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Solitons of trapped waves on jet currents

Envelope solitons over real water

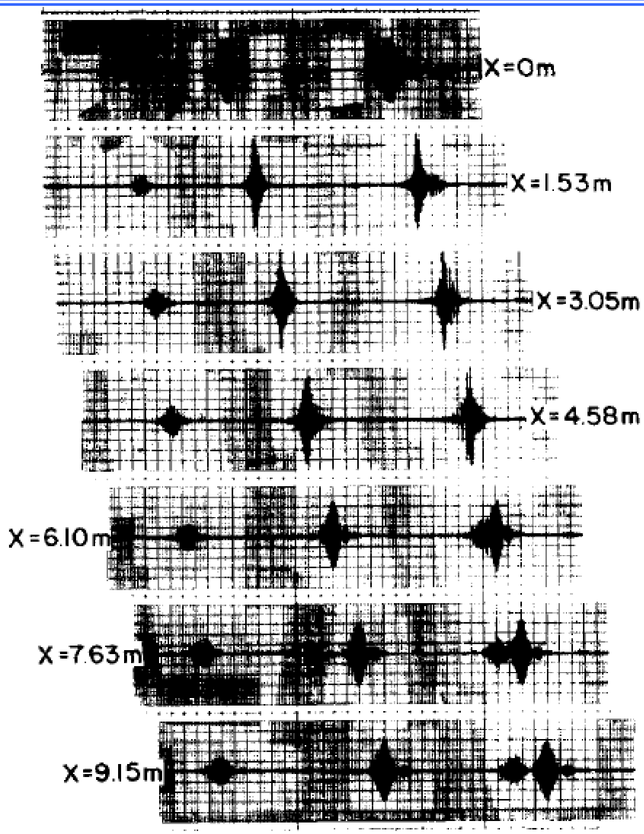
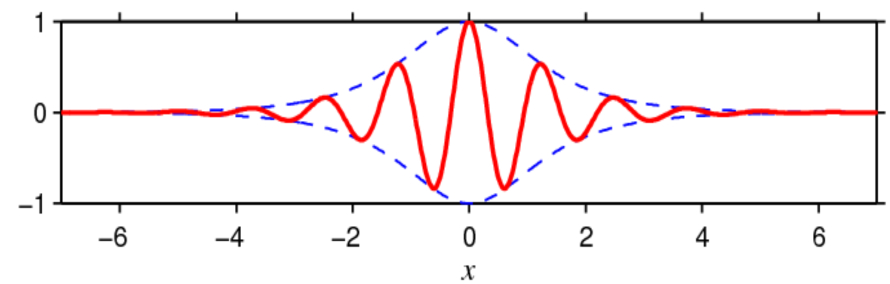


FIG. 3. One wave pulse overtaking and passing through another wave pulse. Left-hand trace: first pulse alone, $\omega_0 \simeq 1.5$ Hz, initial $(ka)_{max} \simeq 0.01$, six-cycle pulse. Center trace: second pulse alone, $\omega_0 = 3$ Hz, initial $(ka)_{max} \simeq 0.2$, 12-cycle pulse which disintegrates into two solitons. Right-hand traces: interaction of the two pulses.

[Yuen & Lake, 1975]

Nonlinear Schrodinger equation

$$i\left(\frac{\partial A}{\partial t} + \frac{\omega}{2k}\frac{\partial A}{\partial x}\right) + \frac{\omega}{8k^2}\frac{\partial^2 A}{\partial x^2} + \frac{\omega k^2}{2}|A|^2 A = 0$$



Envelope solitons:

- **interact elastically;**
- **the asymptotic solution of the Cauchy problem**

Envelope solitons over real water

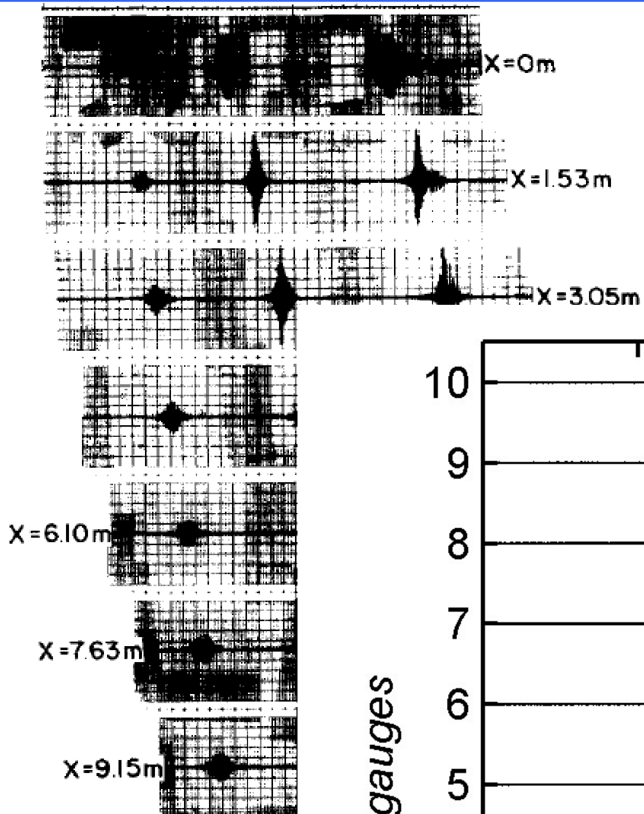
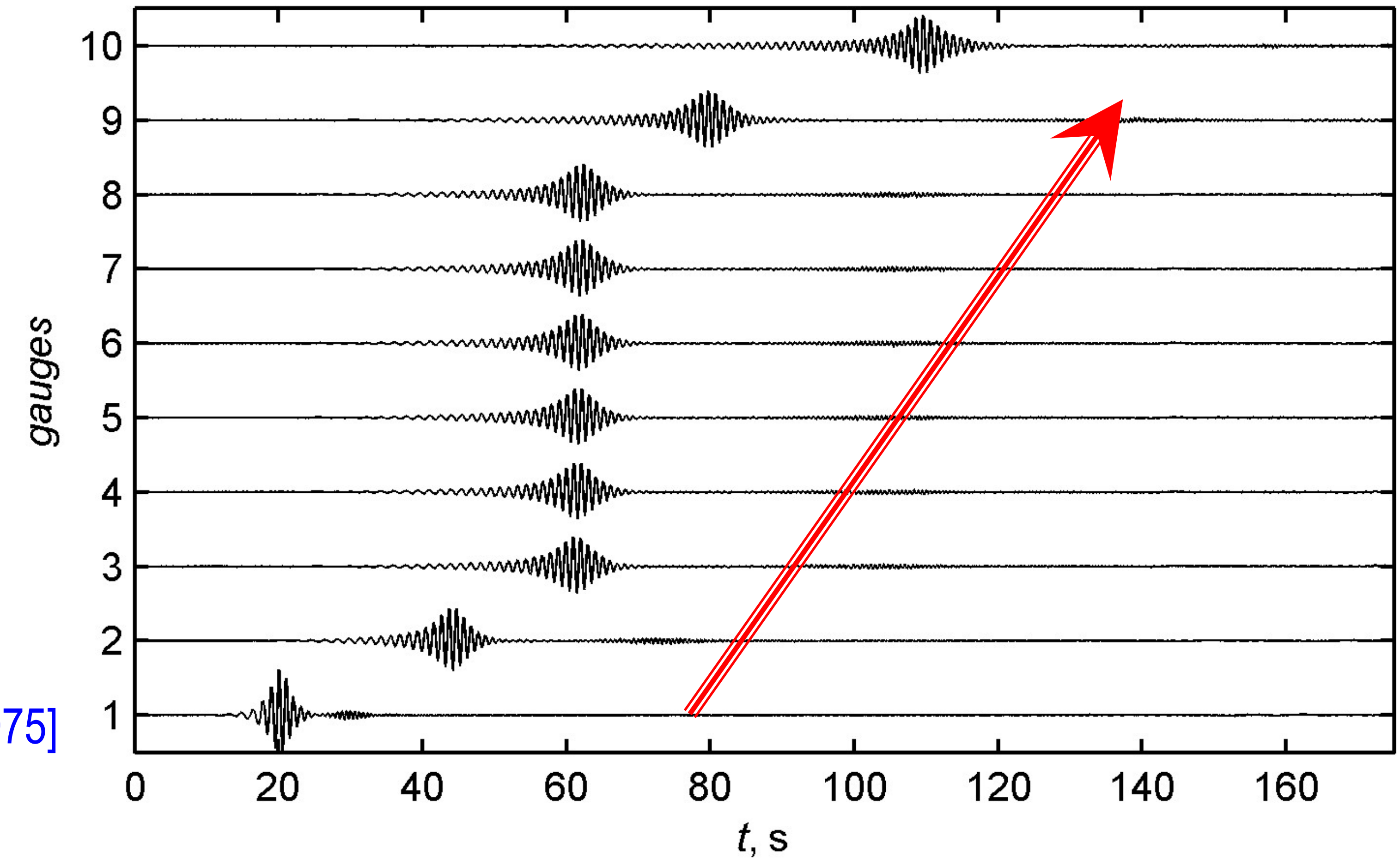


FIG. 3. One wave puls
wave pulse. Left-hand
initial $(ka)_{max} \simeq 0.01$, s
pulse alone, $\omega_0 = 3$ Hz,
disintegrates into two s
of the two pulses.

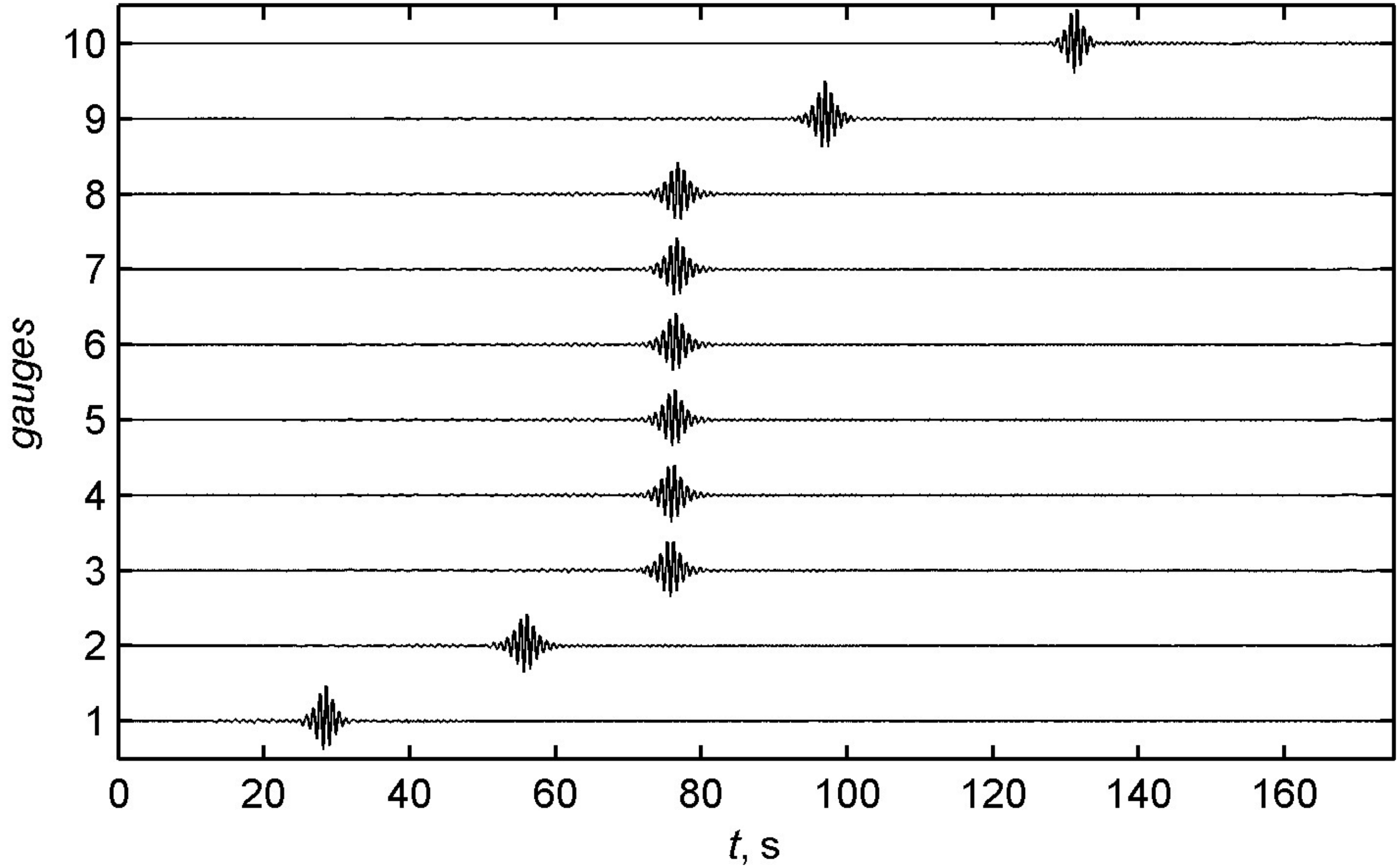
[Yuen & Lake, 1975]

Another unsuccessful attempt to reproduce an intense NLS envelope soliton in a wave flume



Envelope solitons over real water

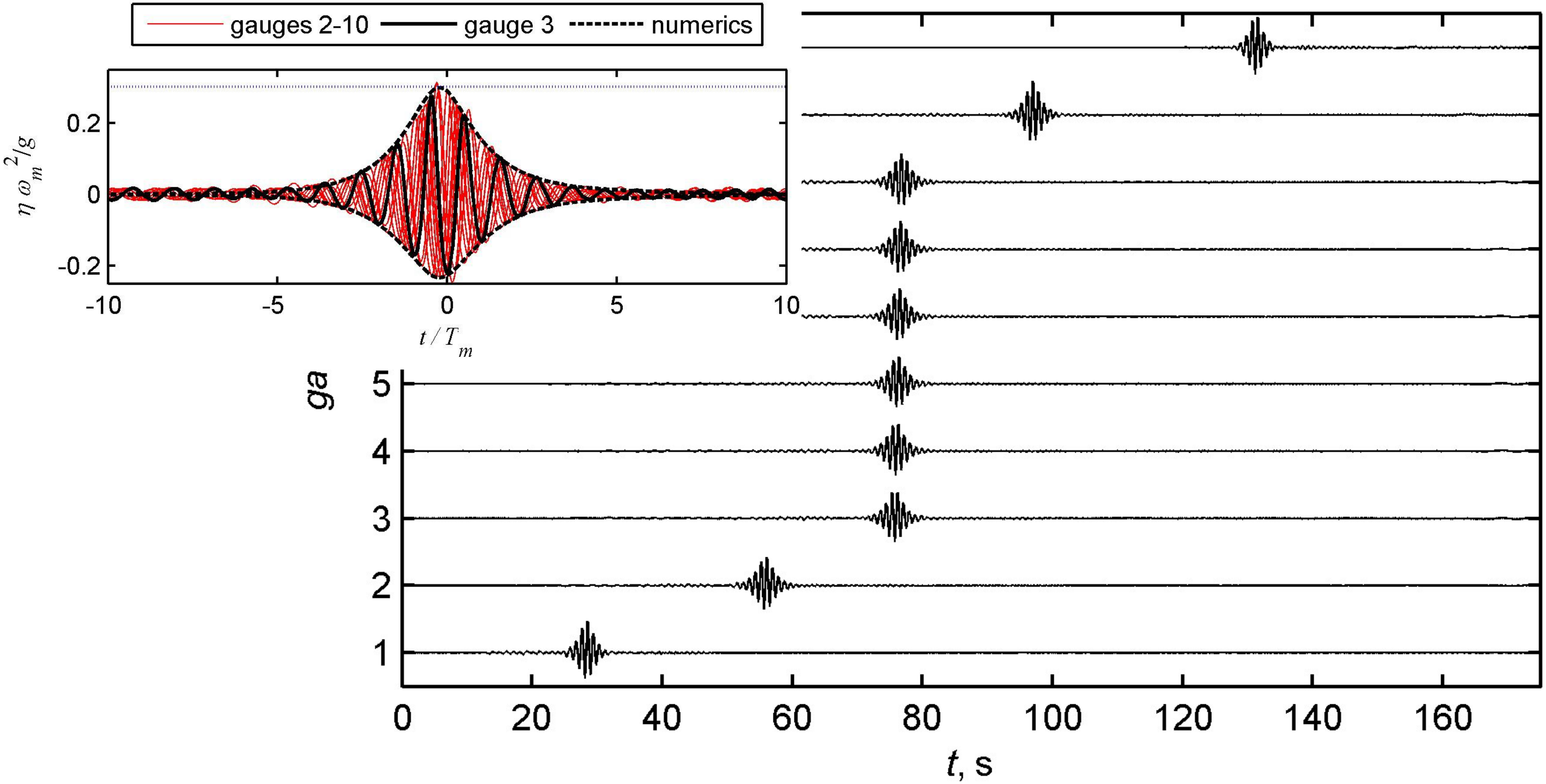
A <u>successful</u> reproduction of an intense NLS envelope soliton in a wave flume



Envelope solitons over real water

A successful attempt to reproduce an intense NLS envelope soliton in a wave flume

Time series

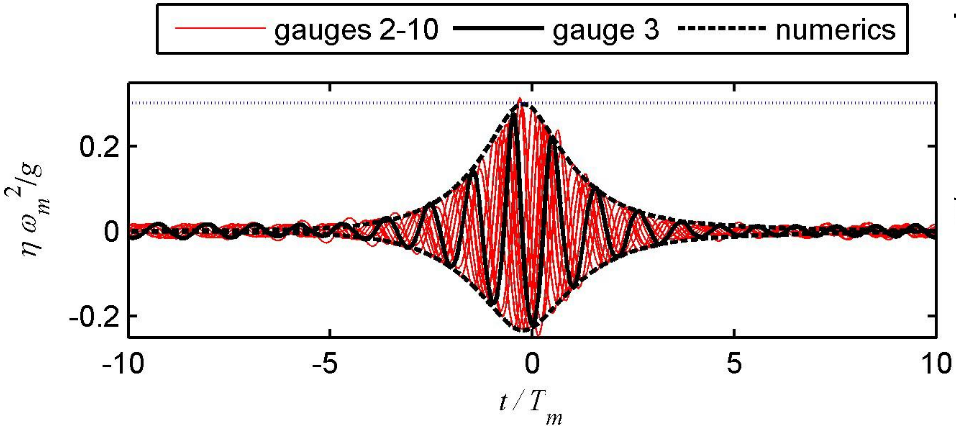


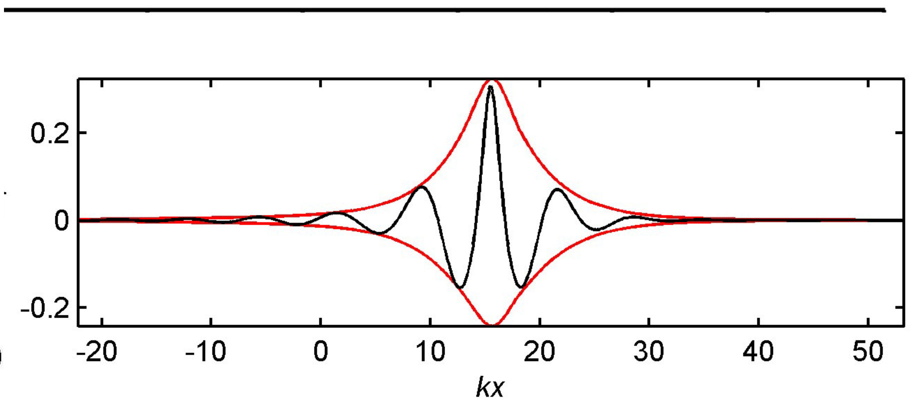
Envelope solitons over real water

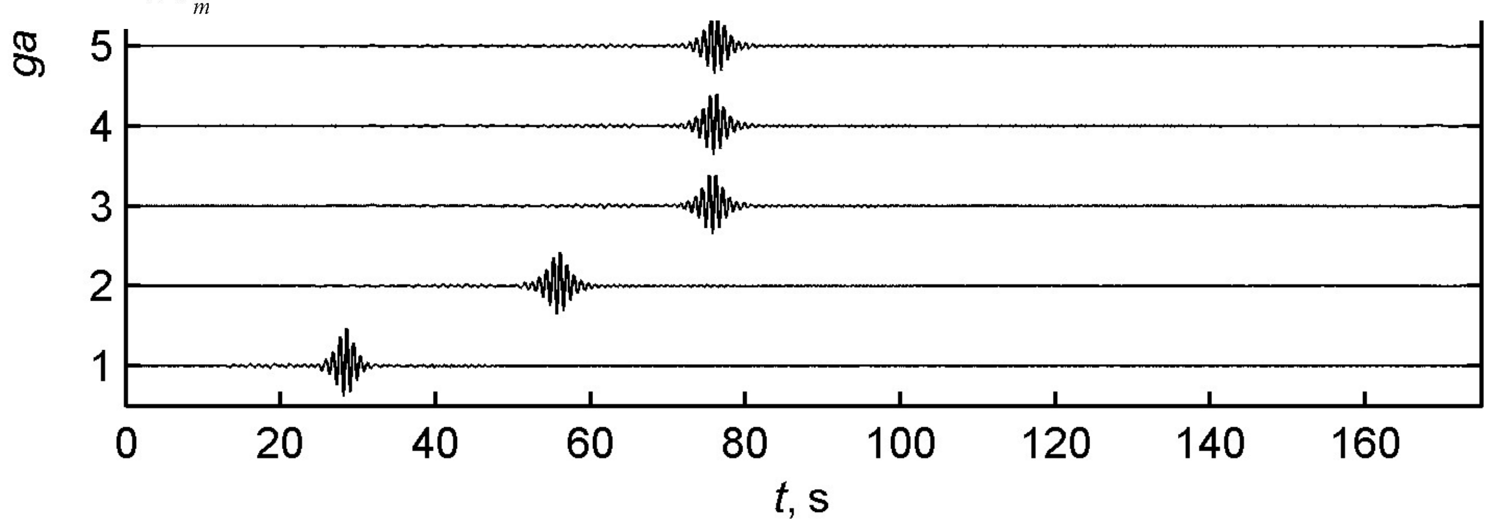
A successful attempt to reproduce an intense NLS envelope soliton in a wave flume

Time series

Snap-shot [Euler eq num sims]



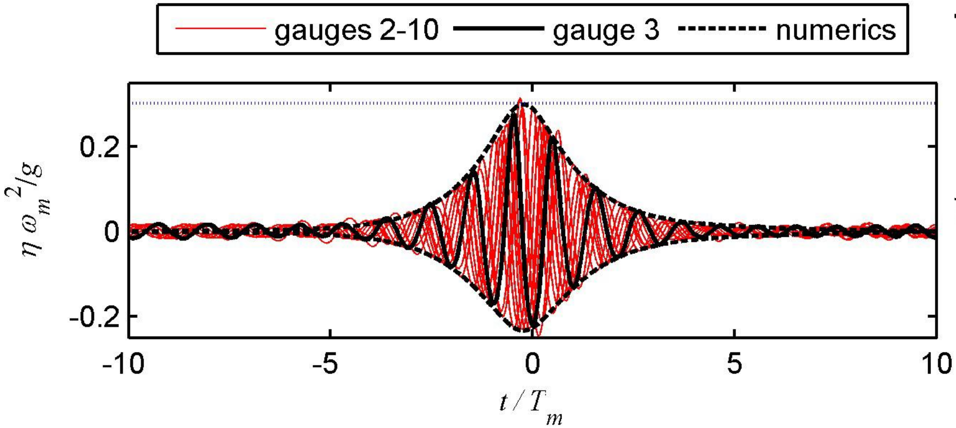




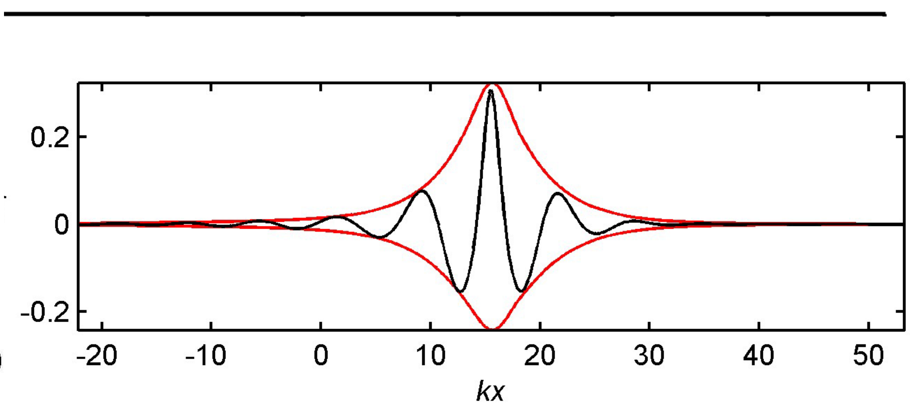
Envelope solitons over real water

A successful attempt to reproduce an intense NLS envelope soliton in a wave flume

Time series



Snap-shot [Euler eq num sims]



Steepness $A_{cr}\omega^2_m/g$ = 0.301 (~kA)

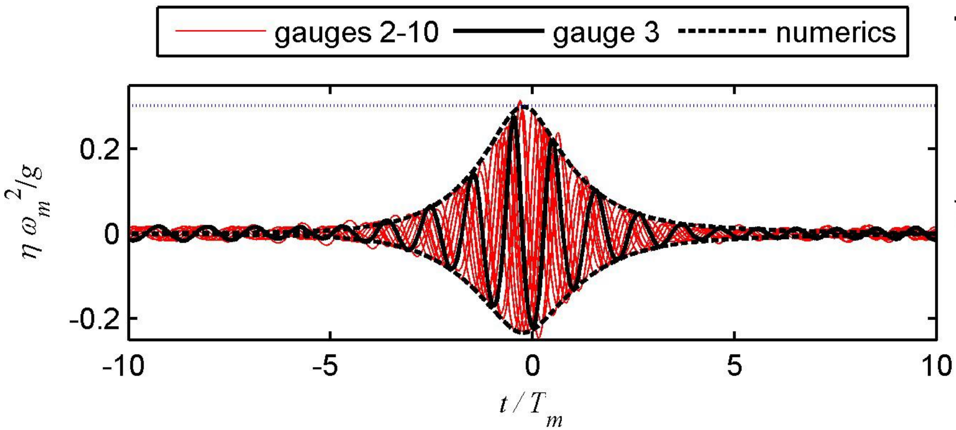
~'Giant breathers' of Dyachenko & Zakharov (2008)

Fig.5. Typical profile of breather

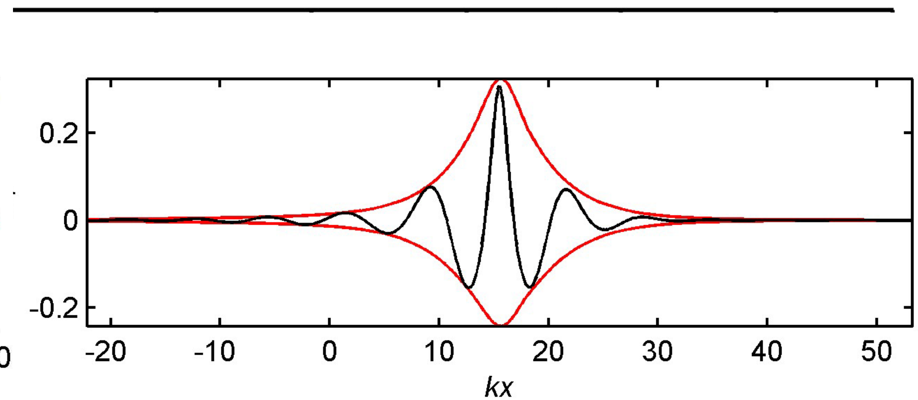
Envelope solitons over real water

A successful attempt to reproduce an intense NLS envelope soliton in a wave flume

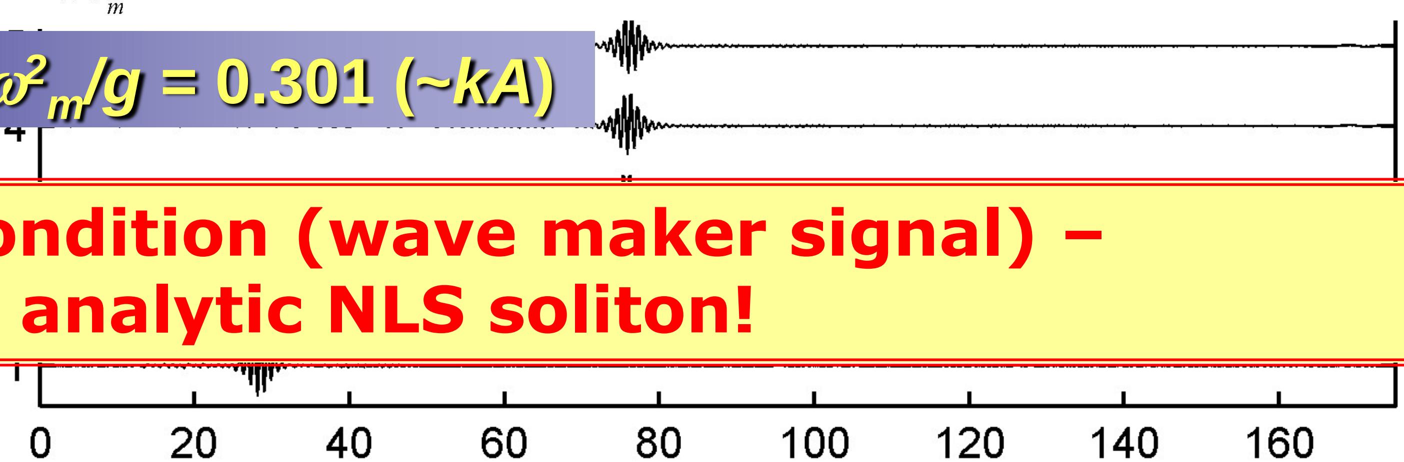
Time series



Snap-shot [Euler eq num sims]



Steepness $A_{cr}\omega^2_m/g = 0.301$ (~kA)

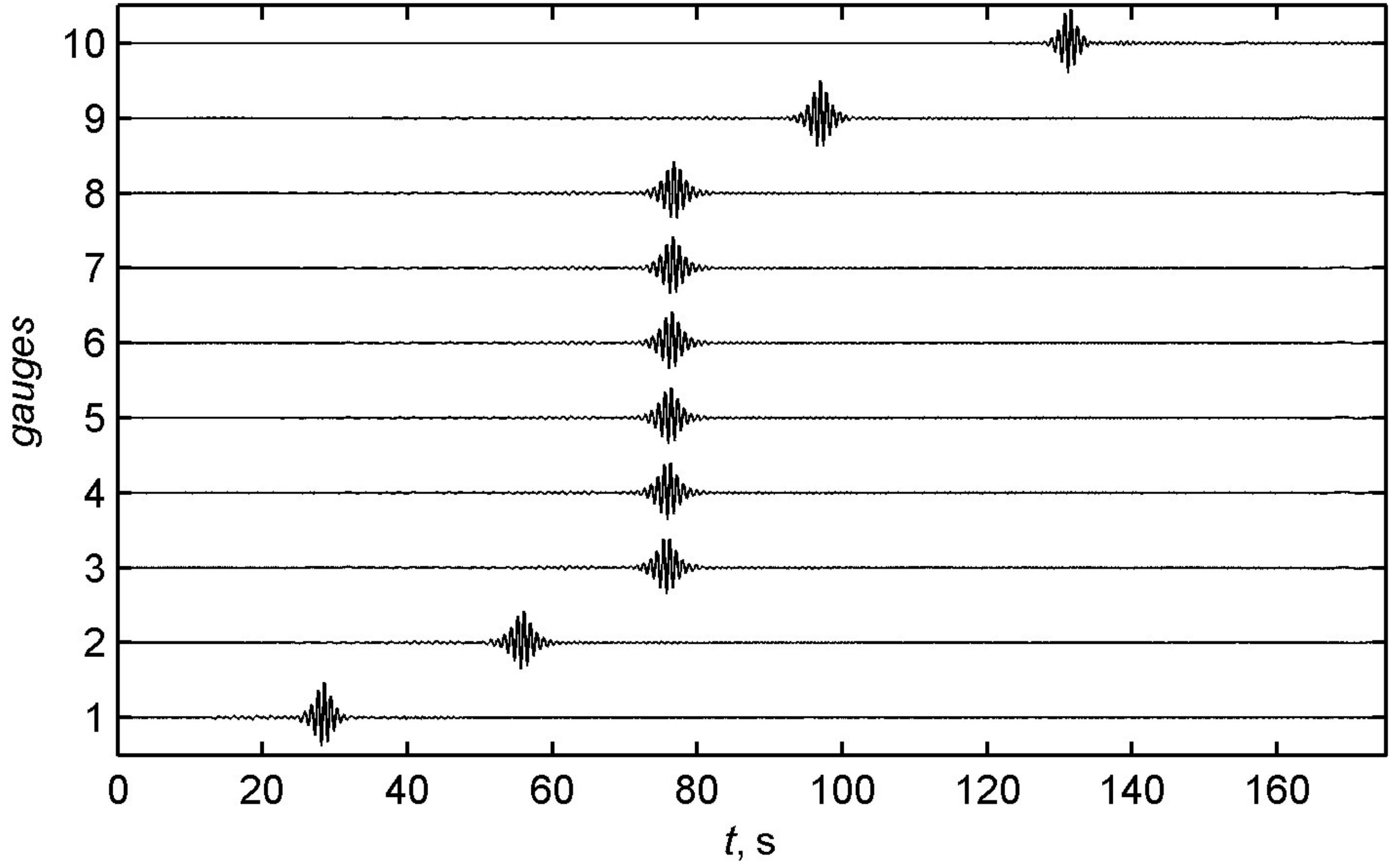


The initial condition (wave maker signal) – was just the analytic NLS soliton!

[Slunyaev et al, PoF2013]

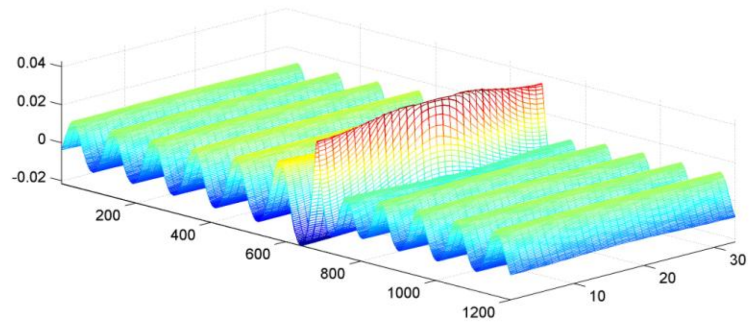
Envelope solitons over real water

3D: Transversally unstable (Zakharov & Rubenchik, 1974)



Rogue waves in 3D seas

Modulated long-crested waves



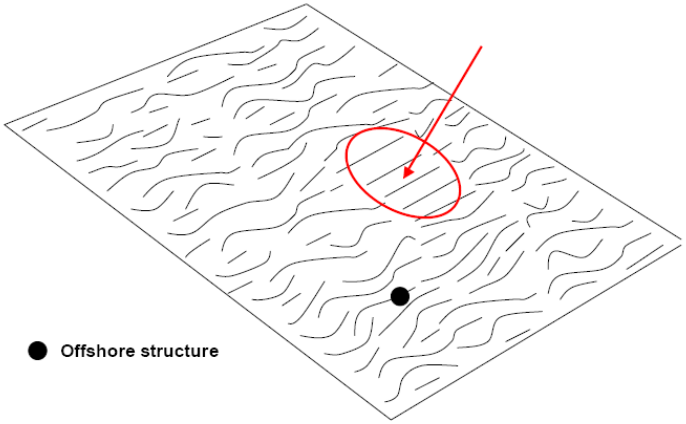
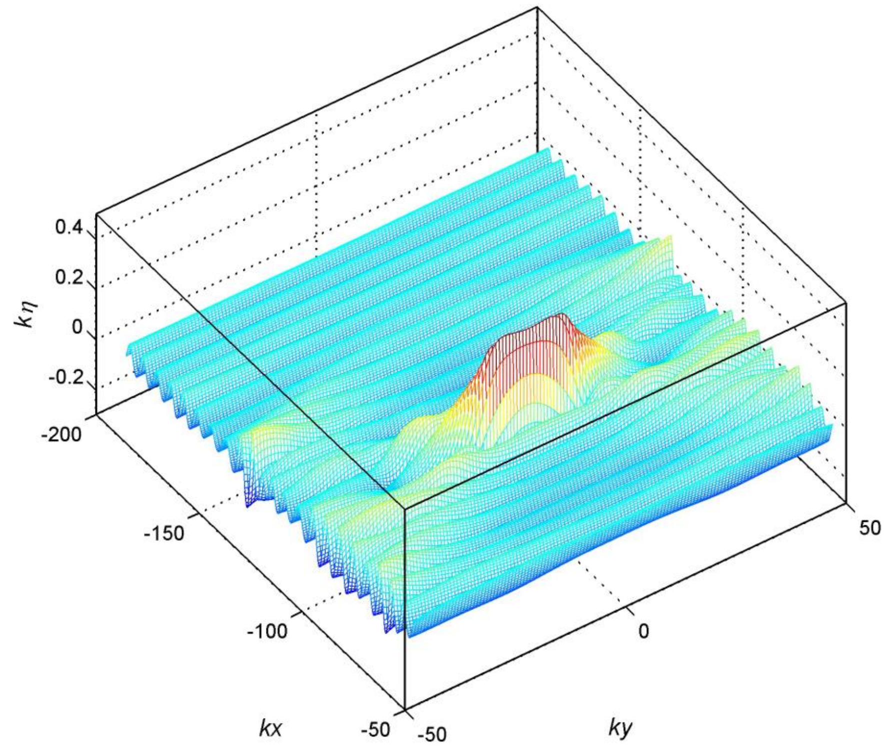
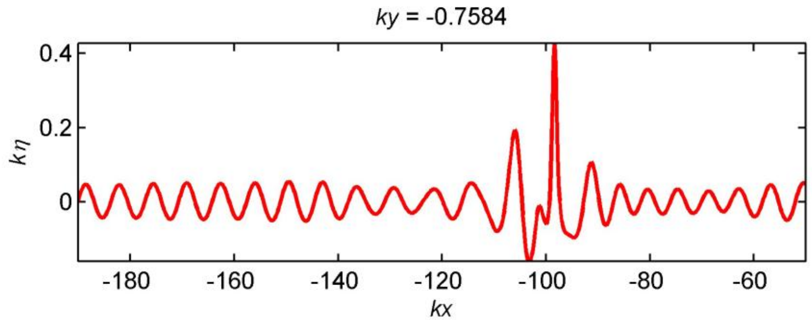


Fig. 6. Illustration of a short crested sea surface with a long crested sub-domain.

[Muller et al, 2005]

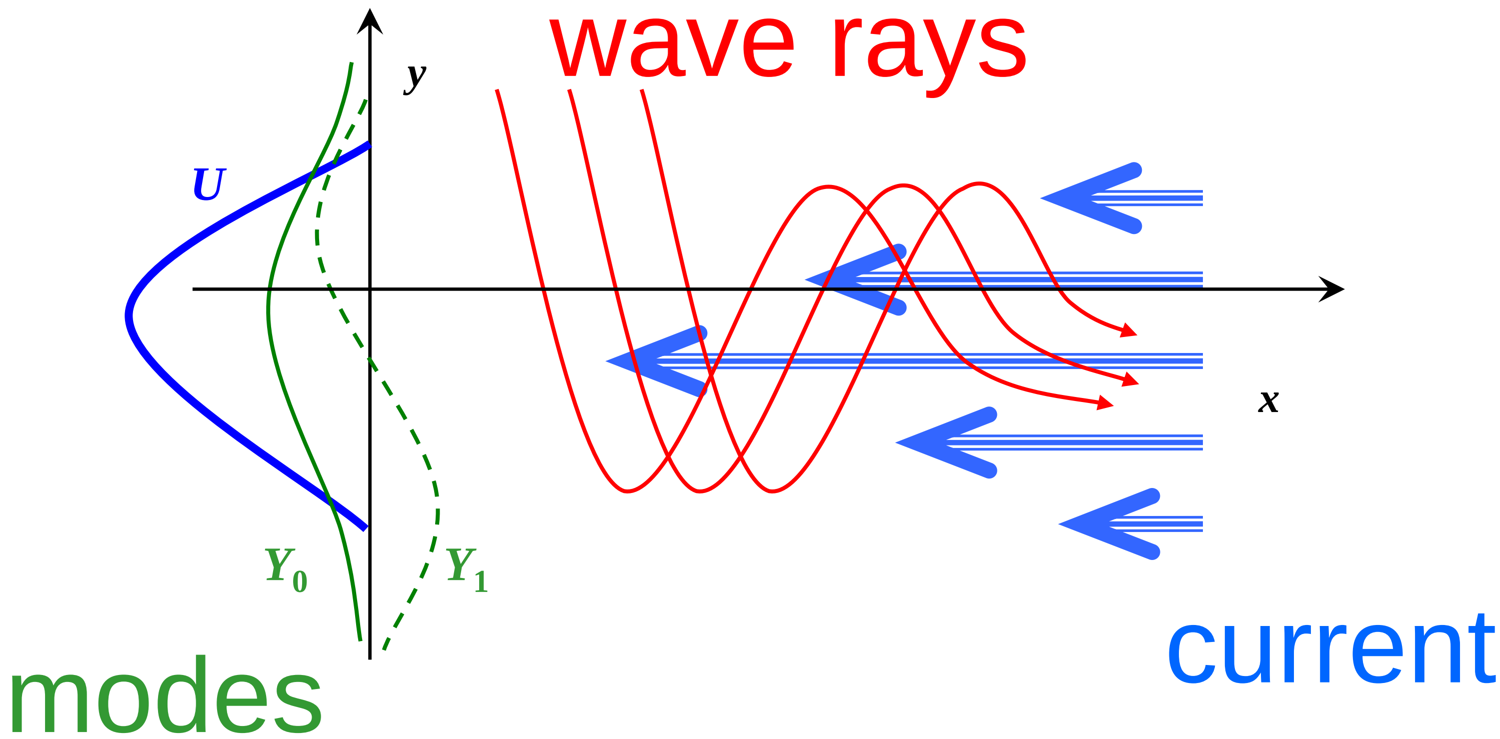




Rogue waves in 3D seas

Waves trapped by a jet current

Top view



Reference to the talk by Prof. V. Shrira (Thursday)

Effectively 1D evolution

Weakly nonlinear theory for modulated mode quartets

$$-i\left(\frac{\partial A}{\partial t}+V\frac{\partial A}{\partial x}\right)+\frac{\omega_n}{8k_0^2}\frac{\partial^2 A}{\partial x^2}+\kappa\frac{\omega_n k_0^2}{2}A|A|^2=0$$

- NLSE for a single trapped mode **(weak current assumption)**

$$V=\frac{\int_{-\infty}^{\infty}\left(\frac{g^2k_0}{2\Omega_n^3}+U\right)Y_n^2dy}{\int_{-\infty}^{\infty}Y_n^2dy}$$

- mode velocity

$$\Omega_n(y)=\omega_n-k_0U>0$$

$$\kappa=\frac{\int_{-\infty}^{\infty}Y_n^4dy}{\int_{-\infty}^{\infty}Y_n^2dy}$$

- attenuation of nonlinearity (κ <1)

$$\frac{d^2Y_n}{dy^2}+4k^2\left[\frac{\omega_n}{\sqrt{gk_0}}-\left(1+\frac{k_0U}{\sqrt{gk_0}}\right)\right]Y_n=0$$

$$Y_n\xrightarrow[y\to\pm\infty]{}0$$

- BVP on modes: Sturm-Liouville problem

Effectively 1D evolution

Wave field composition

Surface elevation

eigenfrequency of the BVP

$$\eta = \mathrm{Re}\left[A(x,t)Y_n(y)\exp(i\omega_n t - ik_0 x)\right]$$

eigenfunction of the BVP

solution of the NLSE

Three examples:

1) Regular wave

2) Modulated wave train

3) Solitary wave group

Simulated by means of current-modified HOSM solver for Euler eqs.
The current is ‘frozen’, wave motions are potential.

A uniform wave train

Euler equations framework

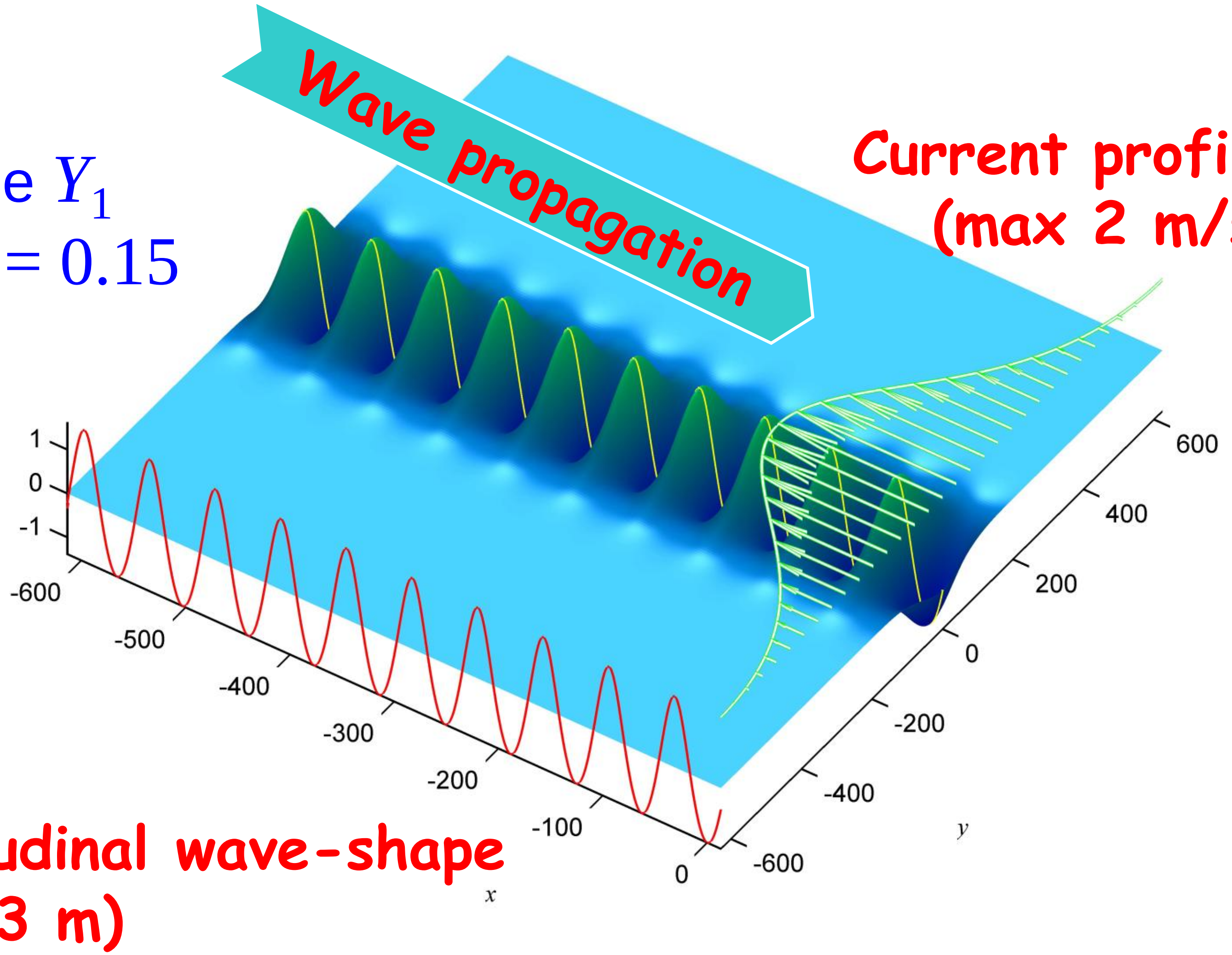
Initial condition: transverse modulated Stokes wave

fundamental mode Y_1

steepness $k_0 H/2 = 0.15$

$C_{ph} \approx 10$ m/s

$$\left|\frac{1}{\omega}\frac{dU}{dy}\right| \sim< 0.008$$

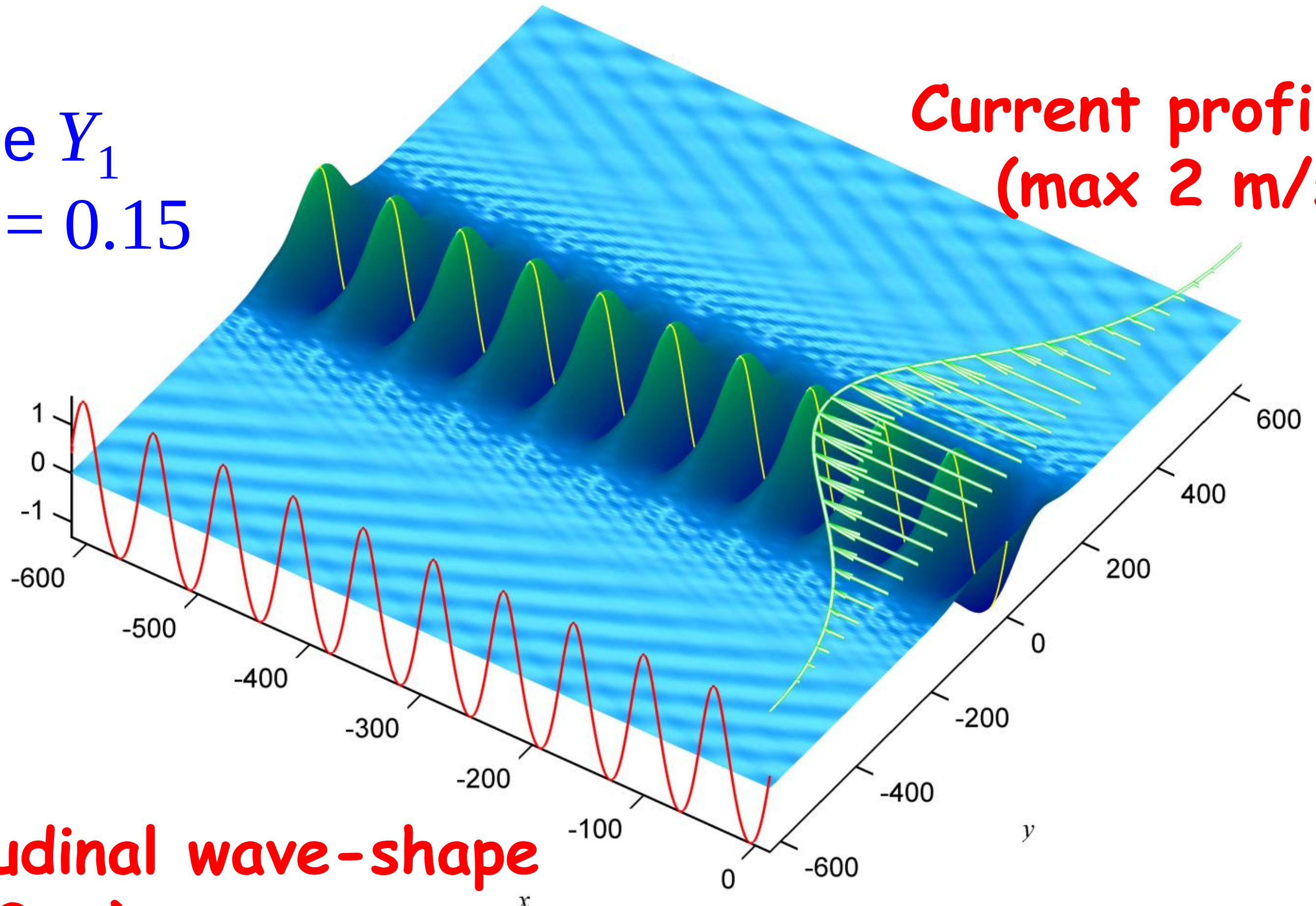


A uniform wave train

Euler equations framework

after 370 wave periods

fundamental mode Y_1
steepness $k_0 H/2 = 0.15$



Current profile (max 2 m/s)

Longitudinal wave-shape (λ = 63 m)

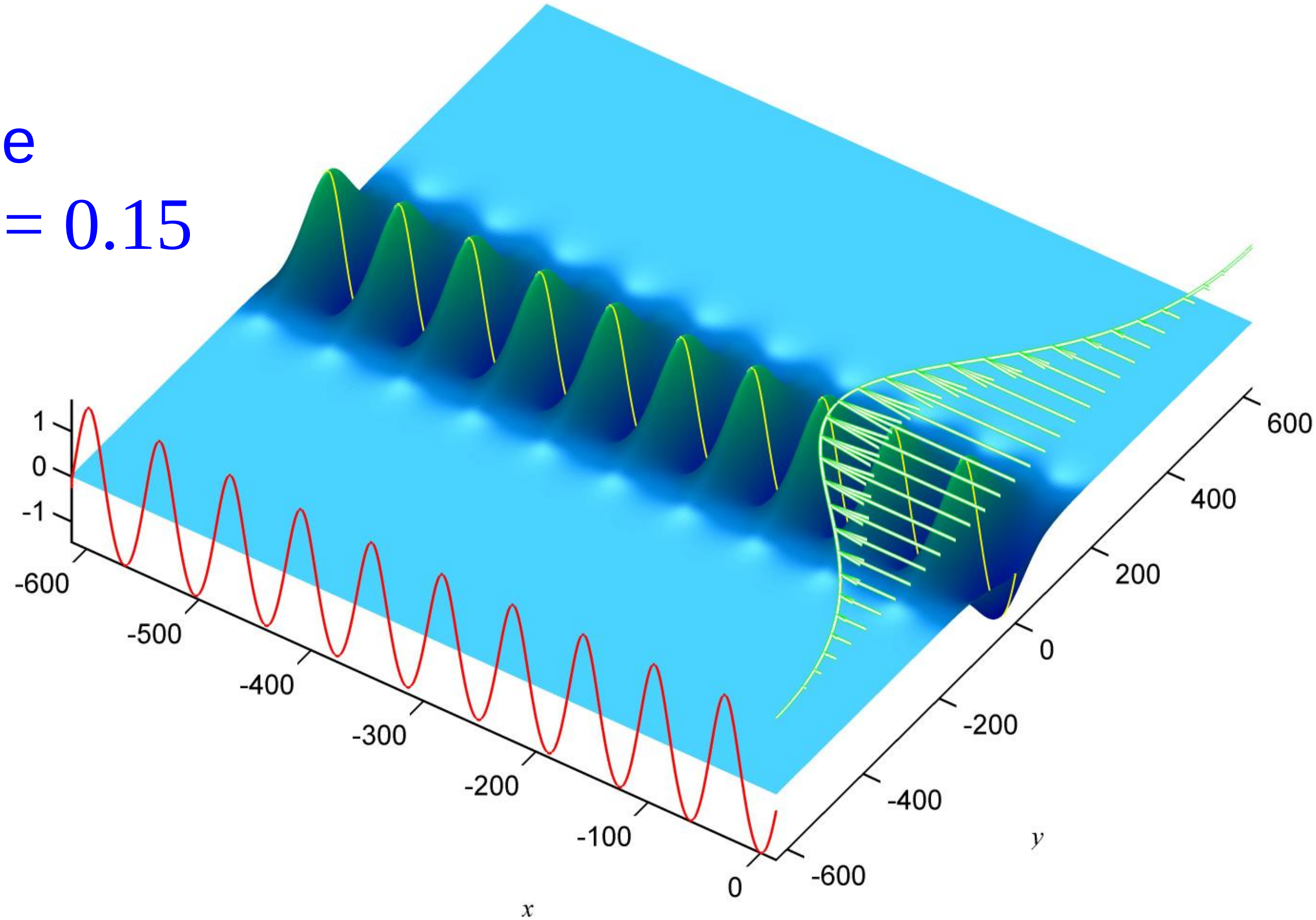
Modulated wave train-> Rogue wave

Euler equations framework

Initial condition: 10 wave periods, 5% amplitude modulation

fundamental mode

steepness $k_0 H/2 = 0.15$

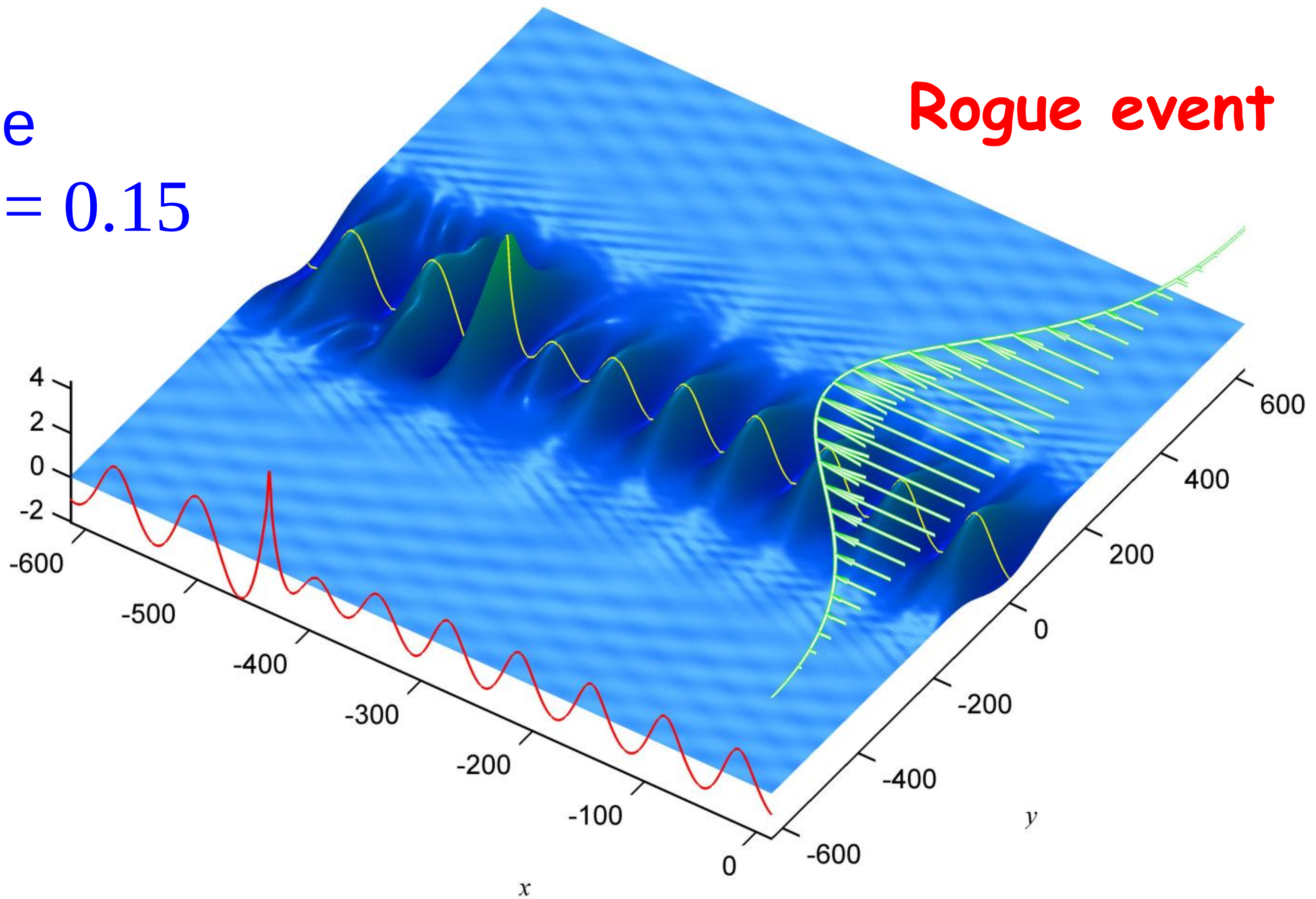


Modulated wave train-> Rogue wave

Euler equations framework

105 wave periods

fundamental mode
steepness $k_0 H/2 = 0.15$



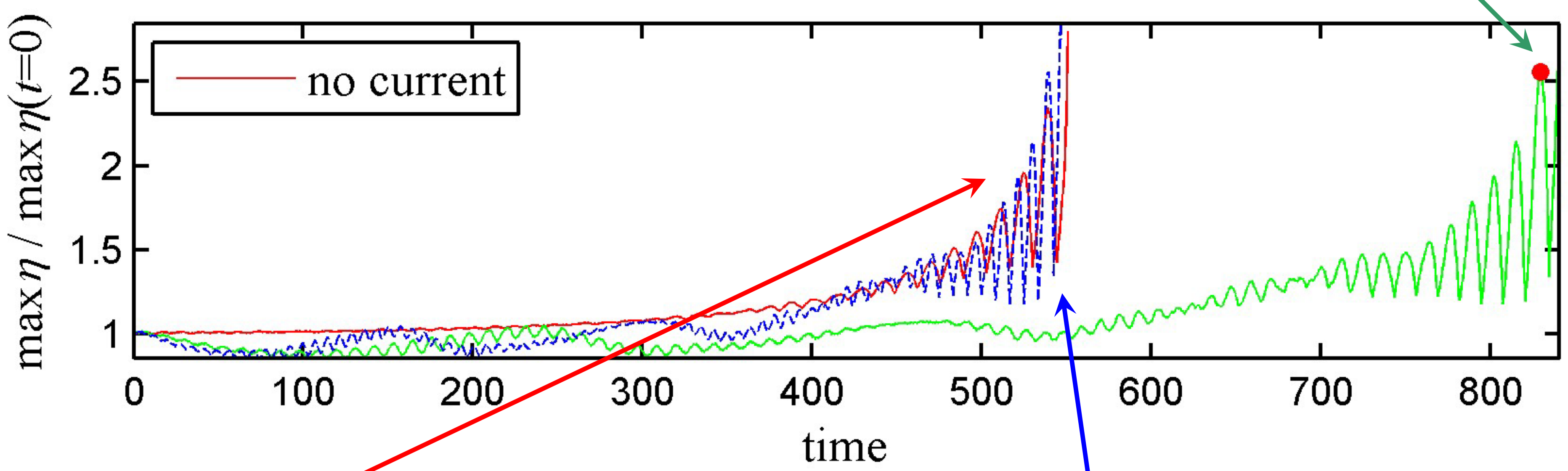
Modulated wave train-> Rogue wave

Euler equations framework

Coefficient which changes the time scale

$$-i\left(\frac{\partial A}{\partial t}+V\frac{\partial A}{\partial x}\right)+\frac{\omega_n}{8k_0^2}\frac{\partial^2 A}{\partial x^2}+\kappa\frac{\omega_n k_0^2}{2}A|A|^2=0$$

Rogue event



Reference simulation (no current)

Scaled time according to κ

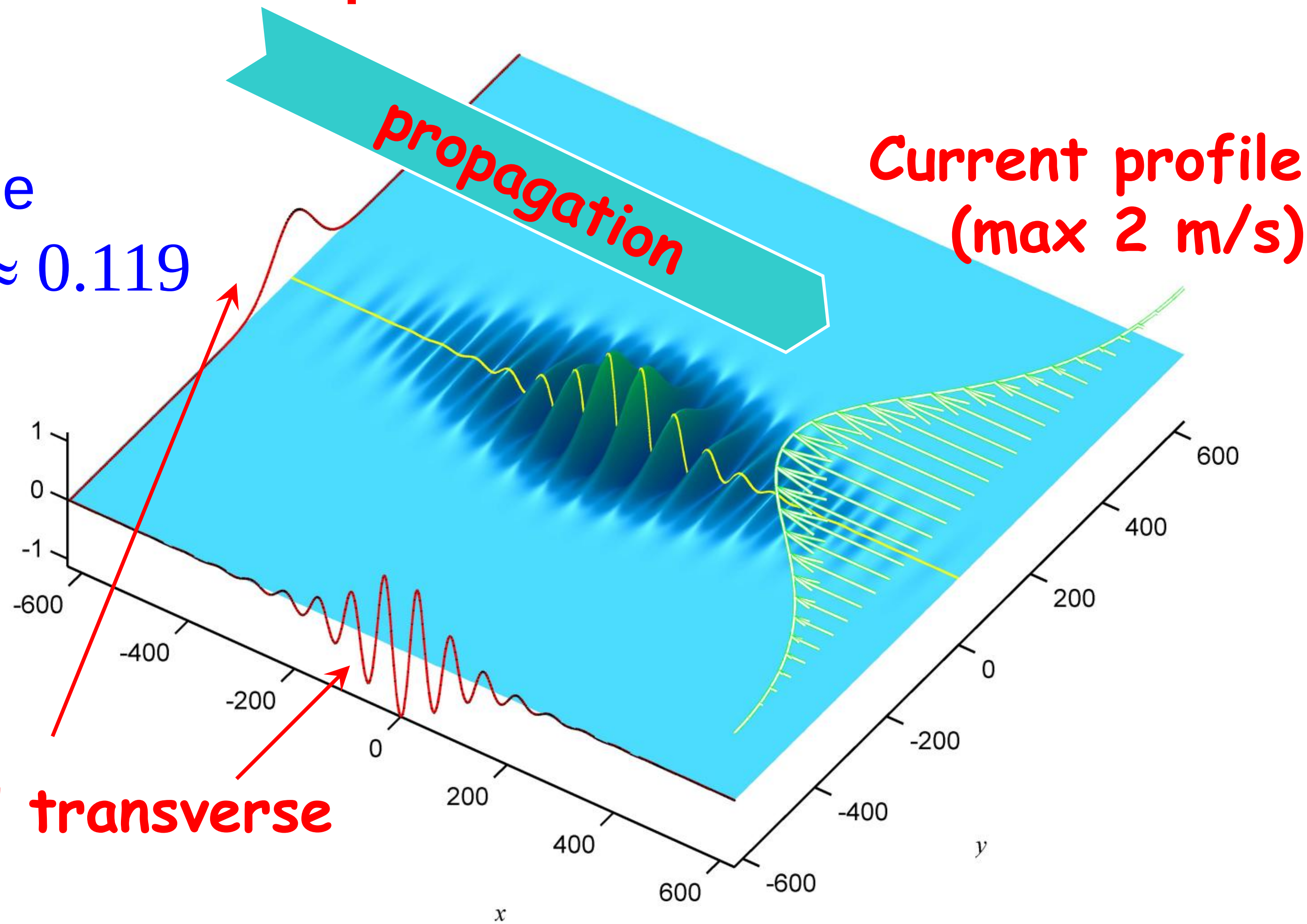
Good fit

Solitons of trapped waves

Euler equations framework

Initial condition: NLS envelope soliton

fundamental mode
steepness $k_0 A_{cr} \approx 0.119$



Longitudinal and transverse wave shapes

Solitons of trapped waves

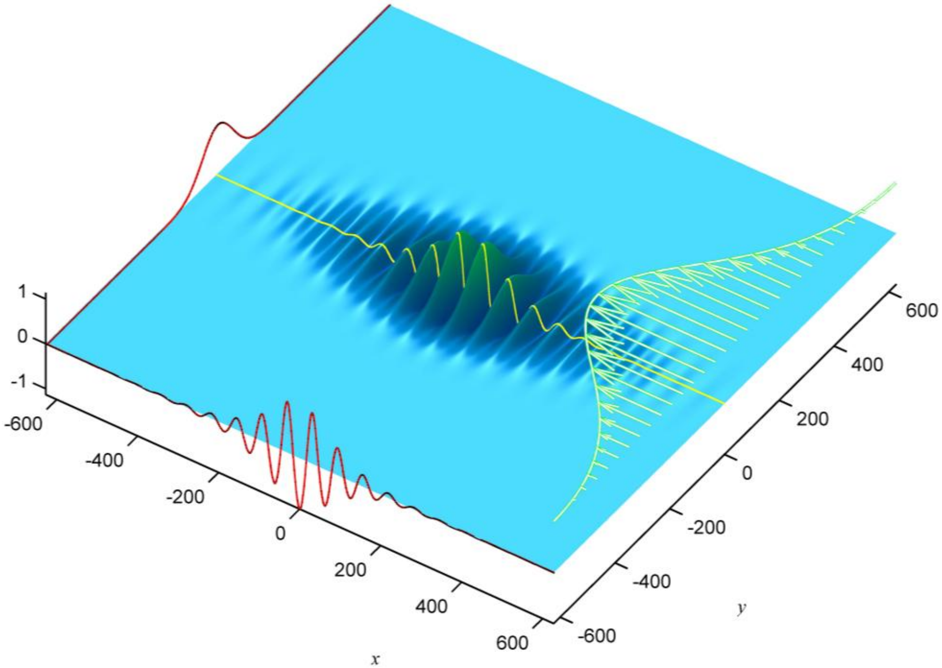
Euler equations framework

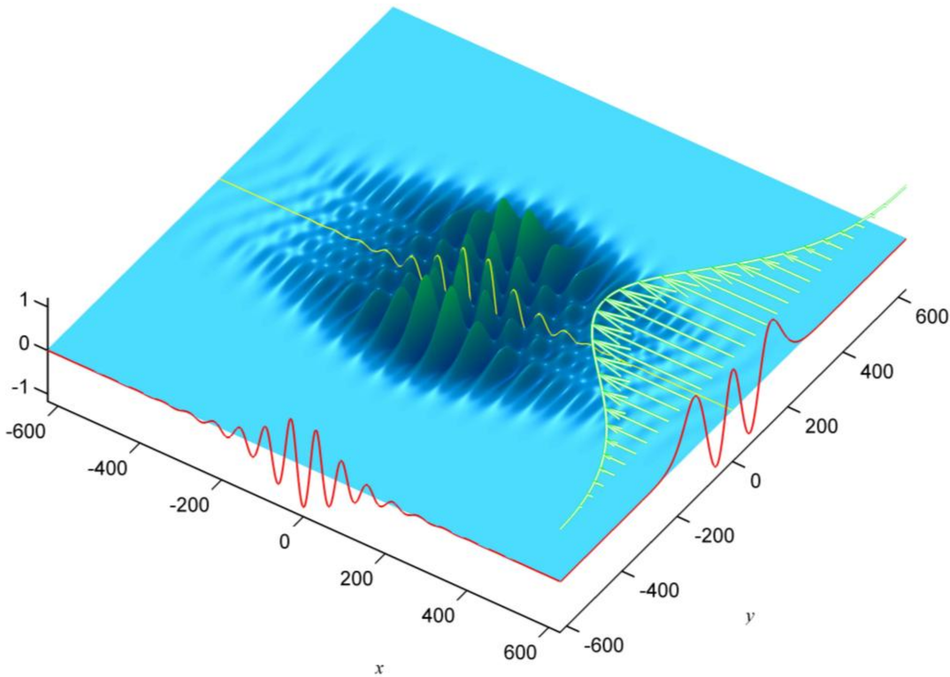
Initial condition: NLS envelope soliton

steepness $k_0 A_{cr} \approx 0.119$

fundamental mode

fifth mode

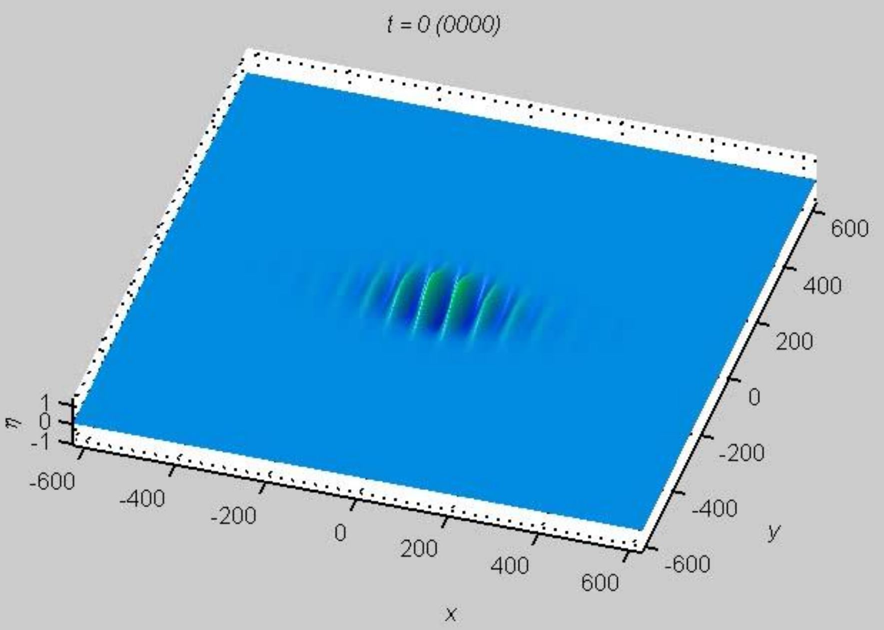




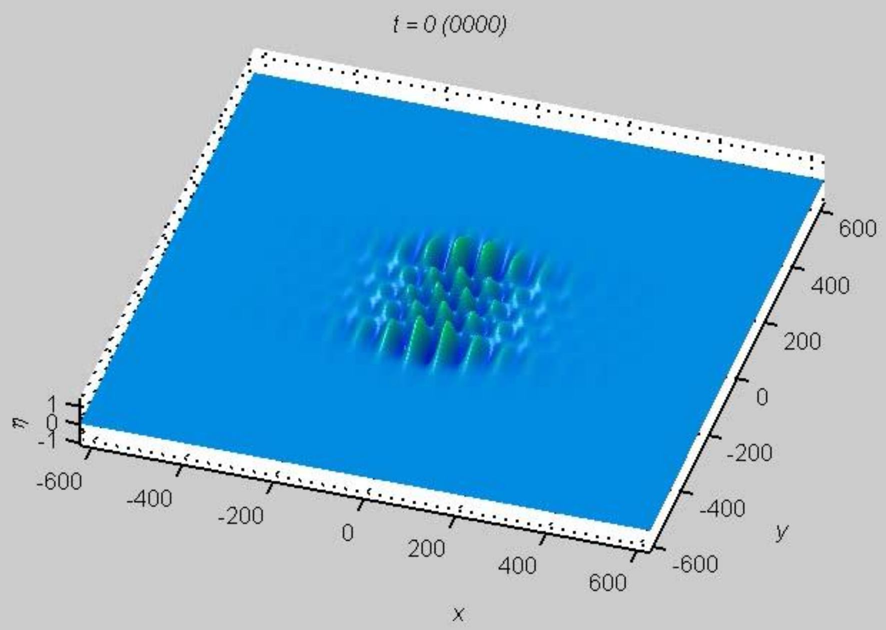
Solitons of trapped waves

Euler equations framework

fundamental mode



fifth mode

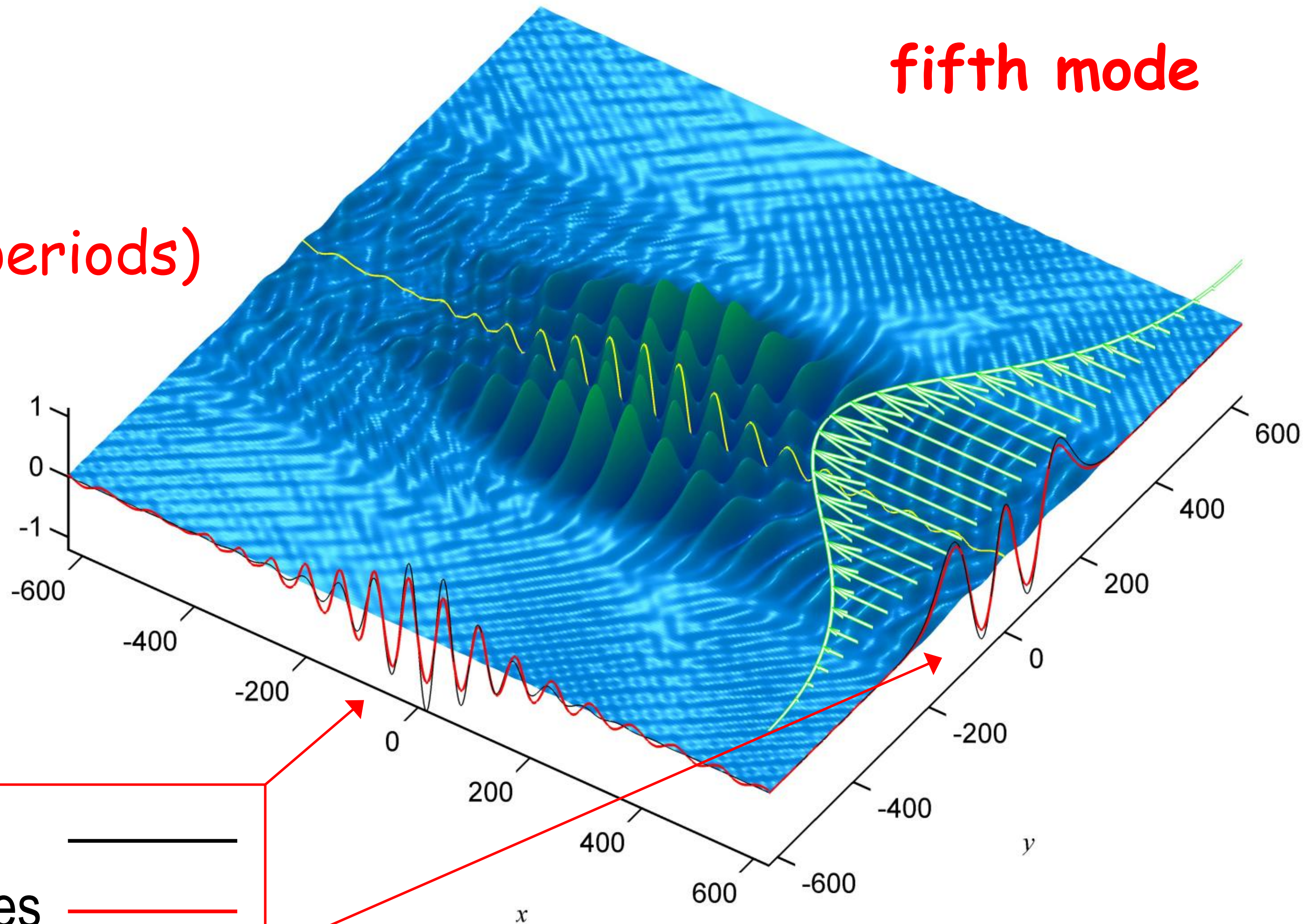


Solitons of trapped waves

Euler equations framework

fifth mode

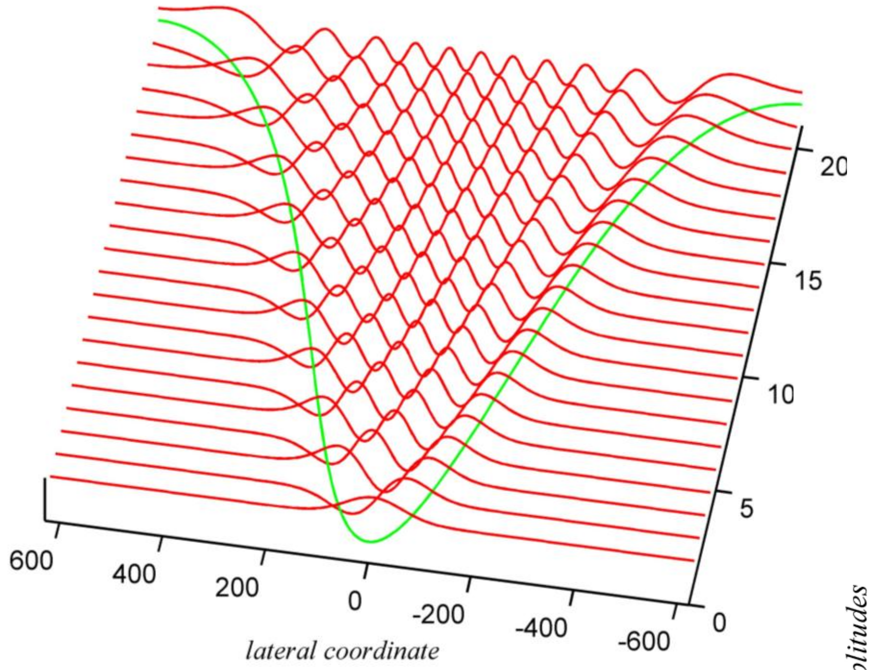
about 10 T_{nl}
(~120 wave periods)



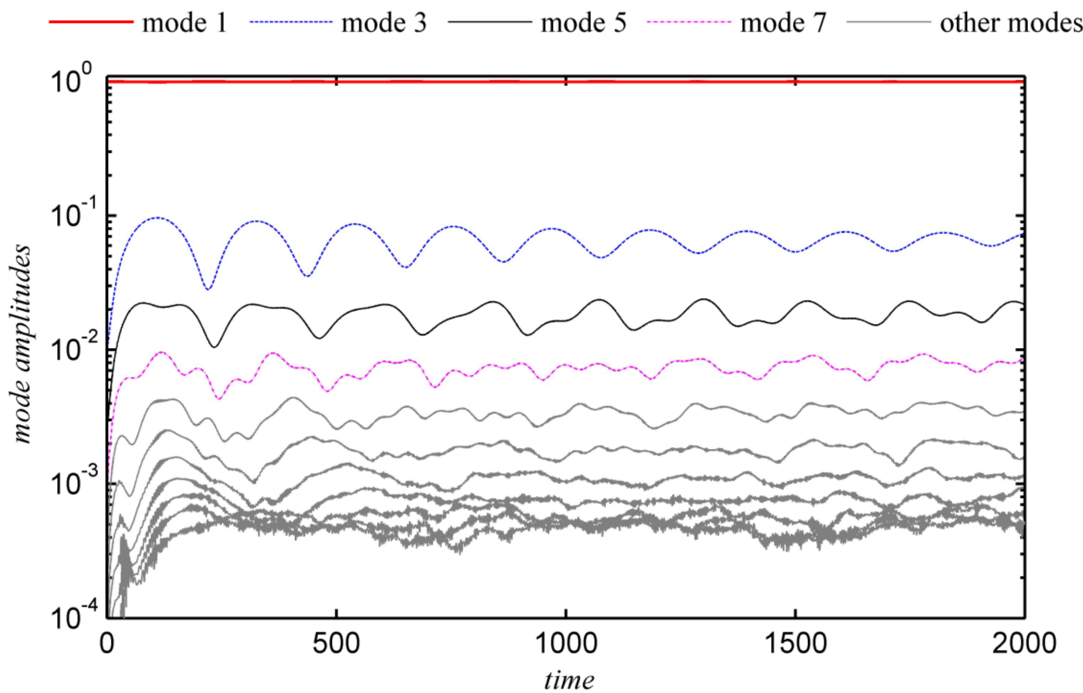
initial condition
after two passages

Solitons of trapped waves

Mode decomposition: trapped modes and mode amplitudes



fundamental mode
steepness $k_0 A_{cr} \approx 0.119$



Mode amplitudes

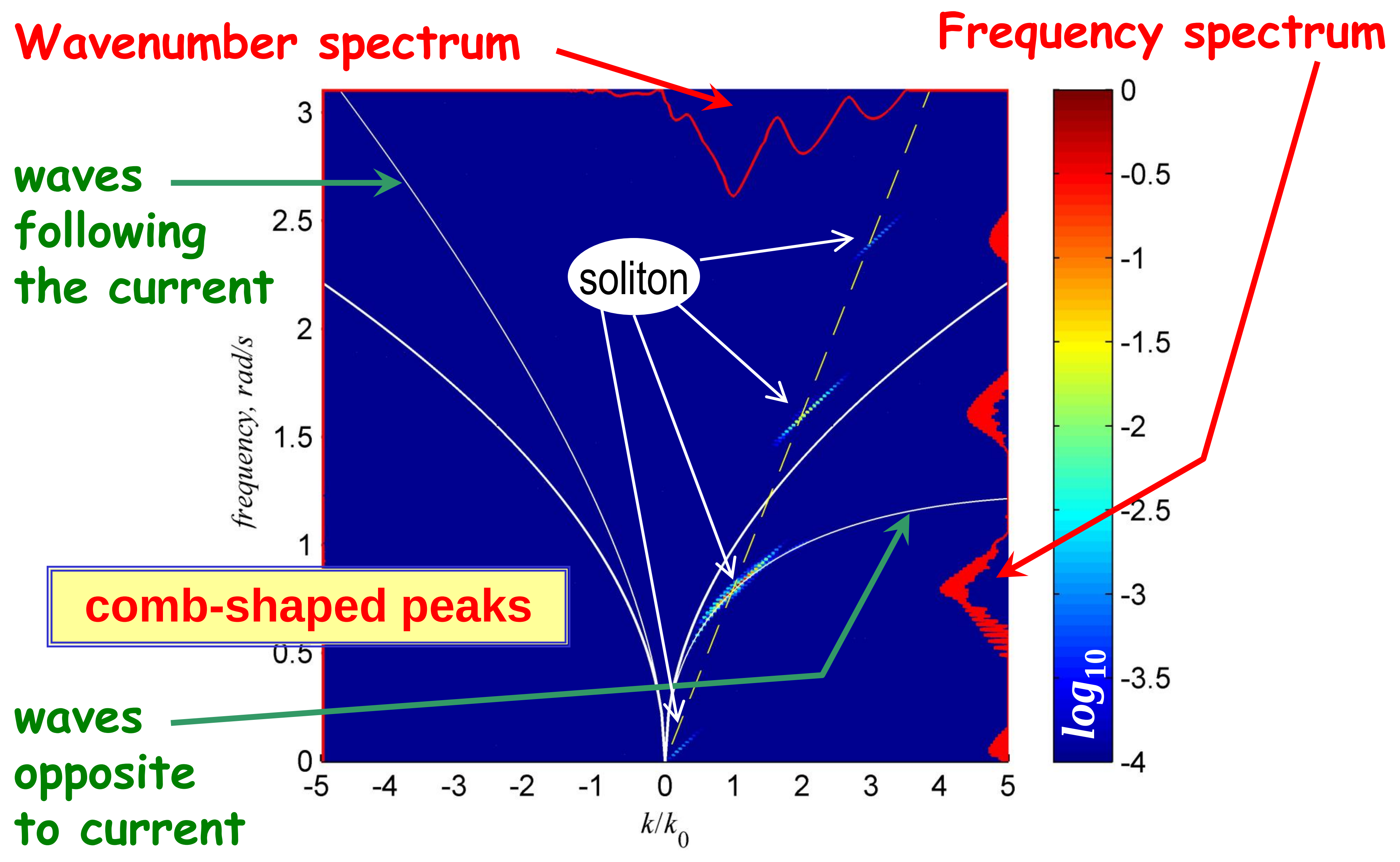
$$\eta_m(x,t) = \int \eta(x,y,t) Y_m(y) dy$$

$$a_m(t) \sim \sqrt{\int \eta_m^2 dx} \qquad \int Y_m^2(y) dy = 1$$

No tendency to disintegrate!

Solitons of trapped waves

Mode decomposition: amplitude Fourier spectra

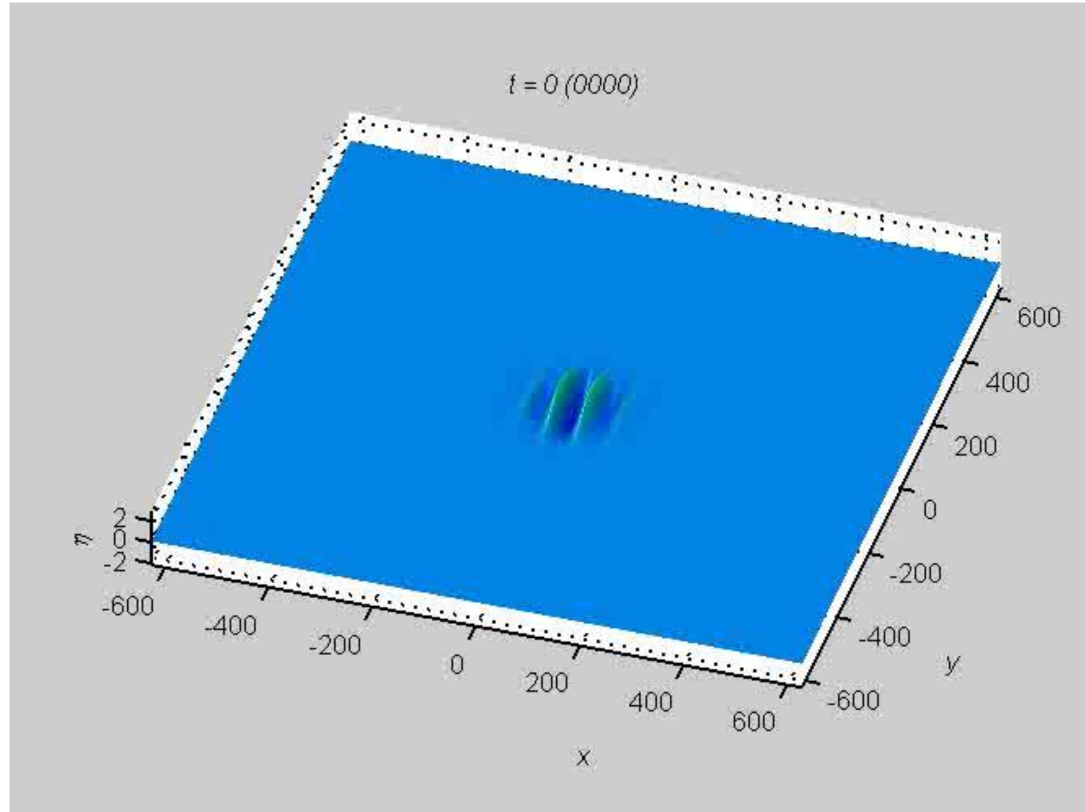


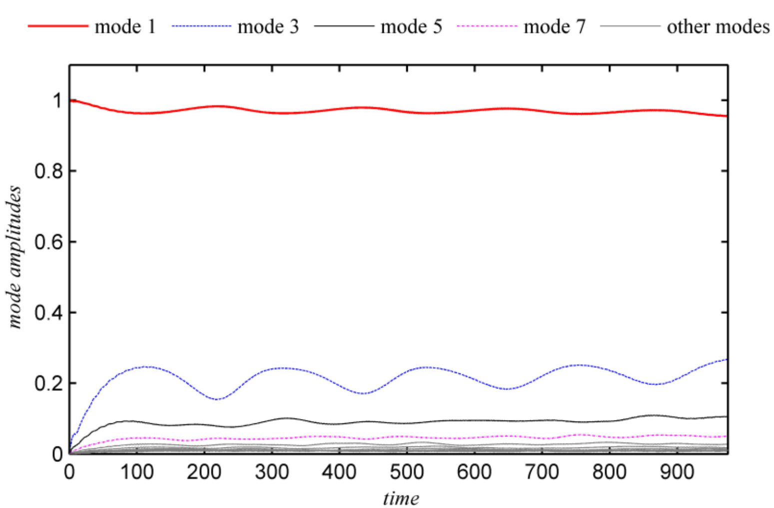
Solitons of trapped waves

Strongly nonlinear sims of weakly nonlinear solutions

fundamental mode
even steeper/shorter wavegroup

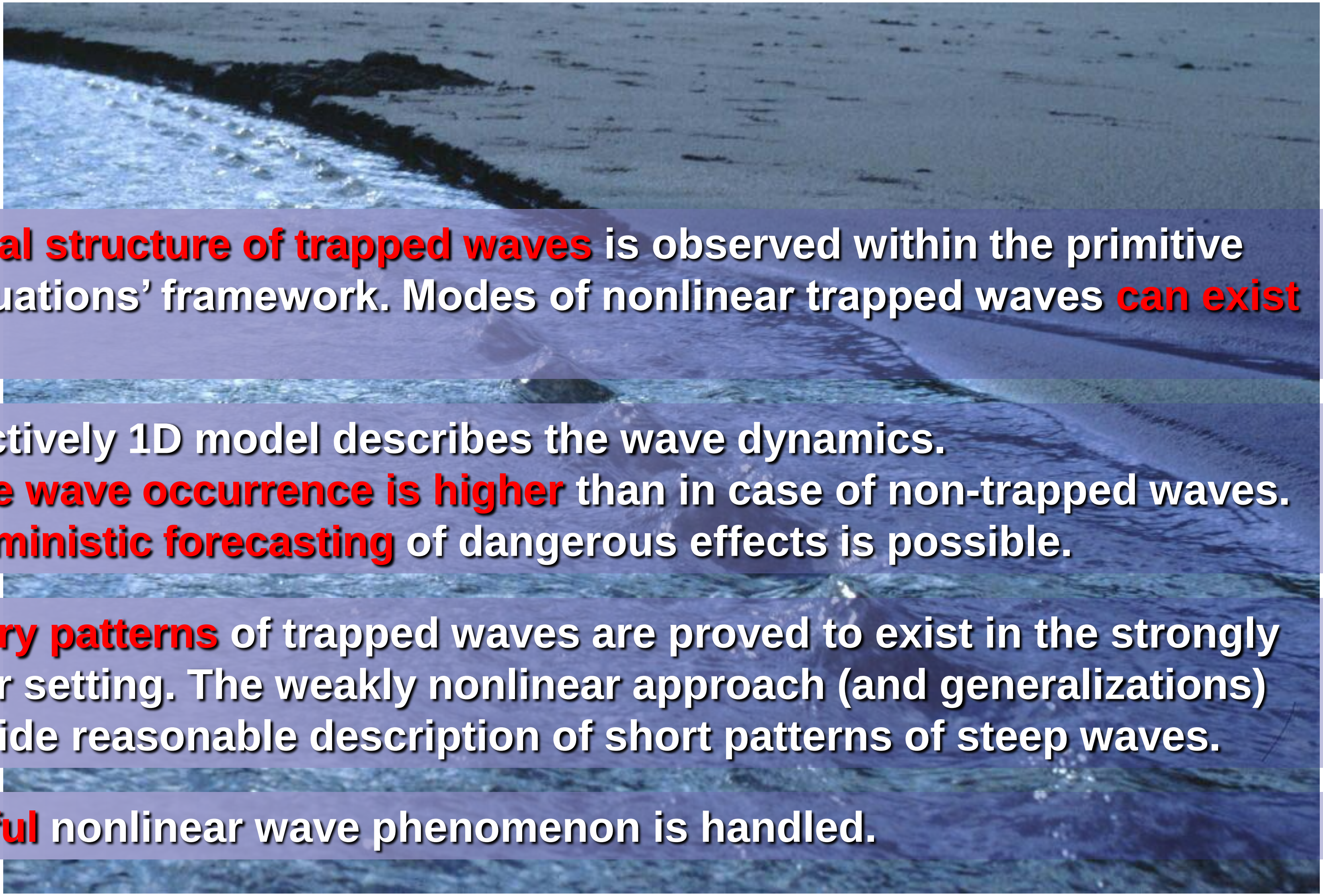
Short 3D wave group of intense waves ($k_0 A_{cr} \sim 0.24$)





mode amplitudes for more than 100 periods

Conclusions



The **modal structure of trapped waves** is observed within the primitive Euler equations' framework. Modes of nonlinear trapped waves **can exist for long**.

The effectively 1D model describes the wave dynamics.
=> **Rogue wave occurrence is higher** than in case of non-trapped waves.
=> **Deterministic forecasting** of dangerous effects is possible.

3D solitary patterns of trapped waves are proved to exist in the strongly nonlinear setting. The weakly nonlinear approach (and generalizations) can provide reasonable description of short patterns of steep waves.

A **beautiful** nonlinear wave phenomenon is handled.

Photo from a presentation by D.H. Peregrine