteration method (n is the number of iteration):

$$(\Omega + Vk - \omega_k)\phi_k^{n+1}$$
$$= M^n \int \tilde{T}_{kk_1}^{k_2k_3} \phi_{k_1}^{*n} \phi_{k_2}^n \phi_{k_3}^n \delta_{k+k_1-k_2-k_3} dk_1 dk_2 dk_3.$$

The Petviashvili coefficient M^n has the form

$$M^n = \left[\frac{\langle \phi_k^n (\Omega + Vk - \omega_k)\phi_k^n \rangle}{\langle \phi_k^n \int \tilde{T}_{kk_1}^{k_2k_3} \phi_{k_1}^{*n} \phi_{k_2}^n \phi_{k_3}^n \delta_{k+k_1-k_2-k_3} dk_1 dk_2 dk_3 \rangle}\right]^{\frac{3}{2}}.$$

In the limit of NLSE breather solution (6) is nothing but well-known NLSE soliton. Space profile of typical breather is shown in Fig. 1.

If Eq. (3) is integrable collisions of breathers would be pure elastic. To study breathers collision we performed the following numerical simulation.

- As initial condition we have used two breathers separated in space (distance was equal to π). Number of space points was 4096. Length of the periodic domain was 2π.
- First breather has the following parameters: $\Omega_1 = 4.01$, $V_1 = 1/16$. Carrier wavenumber appears to be ~64.
- For the second breather, $\Omega_2 = 4.51$, $V_2 = 1/18$. Carrier wavenumber appears to be ~81.

The initial condition and state after 100 breather collisions are shown in Fig. 2. One can see radiation after collisions in this figure that shows the zoomed profile of $|b(x)|$. During numerical simulation the total energy was conserved up to ninth digit after decimal point. Thus numerical simulation shows the collisions are not elastic.

It should be mentioned that in [4, 5] similar collisions simulations seem to be elastic. Thus, it was made suggestion about integrability of compact 1D Zakharov equation. However, in these papers very few collisions were simulated and radiation was hardly observed.

To prove nonintegrability rigorously, we analyze analytically amplitudes of six-wave interactions for this equation.

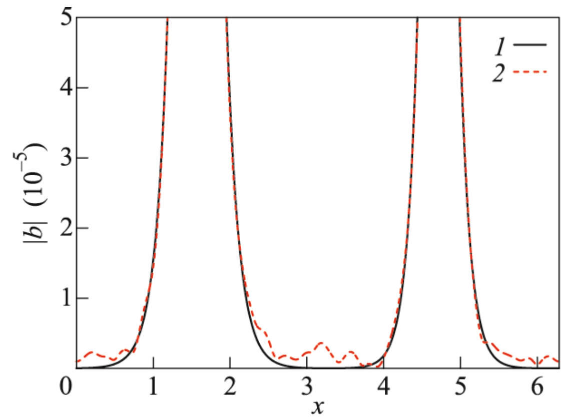

Fig. 3. Modulus of $b(x)$ for two points of time. Solid line corresponds the initial statement ($t = 0$), dashed line corresponds to the state after 100 breather collisions ($t \sim 88000$).

3. SCATTERING MATRIX FOR COMPACT EQUATION

For further consideration we introduce $c_k(t)$ in a following way:

$$b_k(t) = c_k(t)e^{-i\omega_k t}.$$

Then Eq. (5) can be rewritten as

$$i\dot{c}_k(t) = \int T^{kk_1}_{k_2k_3} c^*_{k_1}(t)c_{k_2}(t)c_{k_3}(t)e^{i(\omega_k+\omega_{k_1}-\omega_{k_2}-\omega_{k_3})t} \times \delta(k+k_1-k_2-k_3)dk_1dk_2dk_3 \tag{9}$$

and rewrite it in the Picard form ($\epsilon > 0$)

$$c_k(t) = c^-_k - i\lim_{\epsilon\to 0}\int_{-\infty}^{t} dt\int T^{kk_1}_{k_2k_3}c^*_{k_1}(t)c_{k_2}(t)c_{k_3}(t) \times e^{i(\omega_k+\omega_{k_1}-\omega_{k_2}-\omega_{k_3})t+\epsilon t}\delta(k+k_1-k_2-k_3)dk_1dk_2dk_3. \tag{10}$$

This equation can be solved by iterations:

$$c_k = c^{(0)}_k + c^{(1)}_k(t) + c^{(2)}_k(t) + \ldots, \quad c^{(0)}_k = c_k(-\infty).$$

Following [6] we introduce a so-called formal scattering matrix for Eq. (10) for

$$c^-_k = \lim_{t\to-\infty} c_k(t), \quad c^+_k = \lim_{t\to=+\infty} c_k(t), \quad c^+_k = S[c^-_k].$$

So far as (9) has only four-wave vortex, scattering matrix has the form:

$$S[c^-_k] = c^-_k + S_{22}[c^-_k] + S_{33}[c^-_k] + \ldots.$$

Element S_{22} has already been calculated in [7]. In spite of it has logarithmic divergence (it is why scattering matrix is formal), it does not produce “new” wave vectors as should be in the integrable systems. Below we will calculate S_{33}. Performing two iteration of (10) one can get for $c^{(1)}_k(t)$:

$$c^{(1)}_k(t) = -i\lim_{\epsilon\to 0}\int_{-\infty}^{t} dt\int T^{kk_1}_{k_2k_3}c^{*(0)}_{k_1}c^{(0)}_{k_2}c^{(0)}_{k_3} \times e^{i(\omega_k+\omega_{k_1}-\omega_{k_2}-\omega_{k_3})t+\epsilon t}\delta(k+k_1-k_2-k_3)dk_1dk_2dk_3$$

or

$$c^{(1)}_k(t) = -\int\frac{T^{kk_1}_{k_2k_3}c^{*(0)}_{k_1}c^{(0)}_{k_2}c^{(0)}_{k_3}}{\omega_k+\omega_{k_1}-\omega_{k_2}-\omega_{k_3}} \times e^{i(\omega_k+\omega_{k_1}-\omega_{k_2}-\omega_{k_3})t}\delta(k+k_1-k_2-k_3)dk_1dk_2dk_3.$$

Performing second iteration one can get:

$$\begin{aligned}
c^{(2)}_{p_1}(+\infty) = -i\int_{-\infty}^{+\infty} dt\int drdp_2dp_3dq_1dq_2dq_3 \times c^{*(0)}_{p_2}c^{*(0)}_{p_3}c^{(0)}_{q_1}c^{(0)}_{q_2}e^{i(\omega_{p_1}+\omega_{p_2}+\omega_{p_3}-\omega_{q_1}-\omega_{q_2}-\omega_{q_3})t}\\
\times\Bigg[\frac{T^{p_1r}_{q_1q_2}T^{p_2p_3}_{rq_3}\delta(r+p_1-q_1-q_2)\delta(r+q_3-p_2-p_3)}{\omega_r+\omega_{q_3}-\omega_{p_2}-\omega_{p_3}}\\
+\frac{T^{p_1r}_{q_3q_2}T^{p_2p_3}_{rq_1}\delta(r+p_1-q_2-q_3)\delta(r+q_1-p_2-p_3)}{\omega_r+\omega_{q_1}-\omega_{p_2}-\omega_{p_3}}\\
+\frac{T^{p_1r}_{q_1q_3}T^{p_2p_3}_{rq_2}\delta(r+p_1-q_1-q_3)\delta(r+q_2-p_2-p_3)}{\omega_r+\omega_{q_2}-\omega_{p_2}-\omega_{p_3}}\\
+\frac{T^{p_1p_2}_{rq_1}T^{rp_3}_{q_2q_3}\delta(r+q_1-p_1-p_2)\delta(r+p_3-q_2-q_3)}{\omega_r+\omega_{p_3}-\omega_{q_2}-\omega_{q_3}}\\
+\frac{T^{p_1p_2}_{rq_2}T^{rp_3}_{q_1q_3}\delta(r+q_2-p_1-p_2)\delta(r+p_3-q_1-q_3)}{\omega_r+\omega_{p_3}-\omega_{q_1}-\omega_{q_3}}
\end{aligned}$$

$$+\frac{T^{p_1p_2}_{rq_3}T^{rp_3}_{q_2q_1}\delta(r+q_3-p_1-p_2)\delta(r+p_3-q_1-q_2)}{\omega_r+\omega_{p_3}-\omega_{q_1}-\omega_{q_2}}$$

$$+\frac{T^{p_1p_3}_{rq_1}T^{rp_2}_{q_2q_3}\delta(r+q_1-p_1-p_3)\delta(r+p_2-q_2-q_3)}{\omega_r+\omega_{p_2}-\omega_{q_2}-\omega_{q_3}}$$

$$+\frac{T^{p_1p_3}_{rq_2}T^{rp_2}_{q_1q_3}\delta(r+q_2-p_1-p_3)\delta(r+p_2-q_1-q_3)}{\omega_r+\omega_{p_2}-\omega_{q_1}-\omega_{q_3}}$$

$$\left.+\frac{T^{p_1p_3}_{rq_3}T^{rp_2}_{q_2q_1}\delta(r+q_3-p_1-p_3)\delta(r+p_2-q_1-q_2)}{\omega_r+\omega_{p_2}-\omega_{q_1}-\omega_{q_2}}\right]$$

or

$$c^{(2)}_{p_1}(+\infty) = -2\pi i\int T^{p_1p_2p_3}_{q_1q_2q_3}c^{*(0)}_{p_2}c^{*(0)}_{p_3}c^{(0)}_{q_1}c^{(0)}_{q_2}c^{(0)}_{q_3}$$
$$\times\,\delta(\omega_{p_1}+\omega_{p_2}+\omega_{p_3}-\omega_{q_1}-\omega_{q_2}-\omega_{q_3})$$
$$\times\,\delta(p_1+p_2+p_3-q_1-q_2-q_3)dp_2dp_3dq_1dq_2dq_3.$$

Here $T^{p_1p_2p_3}_{q_1q_2q_3}$ is equal to:

$$T^{p_1p_2p_3}_{q_1q_2q_3} = \left[\frac{T^{p_2p_3}_{p_2+p_3-q_3q_3}T^{q_1+q_2-p_1p_1}_{q_1q_2}}{\omega_{q_1+q_2-p_1}+\omega_{p_1}-\omega_{q_1}-\omega_{q_2}}\right.$$

$$+\frac{T^{p_2p_3}_{p_2+p_3-q_1q_1}T^{q_2+q_3-p_1p_1}_{q_2q_3}}{\omega_{q_2+q_3-p_1}+\omega_{p_1}-\omega_{q_2}-\omega_{q_3}}$$

$$+\frac{T^{p_2p_3}_{p_2+p_3-q_2q_2}T^{q_1+q_3-p_1p_1}_{q_1q_3}}{\omega_{q_1+q_3-p_1}+\omega_{p_1}-\omega_{q_1}-\omega_{q_3}}$$

$$+\frac{T^{p_1p_2}_{p_1+p_2-q_1q_1}T^{q_2+q_3-p_3p_3}_{q_2q_3}}{\omega_{q_2+q_3-p_3}+\omega_{p_3}-\omega_{q_2}-\omega_{q_3}}$$

$$+\frac{T^{p_1p_2}_{p_1+p_2-q_2q_2}T^{q_1+q_3-p_3p_3}_{q_1q_3}}{\omega_{q_1+q_3-p_3}+\omega_{p_3}-\omega_{q_1}-\omega_{q_3}}$$

$$+\frac{T^{p_1p_2}_{p_1+p_2-q_3q_3}T^{q_1+q_2-p_3p_3}_{q_1q_2}}{\omega_{q_1+q_2-p_3}+\omega_{p_3}-\omega_{q_1}-\omega_{q_2}}$$

$$+\frac{T^{p_1p_3}_{p_1+p_3-q_1q_1}T^{q_2+q_3-p_2p_2}_{q_2q_3}}{\omega_{q_2+q_3-p_2}+\omega_{p_2}-\omega_{q_2}-\omega_{q_3}}$$

$$+\frac{T^{p_1p_3}_{p_1+p_3-q_2q_2}T^{q_1+q_3-p_2p_2}_{q_1q_3}}{\omega_{q_1+q_3-p_2}+\omega_{p_2}-\omega_{q_1}-\omega_{q_3}}$$

$$\left.+\frac{T^{p_1p_3}_{p_1+p_3-q_3q_3}T^{q_1+q_2-p_2p_2}_{q_1q_2}}{\omega_{q_1+q_2-p_2}+\omega_{p_2}-\omega_{q_1}-\omega_{q_2}}\right].$$

Element $T^{p_1p_2p_3}_{q_1q_2q_3}$ is the kernel of six waves element of scattering matrix S_{33}. Symbolically it can be represented as 9 diagrams, see for details [8]. Now we calculate 6-waves element $T^{p_1p_2p_3}_{q_1q_2q_3}$ on the resonant manifold

$$\begin{cases} p_1+p_2+p_3 \;=\; q_1+q_2+q_3, \\ \omega_{p_1}+\omega_{p_2}+\omega_{p_3} \;=\; \omega_{q_1}+\omega_{q_1}+\omega_{q_3}. \end{cases}$$

On this manifold expression for $T^{p_1p_2p_3}_{q_1q_2q_3}$ can be simplified:

$$T^{p_1p_2p_3}_{q_1q_2q_3} = \frac{1}{2(\omega_{p_1}\omega_{p_2}\omega_{p_3}-\omega_{q_1}\omega_{q_2}\omega_{q_3})}$$
$$\times\,[\,T^{p_2p_3}_{p_2+p_3-q_3q_3}T^{q_1+q_2-p_1p_1}_{q_1q_2}$$
$$\times\,(\omega_{p_1}-\omega_{q_3})(\omega_{p_1}-\omega_{q_1}-\omega_{q_2}-\omega_{q_1+q_2-p_1})$$
$$+\,T^{p_2p_3}_{p_2+p_3-q_1q_1}T^{q_2+q_3-p_1p_1}_{q_2q_3}$$
$$\times\,(\omega_{p_1}-\omega_{q_1})t(\omega_{p_1}-\omega_{q_2}-\omega_{q_3}-\omega_{q_2+q_3-p_1})$$
$$+\,T^{p_2p_3}_{p_2+p_3-q_2q_2}T^{q_1+q_3-p_1p_1}_{q_1q_3}$$
$$\times\,(\omega_{p_1}-\omega_{q_2})(\omega_{p_1}-\omega_{q_1}-\omega_{q_3}-\omega_{q_1+q_3-p_1})$$
$$+\,T^{p_1p_2}_{p_1+p_2-q_1q_1}T^{q_2+q_3-p_3p_3}_{q_2q_3}$$
$$\times\,(\omega_{p_3}-\omega_{q_1})(\omega_{p_3}-\omega_{q_2}-\omega_{q_3}-\omega_{q_2+q_3-p_3})$$
$$+\,T^{p_1p_2}_{p_1+p_2-q_2q_2}T^{q_1+q_3-p_3p_3}_{q_1q_3}$$
$$\times\,(\omega_{p_3}-\omega_{q_2})(\omega_{p_3}-\omega_{q_1}-\omega_{q_3}-\omega_{q_1+q_3-p_3})$$
$$+\,T^{p_1p_2}_{p_1+p_2-q_3q_3}T^{q_1+q_2-p_3p_3}_{q_1q_2}$$
$$\times\,(\omega_{p_3}-\omega_{q_3})(\omega_{p_3}-\omega_{q_1}-\omega_{q_2}-\omega_{q_1+q_2-p_3})$$
$$+\,T^{p_1p_3}_{p_1+p_3-q_1q_1}T^{q_2+q_3-p_2p_2}_{q_2q_3}$$
$$\times\,(\omega_{p_2}-\omega_{q_1})(\omega_{p_2}-\omega_{q_2}-\omega_{q_3}-\omega_{q_2+q_3-p_2})$$
$$+\,T^{p_1p_3}_{p_1+p_3-q_2q_2}T^{q_1+q_3-p_2p_2}_{q_1q_3}$$
$$\times\,(\omega_{p_2}-\omega_{q_2})(\omega_{p_2}-\omega_{q_1}-\omega_{q_3}-\omega_{q_1+q_3-p_2})$$
$$+\,T^{p_1p_3}_{p_1+p_3-q_3q_3}T^{q_1+q_2-p_2p_2}_{q_1q_2}$$
$$\times\,(\omega_{p_2}-\omega_{q_3})(\omega_{p_2}-\omega_{q_1}-\omega_{q_2}-\omega_{q_1+q_2-p_2})\,].$$

Manifold can be parameterized using three parameters λ, α, and β as following

$$\begin{cases} p_1 = \dfrac{\omega_{p_1}^2}{g} = \left(1 + \lambda\dfrac{1+\alpha}{1+\alpha+\alpha^2}\right)^2, \\ p_2 = \dfrac{\omega_{p_2}^2}{g} = \left[1 + \lambda\dfrac{\alpha(1+\alpha)}{1+\alpha+\alpha^2}\right]^2, \\ p_3 = \dfrac{\omega_{p_3}^2}{g} = \left(1 - \lambda\dfrac{\alpha}{1+\alpha+\alpha^2}\right)^2, \\ q_1 = \dfrac{\omega_{q_1}^2}{g} = \left(1 + \lambda\dfrac{1+\beta}{1+\beta+\beta^2}\right)^2, \\ q_2 = \dfrac{\omega_{q_2}^2}{g} = \left[1 + \lambda\dfrac{\beta(1+\beta)}{1+\beta+\beta^2}\right]^2, \\ q_3 = \dfrac{\omega_{q_3}^2}{g} = \left(1 - \lambda\dfrac{\beta}{1+\beta+\beta^2}\right)^2. \end{cases}$$

Plugging ω_i, p_i, and q_i for different values of λ, α, and β one can check that

$$T_{q_1q_2q_3}^{p_1p_2p_3} \neq 0.$$

This is the proof of nonintegrability of compact 1D Zakharov equation.

Note, that if consider above theory for Nonlinear Schrodinger Equation for which

$$T_{k_3k_4}^{k_1k_2} = 1, \quad \omega_k = k^2$$

simple calculations end up with $T_{q_1q_2q_3}^{p_1p_2p_3} = 0$, as it must be for integrable system.

4. CONCLUSIONS

Compact 1D Zakharov equation (5), or equivalent system with Hamiltonian (1) is nonintegrable system. We have proved it both numerically and analytically. However the question about integrability of fully nonlinear system (1) is still unclear. Exact equation has his own six wave term which could make total six wave coefficient changed.

This work was supported by the Government of the Russian Federation (project no.11.G34.31.0035, program for support of research supervised by leading scientists at Russian universities; leading scientist is V.E. Zakharov at Novosibirsk State University), by the US Army Corps of Engineers (grant no. W912-BU-08-P-0143), by the U.S. Office of Naval Research (Grant no. 00014-10-1-0991), by the U.S. National Science Foundation (grant no. DMS 0404577), by the Russian Foundation for Basic Research (project nos. 12-01-00943-a and 12-05-92004-HHC_a), by the Presidium of the Russian Academy of Sciences (program "Fundamental Problems in Nonlinear Dynamics"), and by the Council of the President of the Russian Federation for Support of Young Scientists and Leading Scientific Schools (project no. NSh-6170.2012.2).

REFERENCES

1. A. I. Dyachenko and V. E. Zakharov, Phys. Lett. A **190**, 144 (1994).
2. V. E. Zakharov, Prikl. Mekh. Tekh. Fiz. **2**, 190 (1968).
3. A. I. Dyachenko and V. E. Zakharov, Eur. J. Mech. B **32**, 17 (2012).
4. http://arxiv.org/abs/1201.4808.
5. F. Fedele and D. Dutykh, JETP Lett. **95**, 622 (2012).
6. V. E. Zakharov and E. I. Shulman, *Integrability of Nonlinear Systems and Perturbation Theory* (Springer, Berlin, 1991).
7. A. I. Dyachenko, Y. V. Lvov, and V. E. Zakharov, Physica D **87**, 233 (1995).
8. See Supplemental Material at URL to be inserted by publisher, www.jetpletters.ac.ru, v. 98, iss. 1.