Nonlinear phenomena with whispering gallery modes

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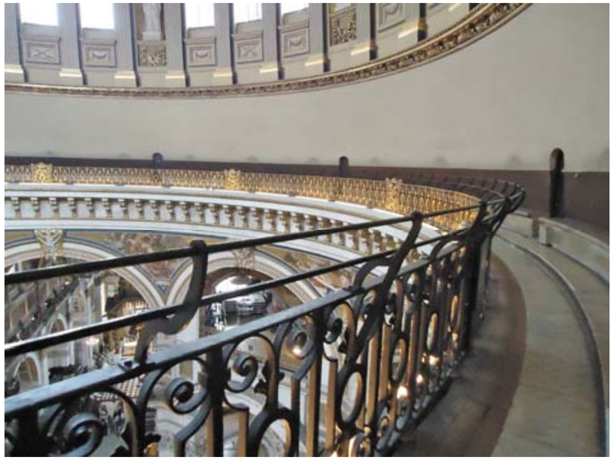
WGMs: What's that?

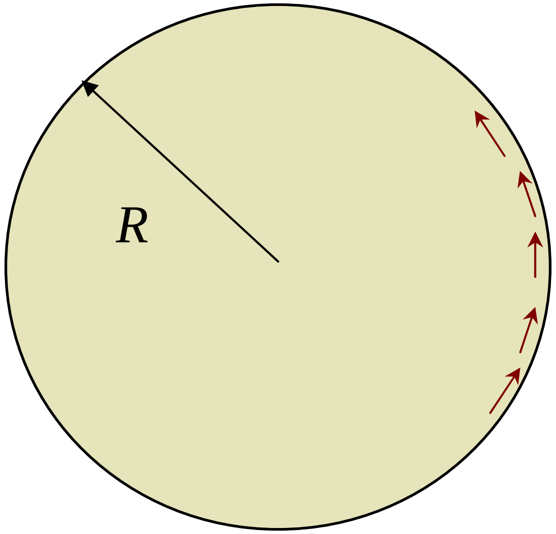
Known since ancient history:

Beijing, Temple of Sky, etc.



Modern history: Rayleigh, 1910

Acoustic modes localized near the equator



A simplified quasi-1D view:

$$\omega_m = k_m v$$

$$k_m \approx \frac{2\pi m}{L} = \frac{m}{R}$$

m is azimuth number

Optical WGMs

Localization owing to the total internal reflection

Exact 3D solutions only for sphere (water droplets)

Three numbers: l (orbital), m (azimuth), and q (radial)

$\omega = \omega_{lq} = cl / (R\, n_{lq})$ – degeneration in m,

$n_{lq} \approx n$ modal refractive index

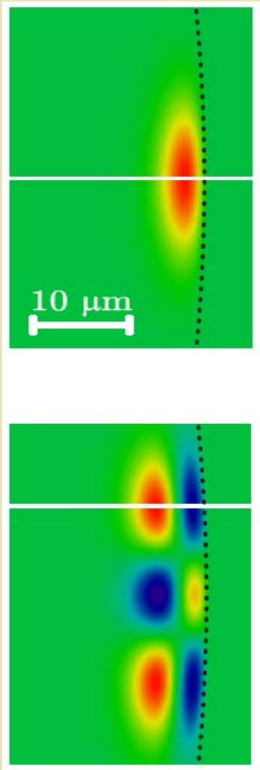
The asymptotics: $J_\nu(x)$ with $\nu = l + 1/2 \gg 1$

and $Y_{lm}(\theta)$ with $(l - m) \ll l$.

The actual numbers: $l = 10^3 - 10^5$, $q \sim 1$.

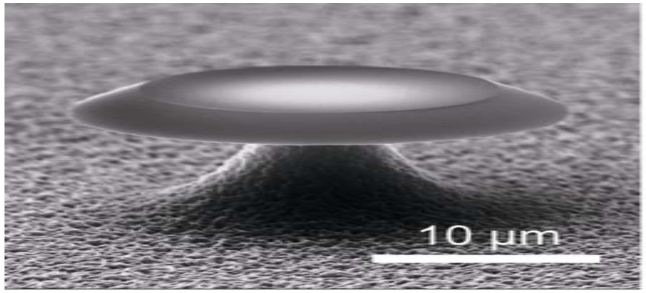
Degree of localization:

$$\delta r_\perp \approx R / l^{2/3}, \quad \delta r_\theta \approx R / l^{1/2}$$



WGM resonators

Toroidal



Edge cut

Fabrication technique: lathe (токарный станок), lithography, polishing

Materials:

SiO_2, CaF_2, GaAs, $LiNbO_3$ ...

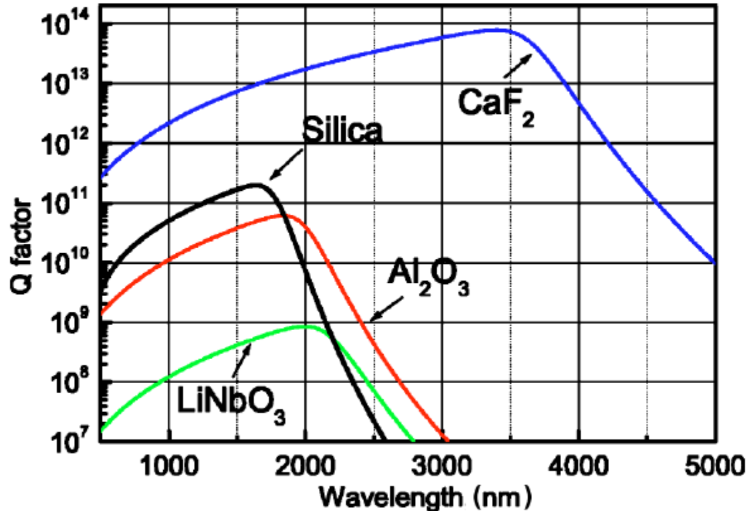
Sizes

$R = (10^{-3} - 10^{-1})$ cm

Figures of merit

Quality factor $Q = \omega / \gamma$
Main losses: absorption, scattering
Radiation losses are negligible
– no coupling with plane waves

$Q = 10^7 – 10^{11}$:
ultra-narrow line widths

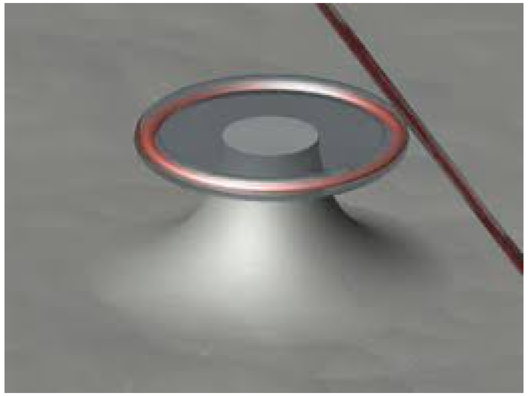


Finesse $F = \lambda Q / 2\pi \, nR$:
The number of circulations during the life time

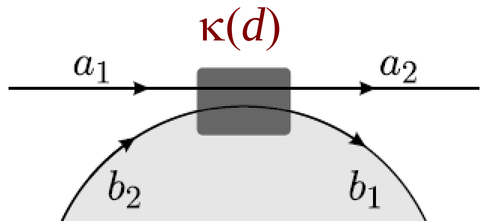
Modal cross-section
$\sigma \sim (10^1 – 10^2)\ \mu m^2$

Evanescent couplers

Turning point in 90-th: the methods to couple light in and out.
Almost 100% of pump power can be delivered into a single WGM



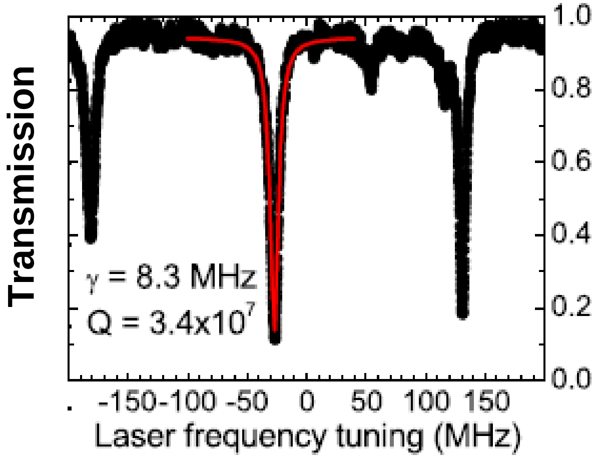
Couplers:
tapered fiber
prism, etc.

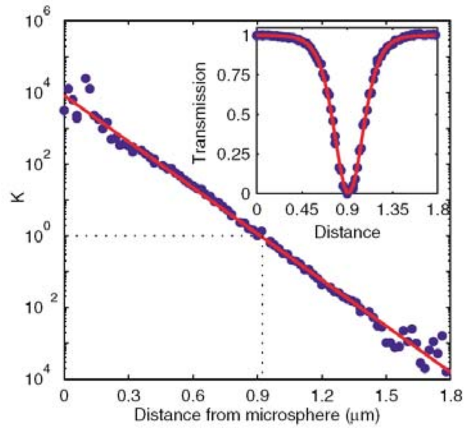


Transmission
$|a_2|^2 / |a_1|^2$

$\gamma_\Sigma = \gamma + \gamma_c(d)$

$\gamma = \gamma_c$: **Critical coupling**, $a_2 = 0$





Analogy with
FP resonators

Why nonlinear applications?

Large values of Q + small modal cross sections → huge enhancement of the pump intensity inside the resonator

The higher the order of the nonlinear process, the stronger is the effect

Low-power continuous-wave light sources provide strong nonlinear effects

Large values of Q allow to work with small nonlinearities

Potentially, even few photons can initiate nonlinear effects

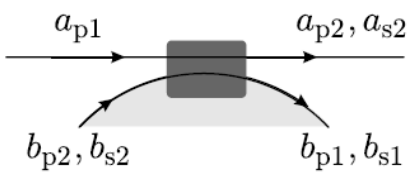
Drawbacks:

Discreteness of the WGM spectrum hampers phase matching

Modal nonlinear losses affect critical coupling

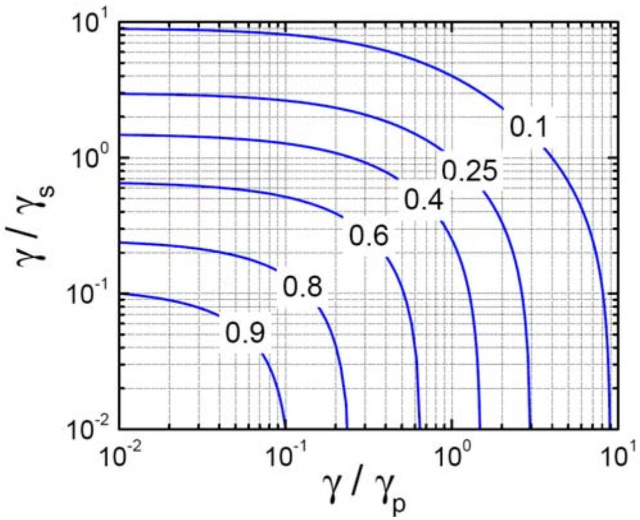
Many linear applications

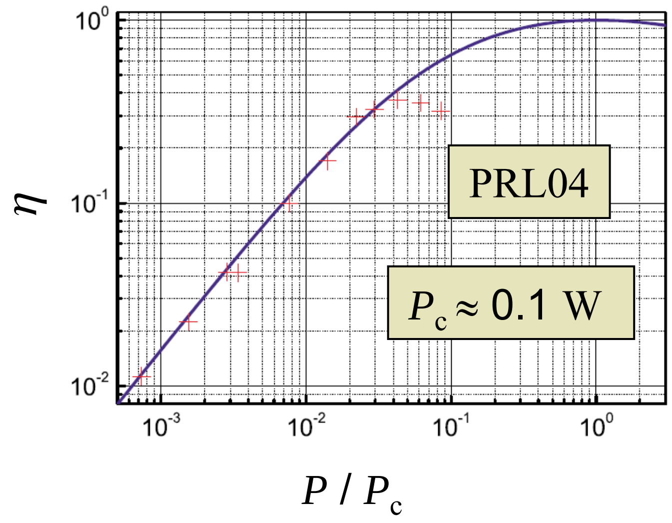
Second harmonic generation, $\chi^{(2)}$



$\eta_{SHG} = |a_{s2}|^2 / |a_{p1}|^2$

$f = \gamma_c / \gamma$





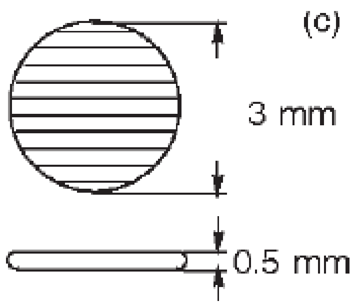
$$P_{opt} = P_{mat} \times \frac{\sigma_{eff}\ R^2}{\lambda^4} \times \frac{1}{Q_p^2 Q_s} \times \frac{(f_p+1)^3 (f_s+1)}{f_p} \geq 1\,\mu\mathrm{W}$$

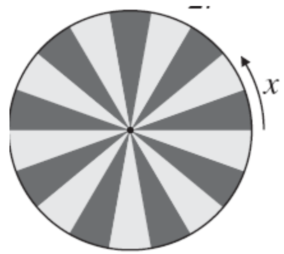
$\sim 10^8$ W $\sim 10^8$ $\sim 10^{-23}$ >10

JOSA B11

P

LiNbO$_3$: Quasi-phase matching

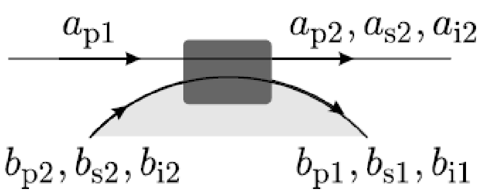




PRL04

PRL11

Optical parametric oscillation, $\chi^{(2)}$



$\omega_p = \omega_s + \omega_i; \quad m_p = m_s + m_i$

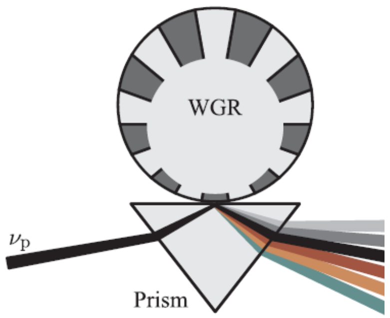
Phase-matching: Radial poling + Temperature tuning

$\eta_{s,i} = P_{s,i} / P_0$

$f = \gamma_c / \gamma$

$$P_{th}^0 = P_{mat} \times \frac{\sigma_{eff} \; R^2}{\lambda^4} \times \frac{1}{Q_p Q_s Q_i} \times \frac{(f_p + 1)^2 (f_s + 1)(f_i + 1)}{f_p} \geq 1 \, \mu\text{W}$$

Experiment, PRL11

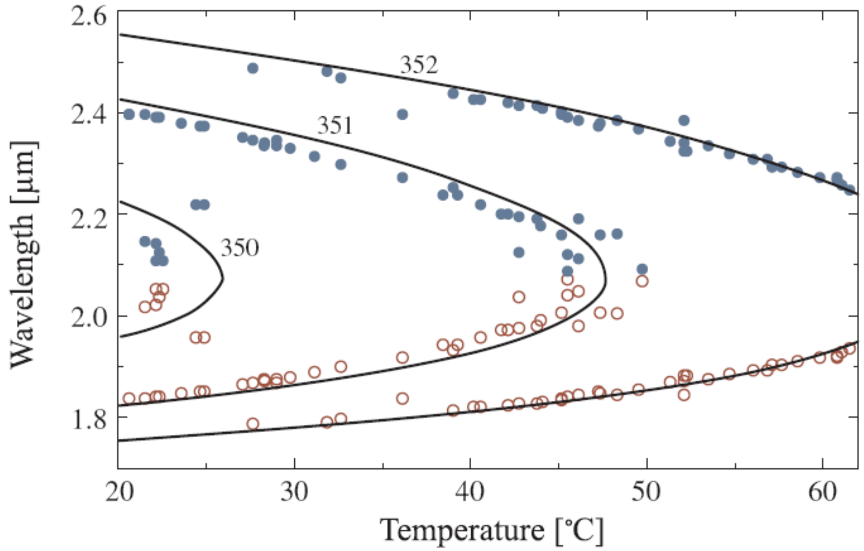


R = 1.58 mm

$\lambda_p = 1.04 \ \mu m$

$Q \sim 10^8$

$P_{th} \sim 1$ mW



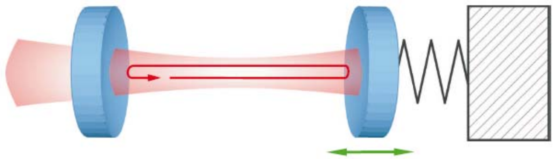
Opto-mechanics

Above a threshold, $P > P_{th}$, resonator is unstable against vibrations with $m = 0$

Mechanism: Light pressure, caused by the surface curvature, and the shift of the resonant eigenfrequency $\omega_0(R)$ during vibration
Predicted by V.B. Braginsky for FP resonators in 1977

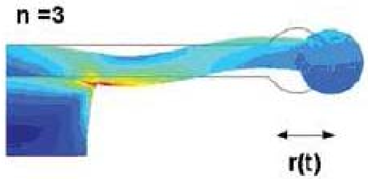
$$\dot{a} + [\gamma + i\Delta(x)]a = ip \quad \longleftarrow \text{optical pump}$$

$$\ddot{x} + \Gamma\dot{x} + \Omega^2 x = f\,|a|^2 / m_* \quad \longleftarrow \text{light pressure}$$



$\Delta(x) = \omega - \omega_0 = \Delta + \omega_0\, x/R$
$a = a_0 + a_+ + a_-$
$\Gamma \ll \Omega < \gamma$

$R = 36\ \mu m$
$\Omega/2\pi = (5\text{-}50)$ MHz
$m_* = (1\text{-}5) \times 10^{-8}$ g
$Q_m = 10^3 - 10^4$

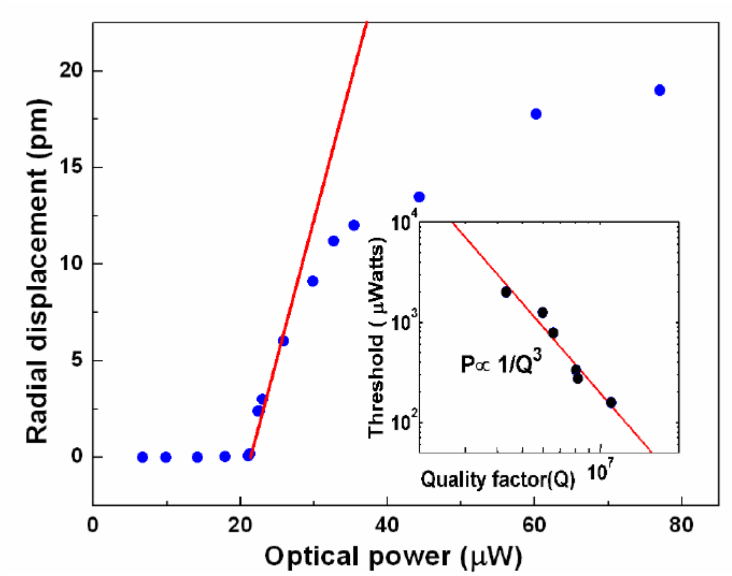


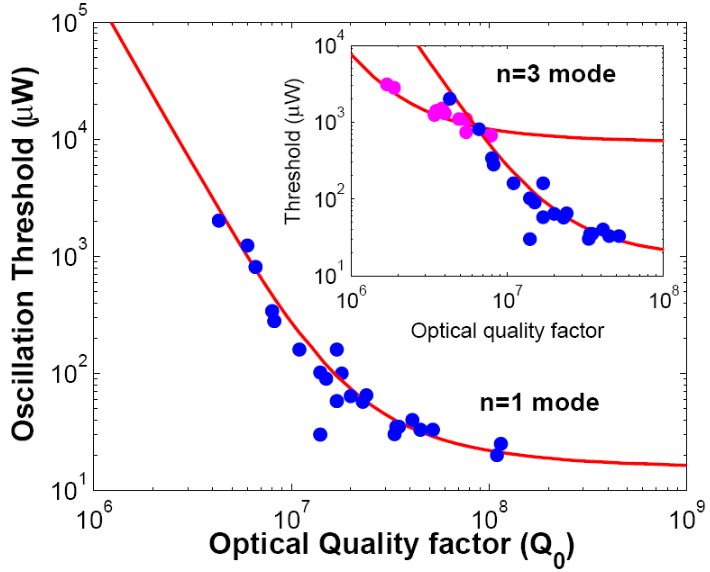
$$P_{th} \approx \frac{\Omega\, m_* c^2 R^2}{2\pi\, Q_m Q^3\, \lambda^2}\,\frac{(\Delta^2/\gamma^2 + 1)^2}{\Delta/\gamma}$$

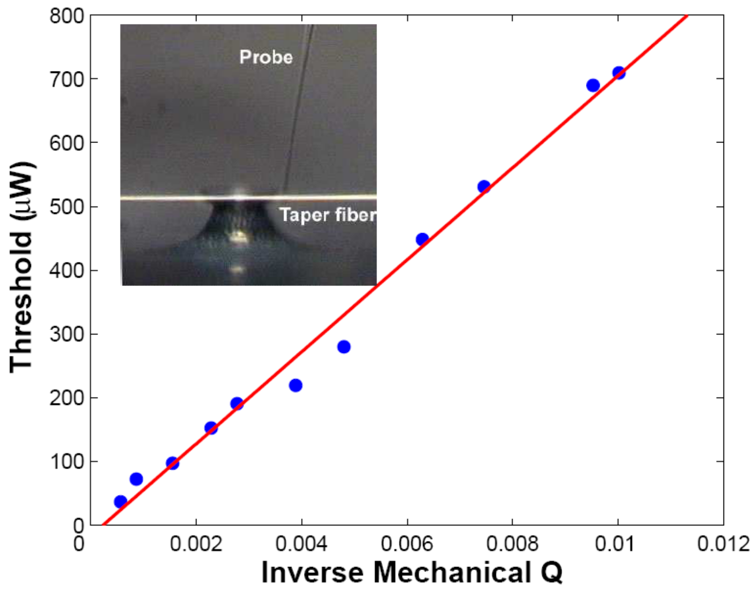
Instability for blue shifts, $\omega > \omega_0$

Parametric instability, experiment

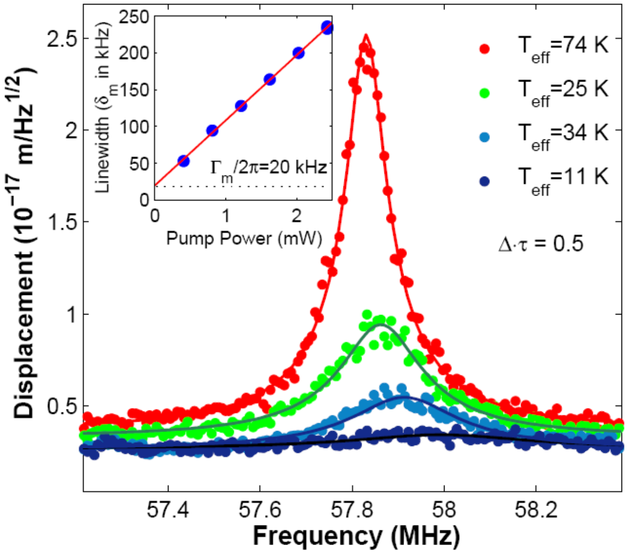
Vahala
Kippenbrg
2005-2007
PRLs, Science







Red detuning: Radiation pressure cooling



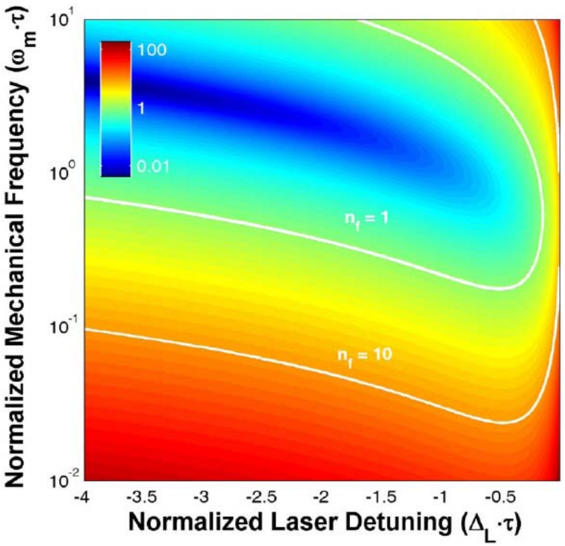
$$(x^2)_\Omega = \frac{2\Gamma T_{eff}}{m_{eff}\,\Omega_0^2[(\Omega-\Omega_0)^2+\Gamma^2]}$$

$$\Gamma = \Gamma_0 + \Gamma_p\,, \qquad \Gamma_p \propto P$$

$$T_{eff} = \frac{\Gamma_0 T_R}{\Gamma_0 + \Gamma_p}$$

Quantum limit, modeling

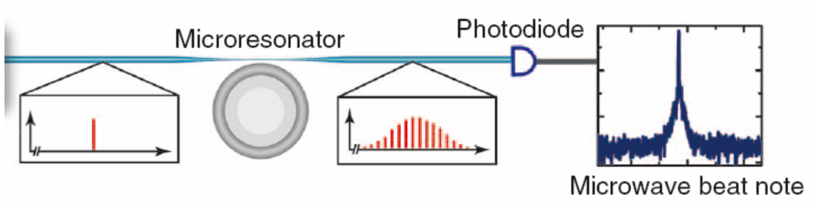
$\Omega / 2\pi$ = 60 MHz
$Q_m = 3 \times 10^4$



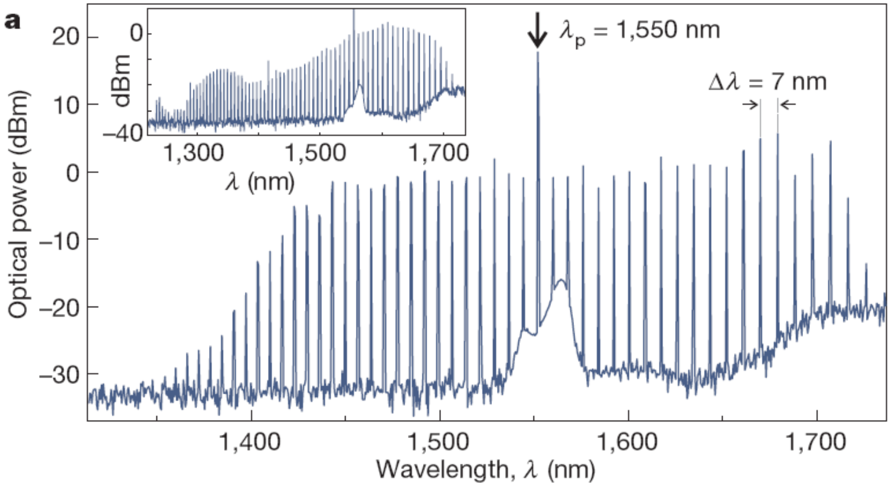
Frequency combs

Nature 07

An important general problem
Earlier: mode-locked lasers
Hänsch, Nobel Prize 2005



SiO_2, toroidal, Q > 10^8, R = (40-180) μm
P_{th} ~ 50 μW, η = (20 – 80)%, λ_p = 1550 nm

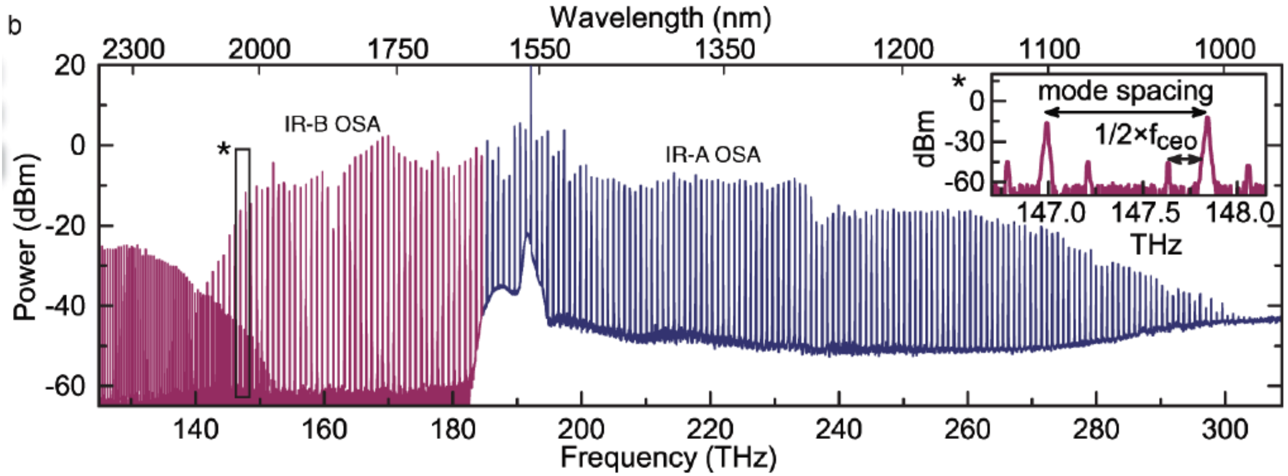


Origin: Cascaded 2→2 processes,
Caused by Kerr nonlinearity

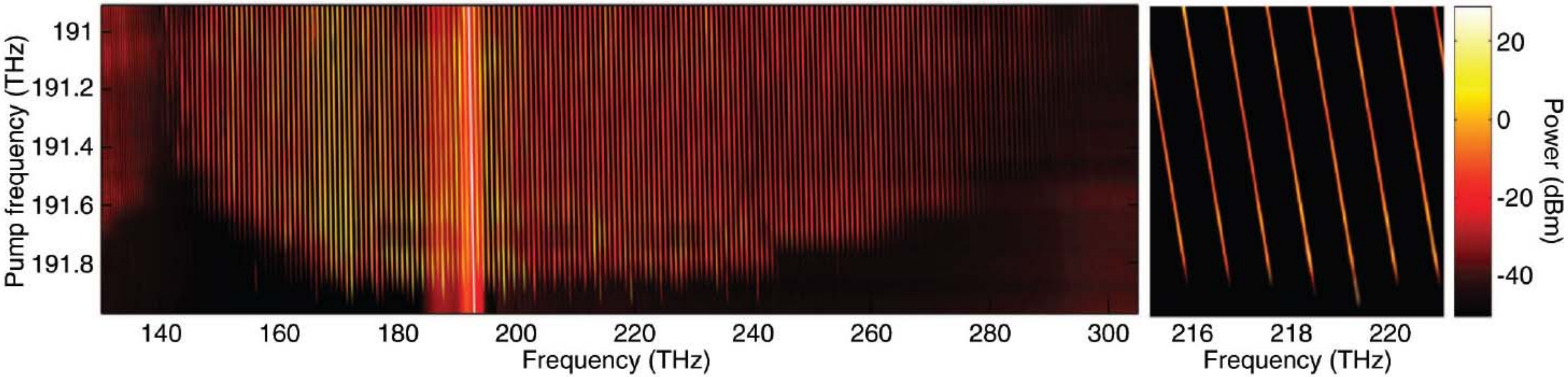
P = 60 μW, Δω ≈ c/nR = FSP
500-nm-wide span

Surprisingly: The peaks are equidistant with a relative accuracy (**10^{-16} - 10^{-17}**) despite the WGM dispersion!

More data on Kerr combs



Octave spanning comb (PRL11), $R = 40$ μm, $\Delta\omega/2\pi = 850$ GHz, $P \sim 1$W

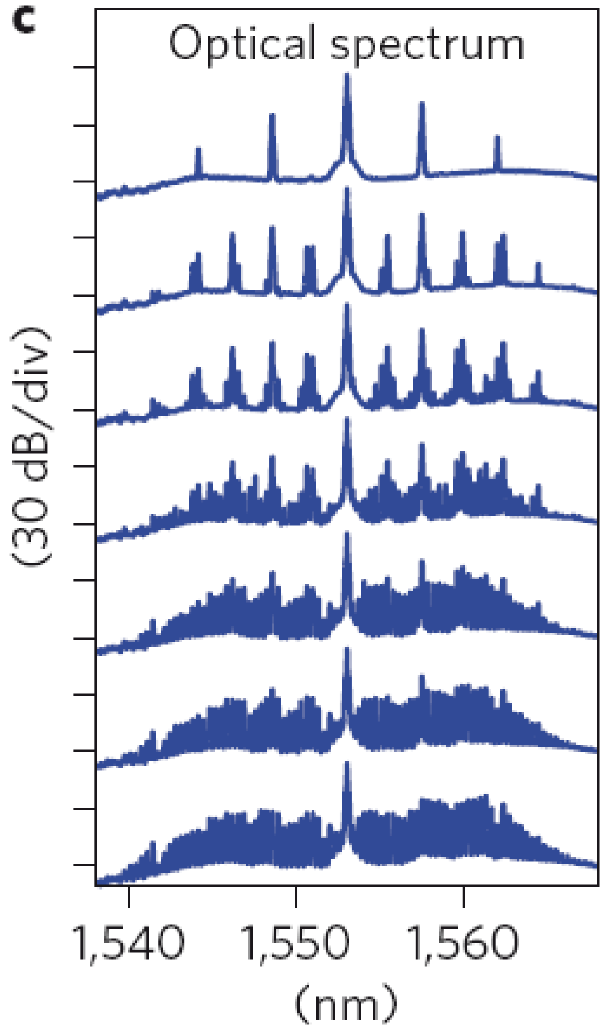


Tunability of the comb

Not as simple as believed first

NP12

The physics of the comb is rich
Different scenarios are possible
Experiment + numericals



The comb can consist of sub-combs with incommensurable spacings and complicated transitions (bifurcations).

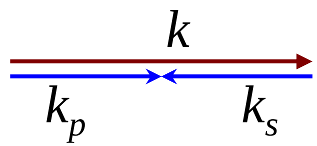
The situation is controlled by Kerr dynamics, and it strongly depends on the modal dispersion

Brillouin excitation of acoustic WGMs

Background:

In the bulk case, the SBS process is one of the strongest

In 1D case (fibers), only the longitudinal sound can be excited



$$\omega_p - \omega_s = \Omega$$

$$k_p - k_s = k$$

In WGM resonators, interaction with acoustic WGMs must be the strongest

Prospective thresholds are close to nW range

Recently found thesholds of SBS lasing (PRLs2009) are much higher (μW range)

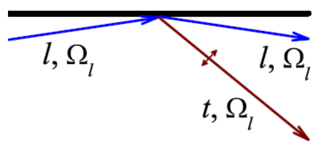
A basic problem: What are acoustic WGMs?

The cases of air (Rayleigh) and solids are different

In solids $v_l \geq \sqrt{2}\, v_t$ **, and the *l*- and *t*-waves are coupled via free surface**

As a result, there no longitudinal acoustic WGMs

The physics of acoustic WGMs is very specific



The end