

SUPERRADIANT LASING: QUASI-REGULAR AND QUASI-CHAOTIC REGIMES

We discuss new nonlinear phenomena inherent to rich dynamics of the class D lasers and their relation to the specific problems of cooperative radiative behavior of many-particle systems.

We analyze physical mechanisms responsible for intriguing superradiance regimes in low-Q cavities.

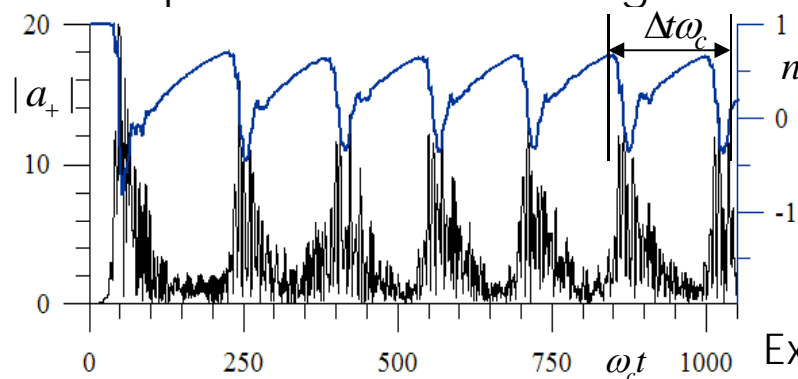
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Classification of lasers (after Arecchi and Khanin)

Dynamical class	A	B	C	D
Relaxations rate	$\gamma_F \ll \gamma_{\parallel}, \gamma_{\perp}$	$\gamma_{\parallel} \ll \gamma_F \ll \gamma_{\perp}$	$\gamma_{\parallel} \leq \gamma_{\perp} \sim \gamma_F$	$\gamma_{\parallel}, \gamma_{\perp} \ll \gamma_F$
Adiabatic elimination	<i>Polarization, inversion</i>	<i>Polarization</i>		<i>Field, if $\omega_R \ll \gamma_F$</i>

$$\omega_R = dE / \hbar$$

Example: Self-mode locking and superradiant bunching in the class D DFB lasers



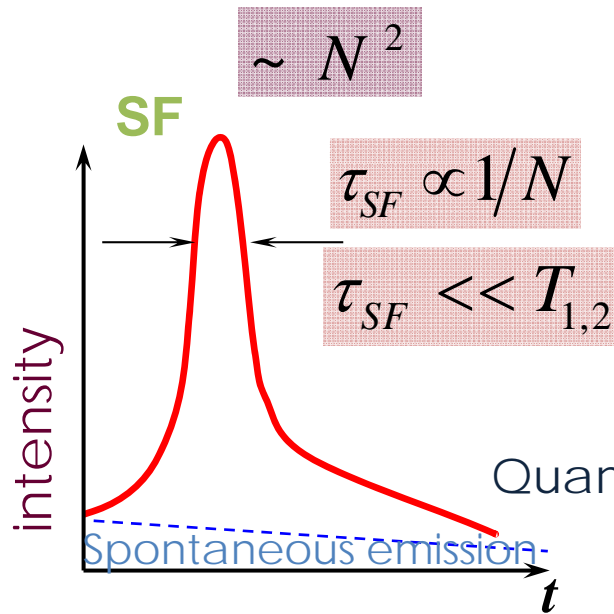
$$\omega_c = \left(2\pi d^2 \omega_0 N / \hbar \right)^{1/2} \quad \text{cooperative frequency}$$

$$\omega_c^2 T_2^* \gg \gamma_{\perp}, \gamma_{\parallel} \quad \text{necessary conditions of superradiance}$$

Extremely high spatial and spectral densities of active centers are required for superradiance. In practice, it means a strong inhomogeneous broadening of an active medium (though exotic systems with homogeneous broadening are also possible).

Superfluorescence (SF) in semiclassical approximation

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Classical Maxwell equation for the field

$$\frac{\partial^2 \mathbf{E}}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = \frac{4\pi}{c^2} \frac{\partial^2 \mathbf{P}}{\partial t^2}$$

Quantum Bloch equations for the polarization \mathbf{P} and the inversion ΔN

relaxation times

$$T_1 \geq 2T_2 \geq 2\pi/\omega_c$$

short sample

$$B \leq B_c = c/(\omega_c \sqrt{\epsilon})$$

$$\frac{\partial^2 \mathbf{P}}{\partial t^2} + \frac{2}{T_2} \frac{\partial \mathbf{P}}{\partial t} + (\omega_0^2 + T_2^{-2}) \mathbf{P} = -\frac{2d^2 \omega_0 \Delta N}{\hbar} \mathbf{E}$$

$$\frac{\partial \Delta N}{\partial t} = -\frac{\Delta N}{T_1} + \frac{2}{\hbar \omega_0} \mathbf{E} \frac{\partial \mathbf{P}}{\partial t}$$

Experimental evidence of superfluorescence

Active media suitable for superradiant lasing:

sub-monolayer quantum-dot heterostructures ?

magnetized quantum wells

excitons in semiconductor traps

active colour centers in solid-state matrices
(e.g., in semiconductors or fibers)

degenerate electron-hole gas in semiconductors

molecular J- and H-aggregates

alkaline-earth-metal cold atomic gases

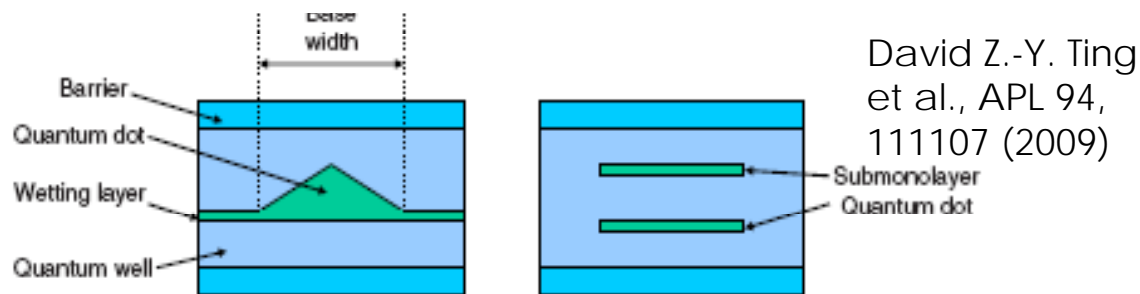
Typical power of a superradiant pulse $\sim 1\text{W}$ corresponds to coherent emission of $10^6 - 10^7$ photons within picosecond timescale

Sub-monolayer quantum-dots heterostructures

TEM images (plan view) of InAs SML insertions in a GaAs matrix stacked with different spacer layers: a) $\text{Al}_{0.3}\text{Ga}_{0.7}\text{As}$ and b) $\text{Al}_{0.6}\text{Ga}_{0.4}\text{As}$

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phys. stat. sol. (a) 183, No. 2 (2001)



David Z.-Y. Ting et al., APL 94, 111107 (2009)

FIG. 1. (Color online) The left panel illustrates a conventional DWELL structure where a SK QD, consisting of pyramidal shape QD resting on a wetting layer, is embedded in a QW structure. The right panel show two stacks of SML QDs embedded in a QW.

$$N_s = 10^{11} - 10^{12} \text{ cm}^{-2}, N_0 = 10^{16} - 10^{17} \text{ cm}^{-3},$$

$$\sqrt{\epsilon} \approx 3.5, d \sim 30 \text{ Debye}, \lambda_{12} \approx 1 \mu\text{m}$$

number of layers 10-30

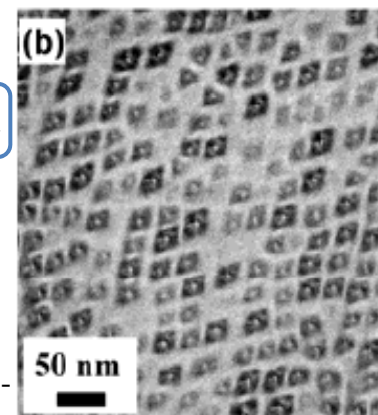
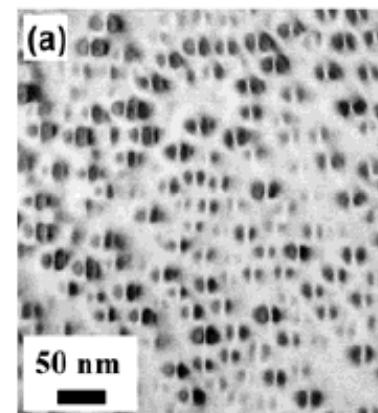
$$\omega_c \sim 3 \cdot 10^{12} - 10^{13} \text{ c}^{-1}, L_c \sim 10 - 30 \mu\text{m}$$

necessary condition of SR lasing

$$\omega_c^2 T_2^* T_E > 1$$

$$T_2 \sim 1 - 10 \text{ ps} \gg T_2^* \sim 25 \text{ fs}, T_1 \sim 0.03 - 1 \text{ ns}$$

$$B \approx 0.1 - 1 \text{ mm}, R = 0.1 - 0.3, T_E \approx 0.5 - 5 \text{ ps}$$



Quasi-monochromatic mode generation in sub-monolayer quantum-dot hetero-lasers was shown about 10 years ago (S.A. Blokhin et al., T.D. Germann et al.)

Superfluorescence in magnetized quantum wells

PRL 96 (2006) 237401, PRB 81 (2010) 155314

Cooperative Recombination of a Quantized High-Density Electron-Hole Plasma

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³*National High Magnetic Field Laboratory, Florida State University, Tallahassee, FL 32310*

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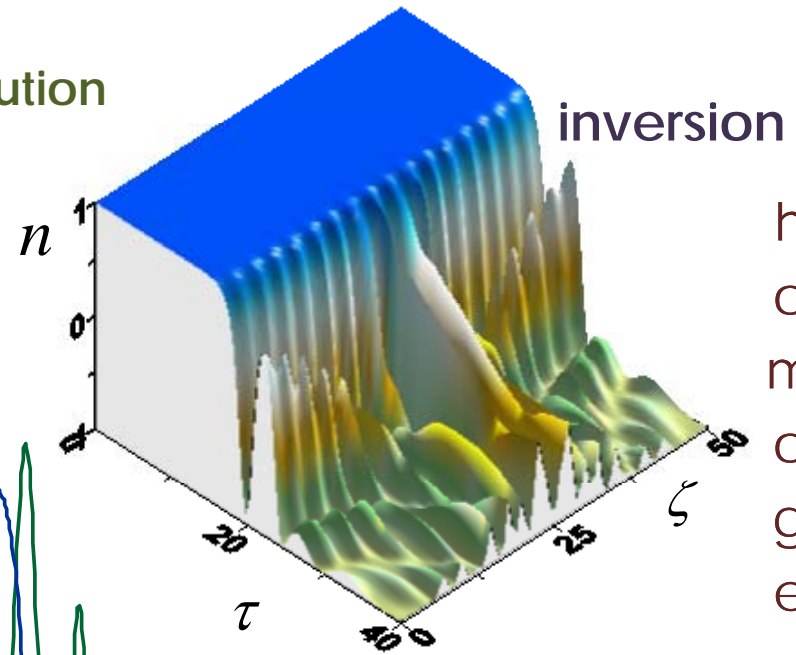
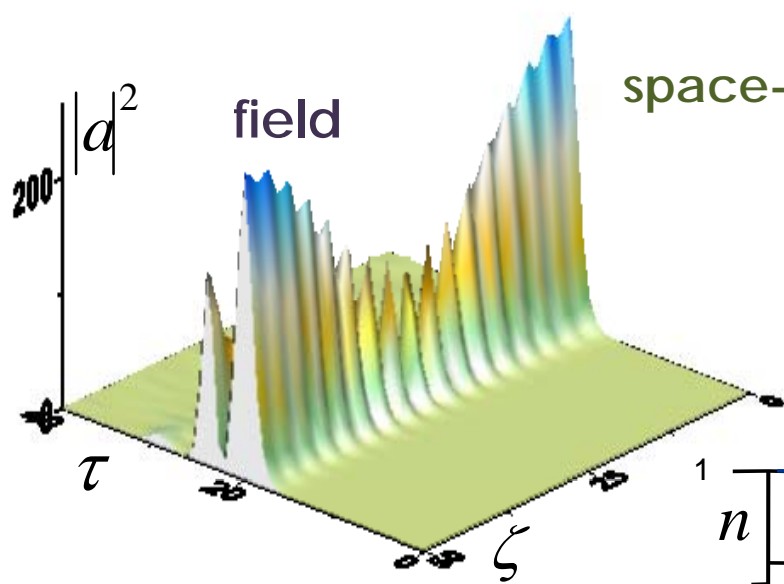
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We investigate photoluminescence from a high-density electron-hole plasma in semiconductor quantum wells created via intense femtosecond excitation in a strong perpendicular magnetic field, a fully-quantized and tunable system. At a critical magnetic field strength and excitation fluence, we observe a clear transition in the band-edge photoluminescence from omnidirectional output to a randomly directed but highly collimated beam. In addition, changes in the linewidth, carrier density, and magnetic field scaling of the PL spectral features correlate precisely with the onset of random directionality, indicative of cooperative recombination from a high density population of free carriers in a semiconductor environment.

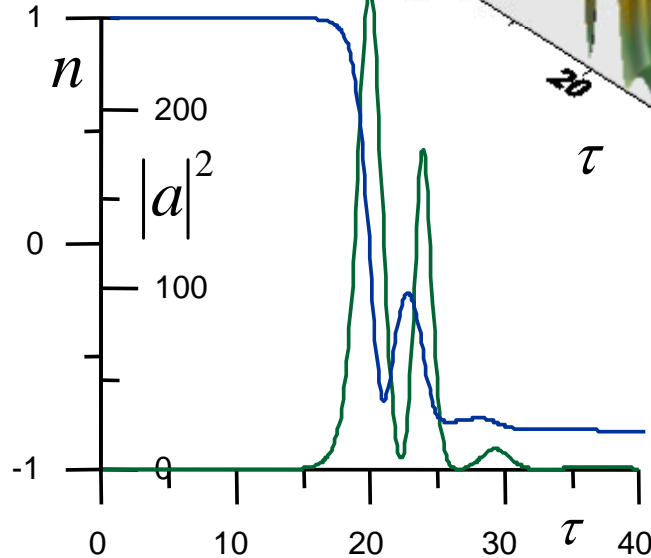
Superfluorescence of a long active sample

Numerical simulation of complete Maxwell-Bloch equations



$$B \leq B_c \ln |R|^{-1} / \sqrt{n_p},$$

$$i.e., T_E \leq 1 / (\omega_c \sqrt{n_p}).$$



$$B = 8\lambda$$

$$I = 0.01$$

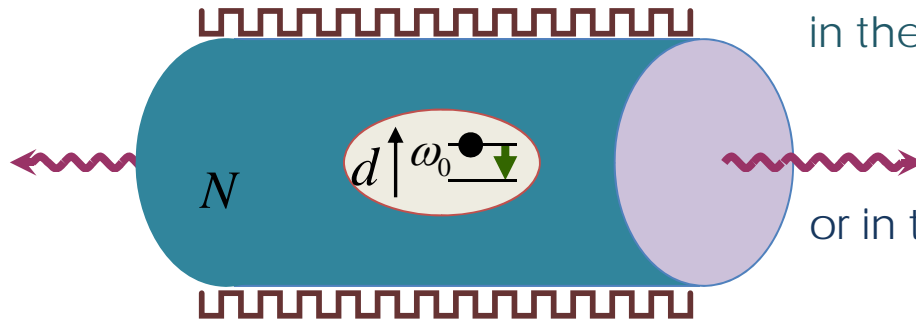
$$p_0 \approx 10^{-6}$$

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Superradiant pulses in class D lasers

cooperative frequency

$$\omega_c = \left(2\pi d^2 \omega_0 N / \hbar \right)^{1/2}$$



necessary conditions of superradiance in an active media

$$\omega_c^2 T_2^2, \omega_c^2 T_2 T_2^* \gg 1$$

Mode superradiance requires:
in the case of homogeneous broadening

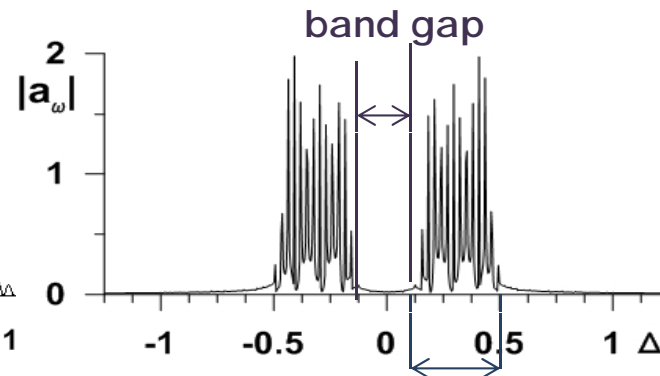
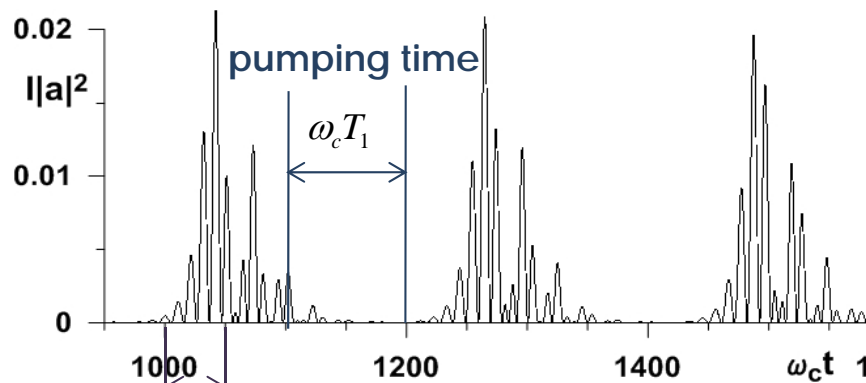
$$\frac{1}{T_2} > \frac{1}{T_2^*}$$

$$\omega_c^2 T_2 T_E > 1$$

or in the case of inhomogeneous broadening

$$\frac{1}{T_2} \ll \frac{1}{T_2^*}$$

$$\omega_c^2 T_2^* T_E > 1$$



$\omega_c T_2$ polarization lifetime

$$T_2 \gg T_E > 1 / \omega_c^2 T_2^*$$

$1 / \omega_c T_E$ photon lifetime

Dimensionless equations for the field, polarization and inversion in a 1D active sample with distributed feedback

approximation of two counter-propagating waves

field $E = \text{Re} \left[x_0 \left(A_+(z,t) e^{ikz} + A_-(z,t) e^{-ikz} \right) e^{-i\omega_0 t} \right],$

polarization $P = \text{Re} \left[x_0 \left(P_+(z,t,\Delta) e^{ikz} + P_-(z,t,\Delta) e^{-ikz} \right) e^{-i\omega_0 t} \right],$

inversion $N / (N_0 f(\Delta)) = n(\Delta) + \text{Im} \left[n_z(\Delta) e^{2ikz} \right]$

dielectric constant

$$\varepsilon = \bar{\varepsilon} \text{Re} [1 + 4\bar{\beta} \exp(2iK\zeta)]$$

band gap width, $\beta = \bar{\beta} / \sqrt{I}$

dimensionless amplitude of the modulation of permittivity, and coupling factor for the counter-propagating waves

$$\left[\frac{\partial}{\partial \tau} \pm \frac{\partial}{\partial \zeta} \right] a_{\pm} = i\beta a_{\mp} + i \int_{-2\Delta_0}^{2\Delta_0} p_{\pm}(\Delta) f(\Delta) d\Delta,$$

$$\left[\frac{\partial}{\partial \tau} + \Gamma_2 + i\Delta \right] p_{\pm}(\Delta) = -\sqrt{I} \left(in(\Delta) a_{\pm} \pm \frac{n_z^{1,*}(\Delta)}{2} a_{\mp} \right),$$

Initial conditions:

$$n|_{\tau=0} = 1, \quad p_{\pm}|_{\tau=0} = p_0$$

$$n_z|_{\tau=0} = 0, \quad a_{\pm}|_{\tau=0} = a_0$$

$$\left[\frac{\partial}{\partial \tau} + \Gamma_1 \right] (n(\Delta) - n_p) = -\sqrt{I} \text{Im} (a_+ p_+^*(\Delta) + a_- p_-^*(\Delta)),$$

boundary conditions:

$$a_+(\tau, -L/2) = R a_-(\tau, -L/2)$$

$$a_-(\tau, L/2) = R a_+(\tau, L/2)$$

$$\left[\frac{\partial}{\partial \tau} + \Gamma_1 \right] n_z(\Delta) = \sqrt{I} (a_-^* p_+(\Delta) - a_+ p_-^*(\Delta)).$$

normalized time, coordinate and length of a sample $\tau = t\omega_c$, $\zeta = z\omega_c \sqrt{\varepsilon} / c$, $L = B\omega_c \sqrt{\varepsilon} / c$

$\Delta = \frac{\omega - \omega_0}{\omega_c}$ a frequency variable along an inhomogeneously broadened spectral line

$p_{\pm} = P_{\pm} / (dN_0 f(\Delta))$, $a_{\pm} = A_{\pm} \bar{\varepsilon} / 2\pi dN_0$, polarization, field inversion

$$K = \frac{2\pi c}{\omega_c \lambda \bar{\varepsilon}}, \quad \Gamma_{1,2} = (T_{1,2} \omega_c)^{-1}$$

relaxation rates of inversion, polarization

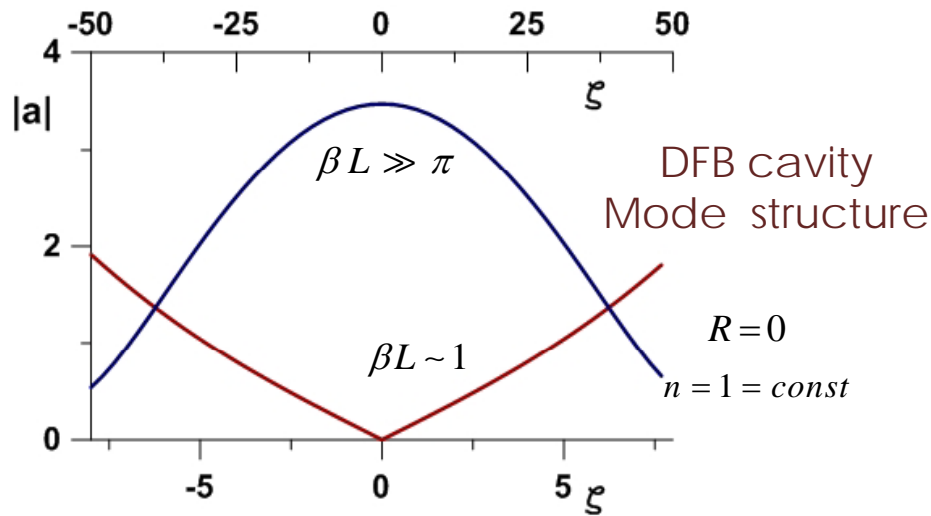
$$I = \frac{\omega_c^2}{\omega_0^2} \ll 1$$

Mode superradiance

$$a_{\pm} = \left(A'_{\pm} e^{i\kappa\zeta} + A''_{\pm} e^{-i\kappa\zeta} \right) e^{-i\Omega\tau}$$

Electromagnetic waves propagate along an active sample and reflect from the Bragg periodic structure with integral reflection factor

$$\sqrt{R} = \tanh(\beta L)$$



$$\beta^2 + \kappa^2 = \left(i\Sigma + \Omega + \frac{n}{\Omega + \Phi + i(\Delta_0 + \Gamma_2)} \right)^2$$

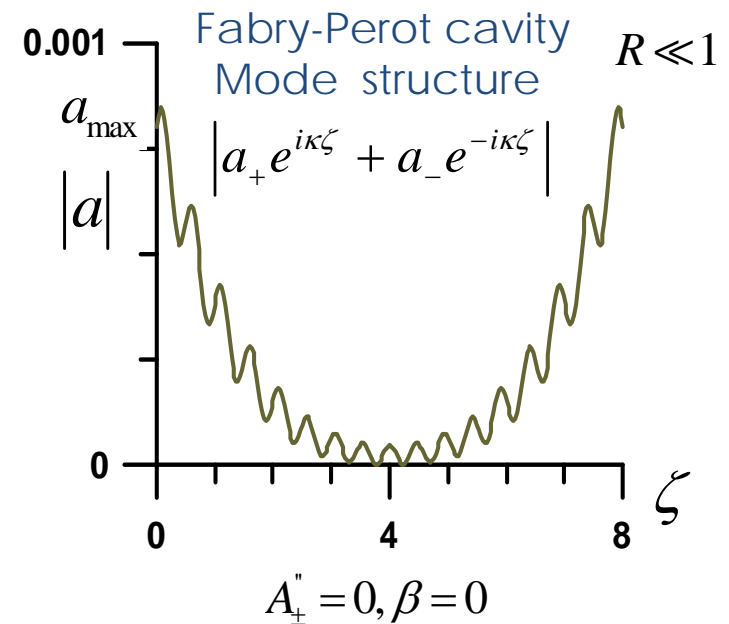
dispersion equation

$$\frac{(\sqrt{R_1} + \sqrt{R_2})\beta}{1 + \sqrt{R_1 R_2}} \pm \sqrt{\kappa^2 + \beta^2} + \frac{\kappa(1 - \sqrt{R_1 R_2})}{1 + \sqrt{R_1 R_2}} (1 + e^{2i\kappa L}) / (1 - e^{2i\kappa L}) = 0$$

characteristic equation

Joint solving of the dispersion and characteristic equations for an active sample with inhomogeneous broadening shows that it is possible an excitation of electromagnetic modes with growth rate exceeding the homogeneous broadening, though less than the intermode frequency spacing.

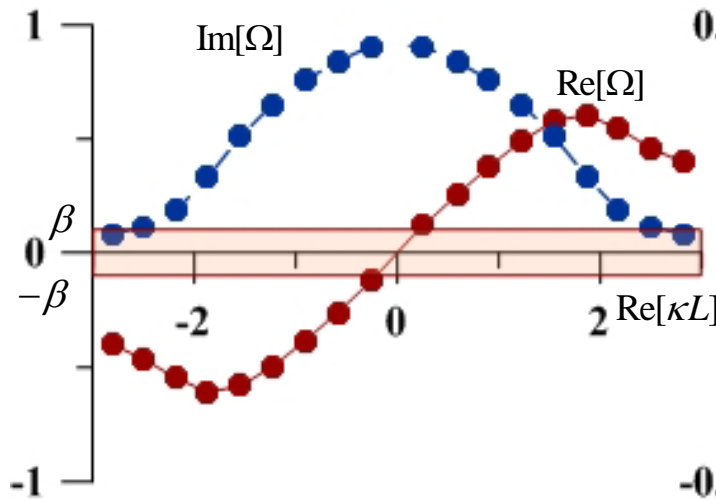
It is individual mode superfluorescence.



Modes of a DFB active sample

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m o
o a
g d
e e
n n
e i
o n
g u
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$$\omega_c \gg 1/T_2$$

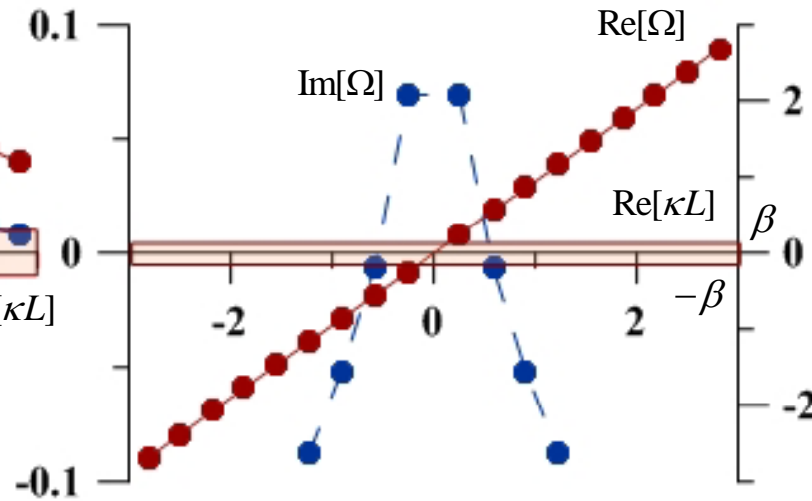


Maximal growth rate

$$\text{Im}[\omega] = \omega_c^2 \cdot T_f - 1/T_2$$

$$L = 10, \beta = 0.1, \beta L = 1, \Gamma_2 = 0.02, \Delta_0 = 0$$

$$\omega_c \ll 1/T_2^*$$



Maximal growth rate

$$\text{Im}[\omega] = \omega_c^2 \cdot T_2^* - 1/T_f$$

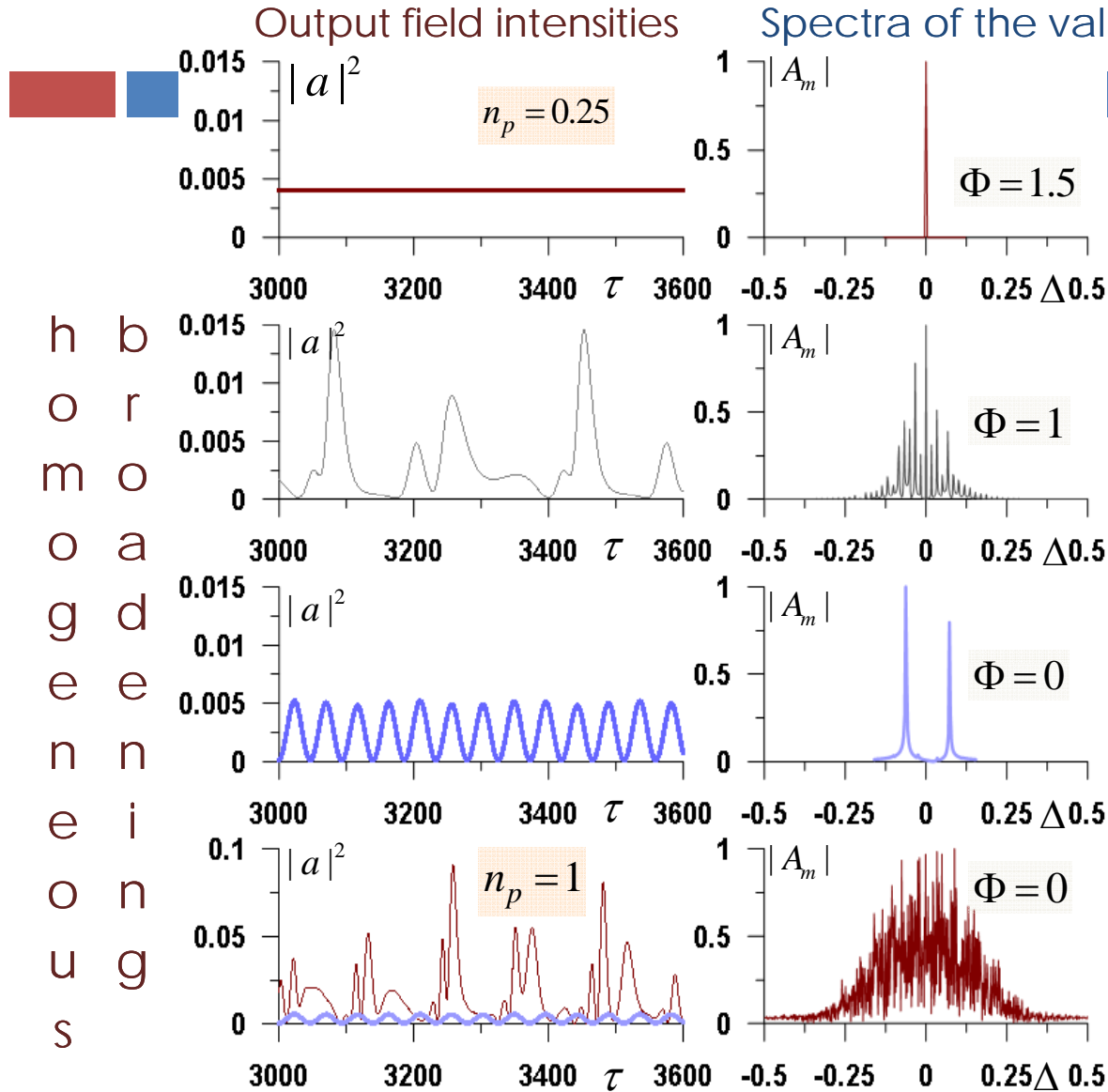
$$L = 10, \beta = 0.1, \beta L = 1, \Gamma_2 = 0.02, \Delta_0 = 4$$

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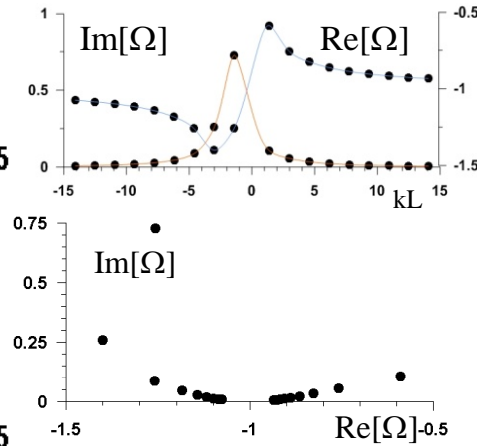
$$(\omega_m - \omega_{21})T_2 = n\omega_c^2 T_2 T_E x_m / (1 + x_m^2) + i \left[n\omega_c^2 T_2 T_E / (1 + x_m^2) - 1 \right]$$

$$x_m = (m - m_0 + \delta) \pi / \ln |R|^{-1}, \quad \ln |R|^{-1} \rightarrow \ln \left| R + \beta L / 2 (x_m + i) \ln |R|^{-1} \right|^{-1}$$

One and a few polariton mode lasing



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Normalized detuning between the transition frequency and the Bragg resonance for distributed feedback

$$\Phi = \left(\omega_0 - 2\pi c / \lambda \sqrt{\epsilon} \right) / \omega_c$$

Spontaneous (nonlinear) symmetry breaking of the hot polariton modes

Lasing (1st and 2nd) and superradiant lasing thresholds are of the same order

$$L = 2, R = 0, \beta L = 2, \Delta_0 = 0.002$$

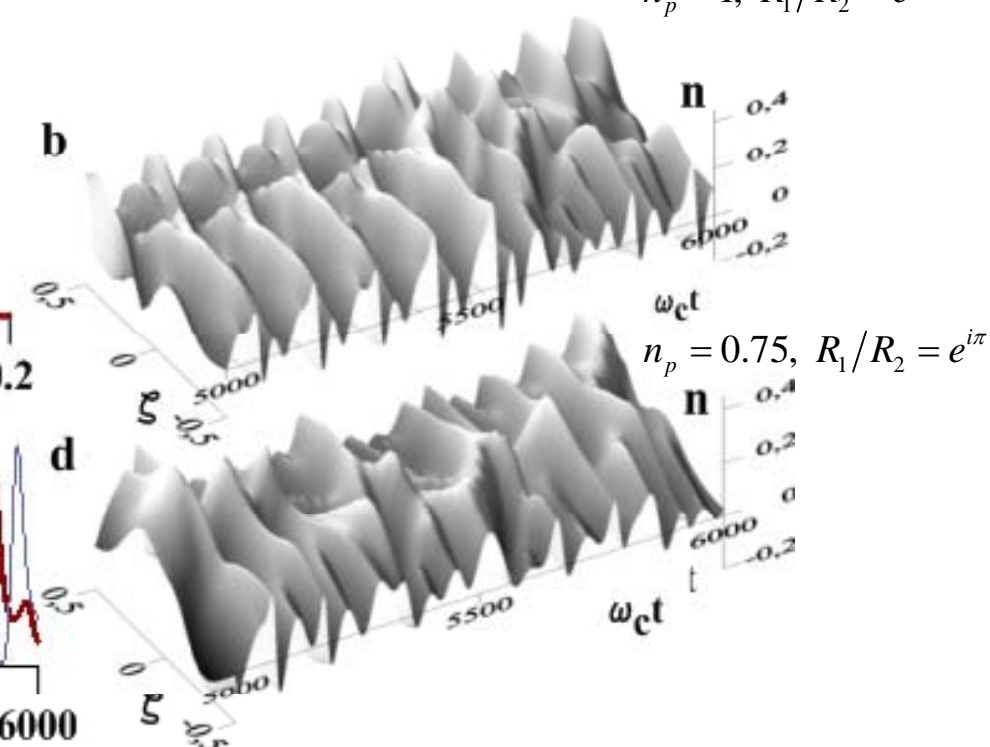
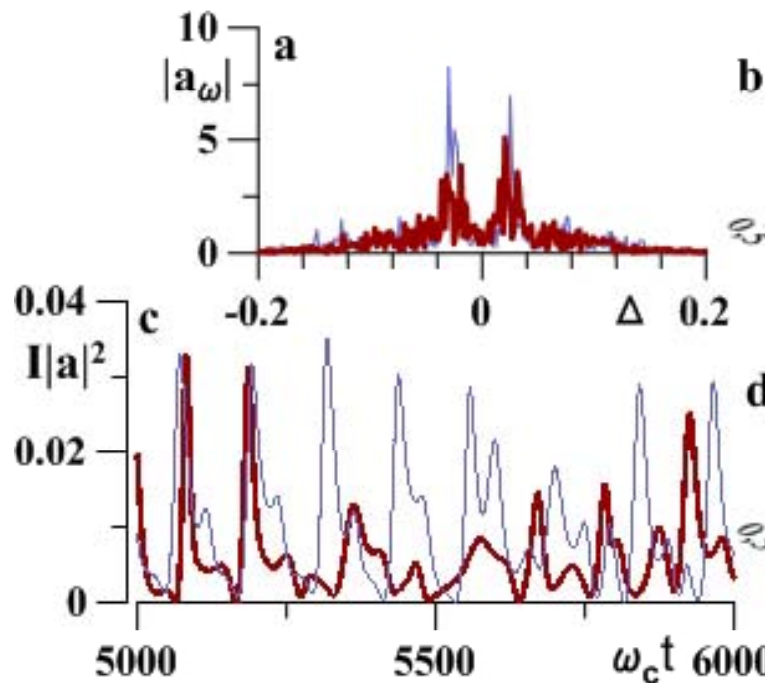
$$\Gamma_1 \approx 0.01, \Gamma_2 \approx 0.02$$

From quasi-stationary to superradiant regime

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Self-modulation and coherent pulsation

$$n_p = 1, R_1/R_2 = e^{i0.8\pi}$$



$$n_p = 0.75, R_1/R_2 = e^{i\pi}$$

$$I|a|^2 = \left(\frac{\omega_R}{\omega_c} \right)^2 \quad \omega_R = dE/\hbar \text{ Rabi frequency}$$

$$L=1, \Delta_0=4, I=25 \cdot 10^{-6}, b=0 \\ \Gamma_1=0.01, \Gamma_2=0.02, |R|=0.25$$

Lasing thresholds and instability estimates

Maximum intensity of SR pulse is achieved for

$$\beta L \sim 1$$

Facet reflections $\sqrt{R_{1,2}} \sim 0.1$ could favor mode selection.

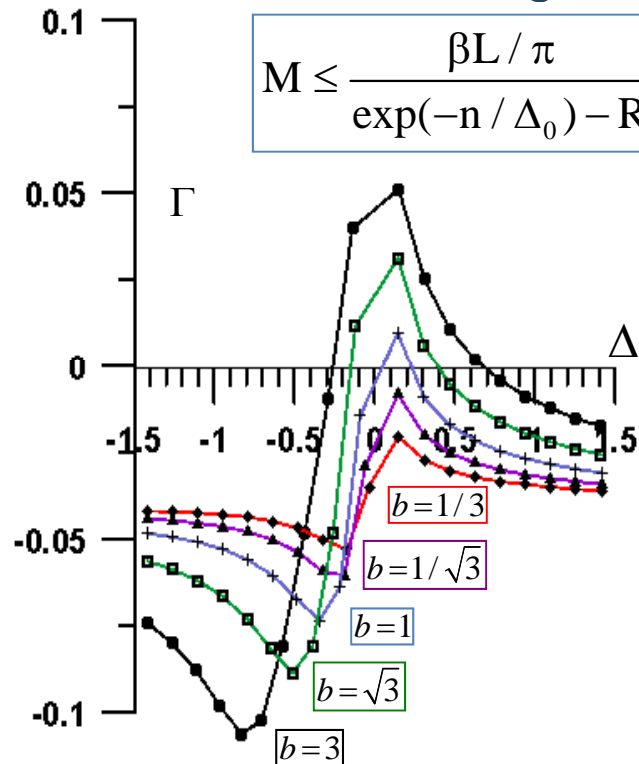
number of modes in lasing

$$M \leq \frac{\beta L / \pi}{\exp(-n / \Delta_0) - R}$$

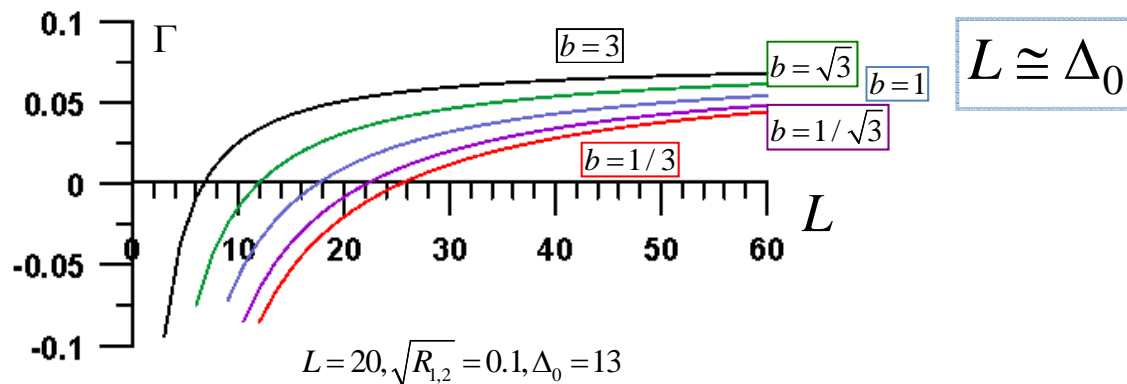
when the integral reflection factor is of the order of unity

estimate of the growth rate of central modes

$$\Gamma \cong \frac{n}{\Delta_0} - \frac{\ln(R + \beta L / 2\pi m)^{-1}}{L}$$



threshold length of an active medium is defined by the effective cooperative length



Efficient superradiant lasing takes place if the (photonic) band gap is not less than the so-called active cooperative frequency of the lasing medium and both these quantities are in between the values of homogeneous and

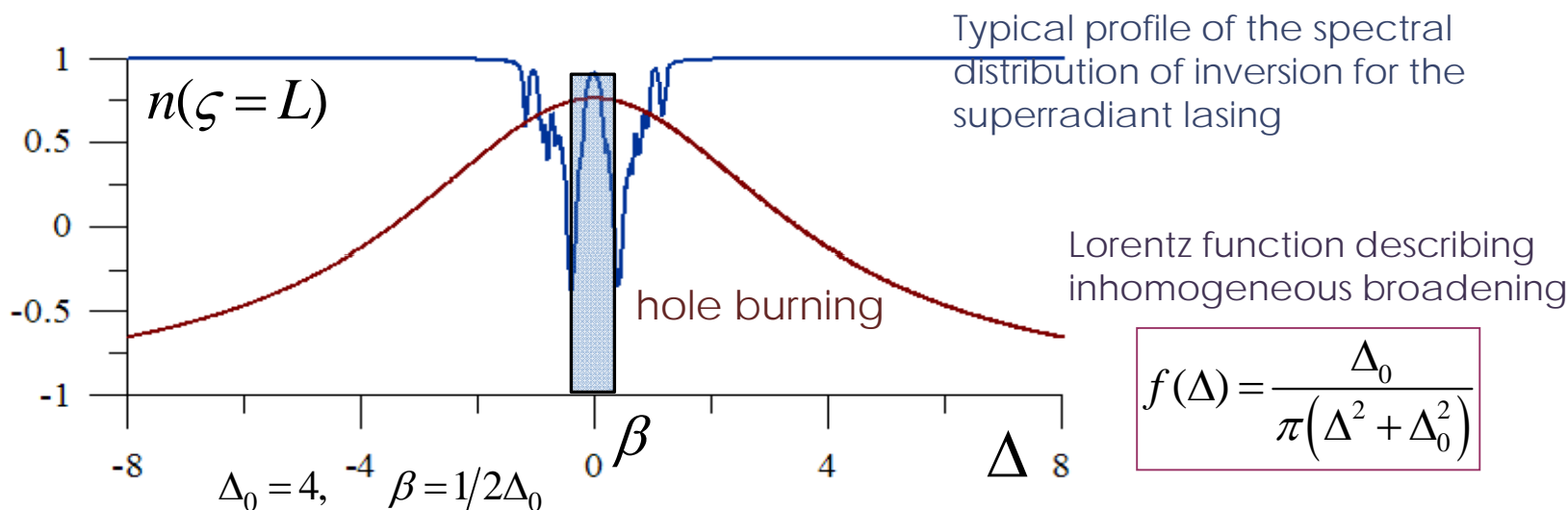
inhomogeneous broadening

(all values are normalized by means of ω_c):

$$\Delta_0 = \frac{1}{T_2^* \omega_c} \gg 1 \geq 2\beta = \frac{2\omega_0 \bar{\beta}}{\omega_c} \geq \frac{1}{\Delta_0} \geq \Gamma_2 = \frac{1}{T_2 \omega_c}$$

$$\bar{\nu}_c = \omega_c / \Delta_0, \bar{L}_c = c / (\bar{\nu}_c \sqrt{\epsilon})$$

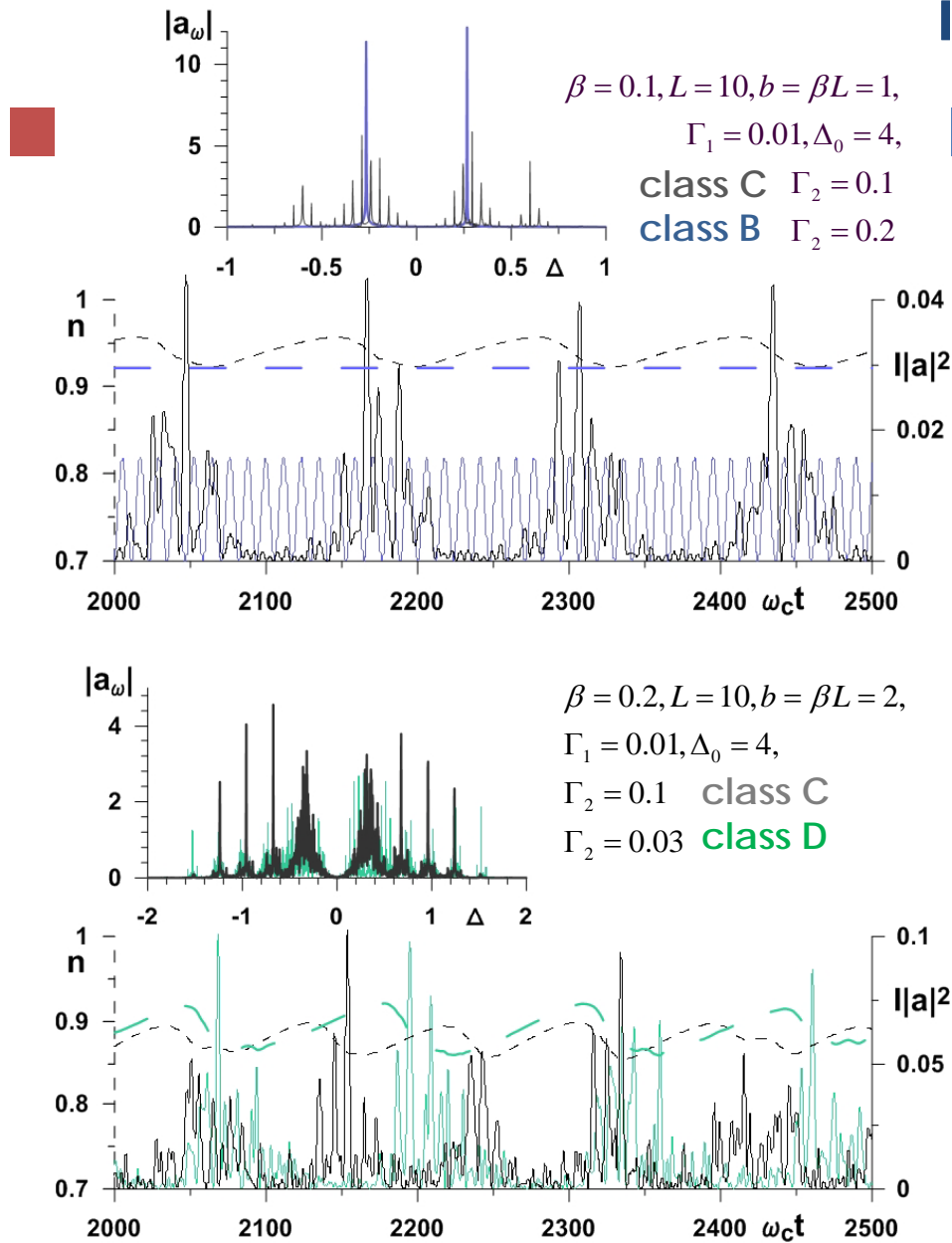
The effective cooperative length, \bar{L}_c , is defined by an active cooperative frequency, $\bar{\nu}_c$, and gives the lower limit of the required cavity length.



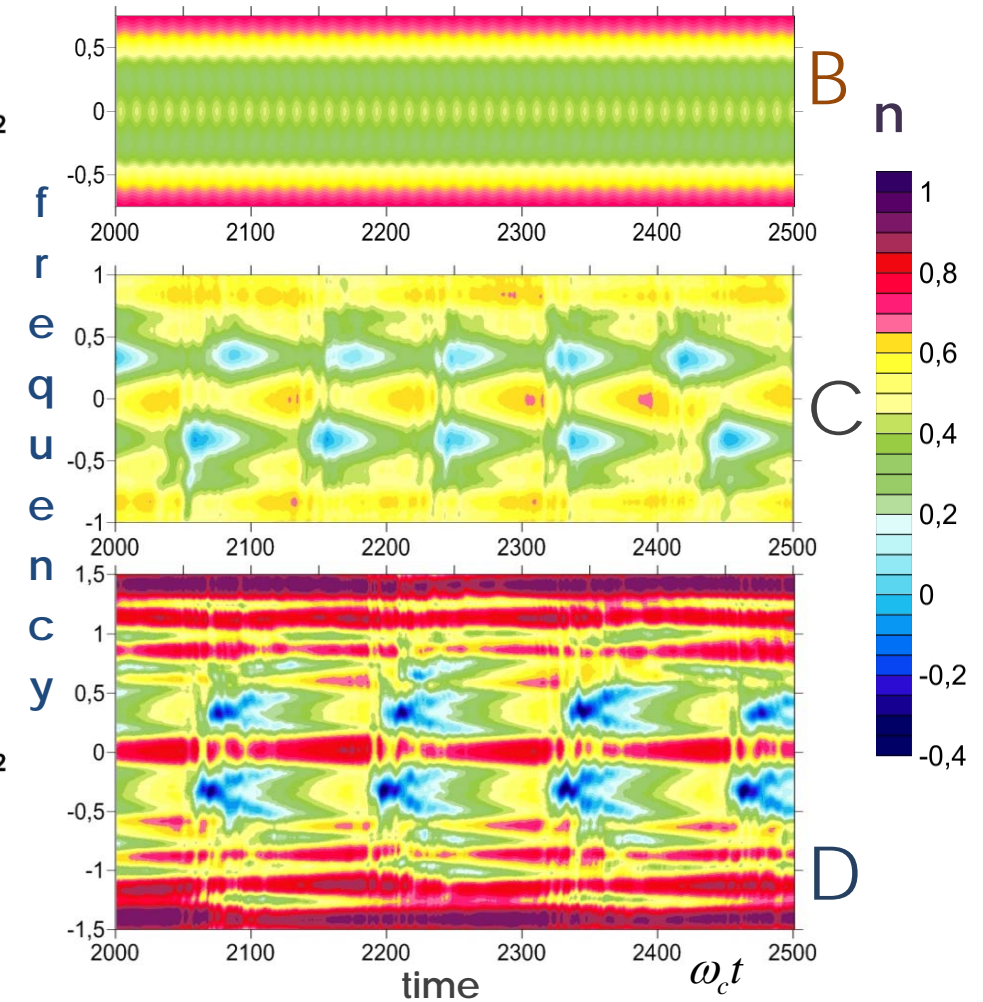
l b
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$R = 0$

Comparison of generation patterns in lasers of various classes



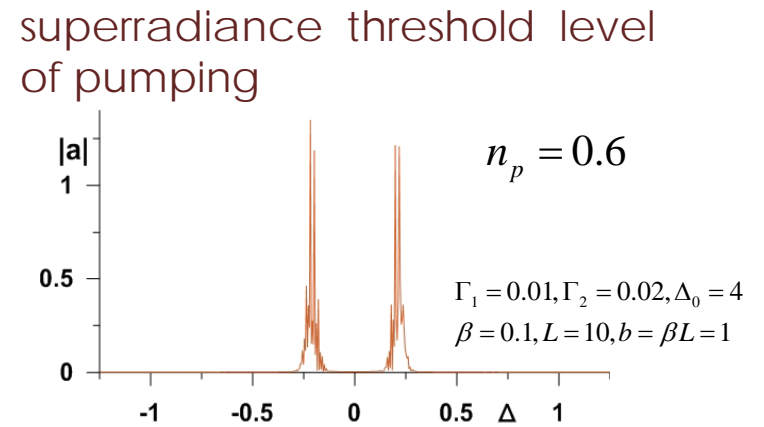
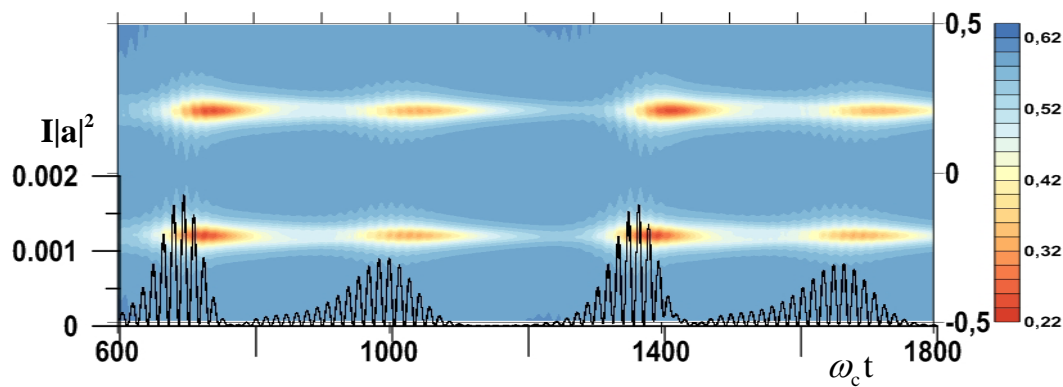
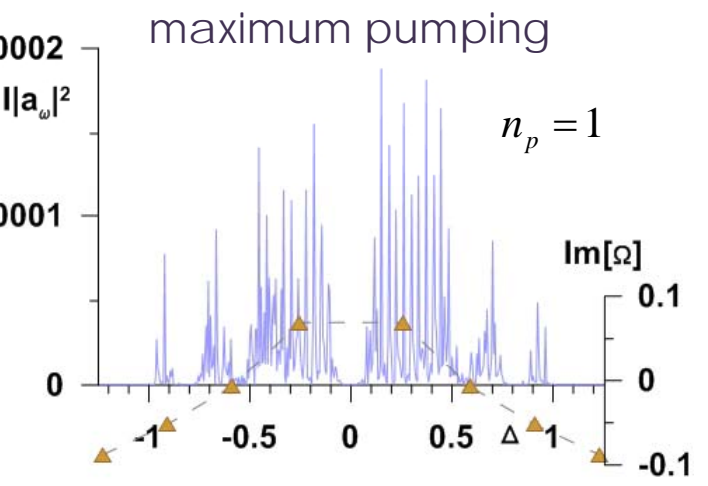
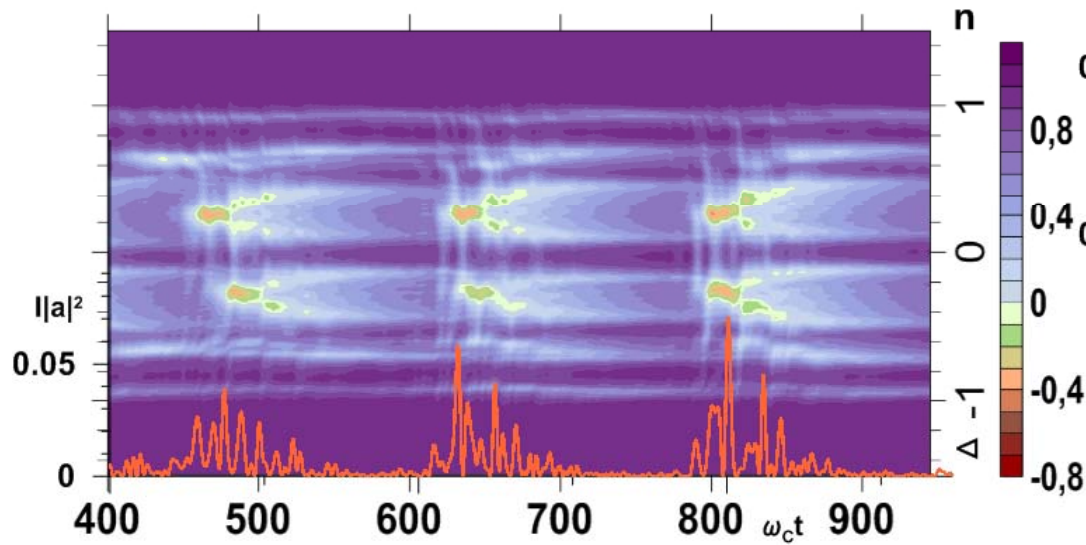
dynamical spectra of inversion at the sample edge



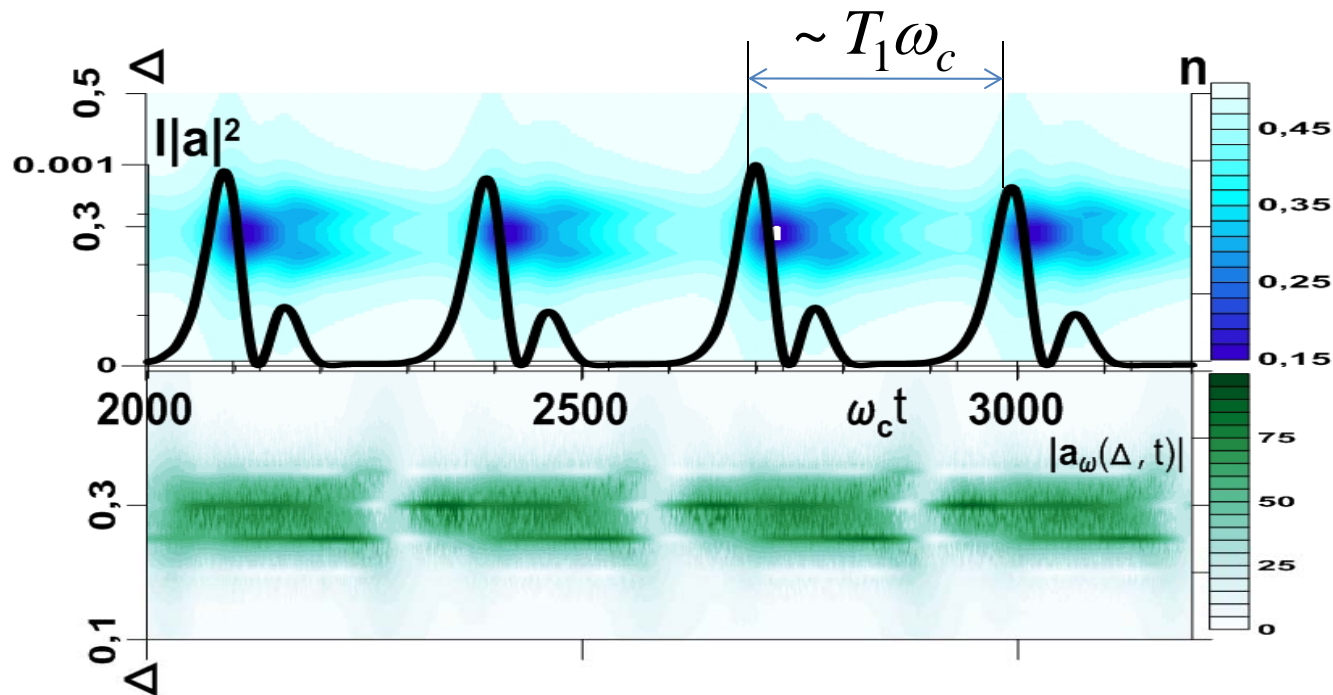
Sequence of bunches of the mode superradiance pulses

Dynamical spectra and oscillogramms

Spectra



Single mode superradiant lasing

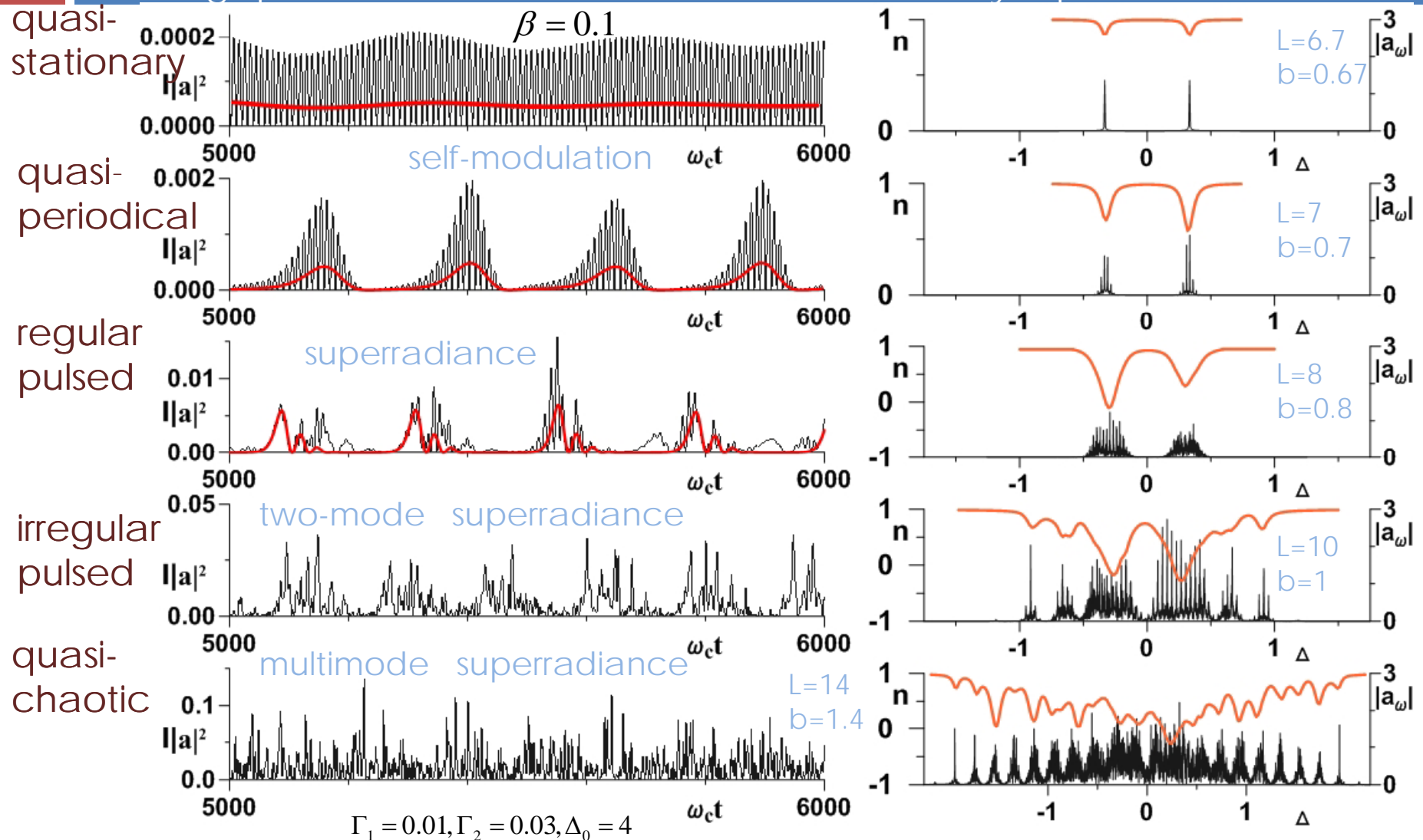


Combined distributed feedback Fabry-Perot laser in the case of an active medium with a strong inhomogeneous broadening. Holes burned in the dynamical spectrum of the inversion (*blue*) correspond to peaks in the dynamical spectrum of the field (*green*) and peaks in the oscillogram of intensity of the output field (*black thick line*). All values are dimensionless, including the frequencies of the field harmonics and active centers, Δ .

Typical regimes of generation and spectral properties of emission for the DFB class D laser

$$b = \beta L$$

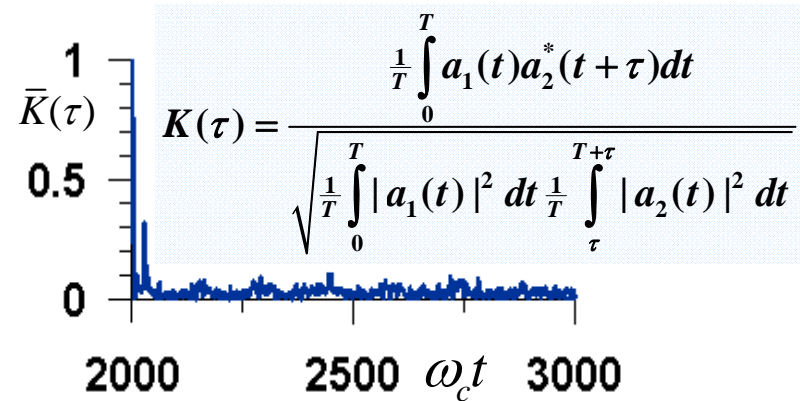
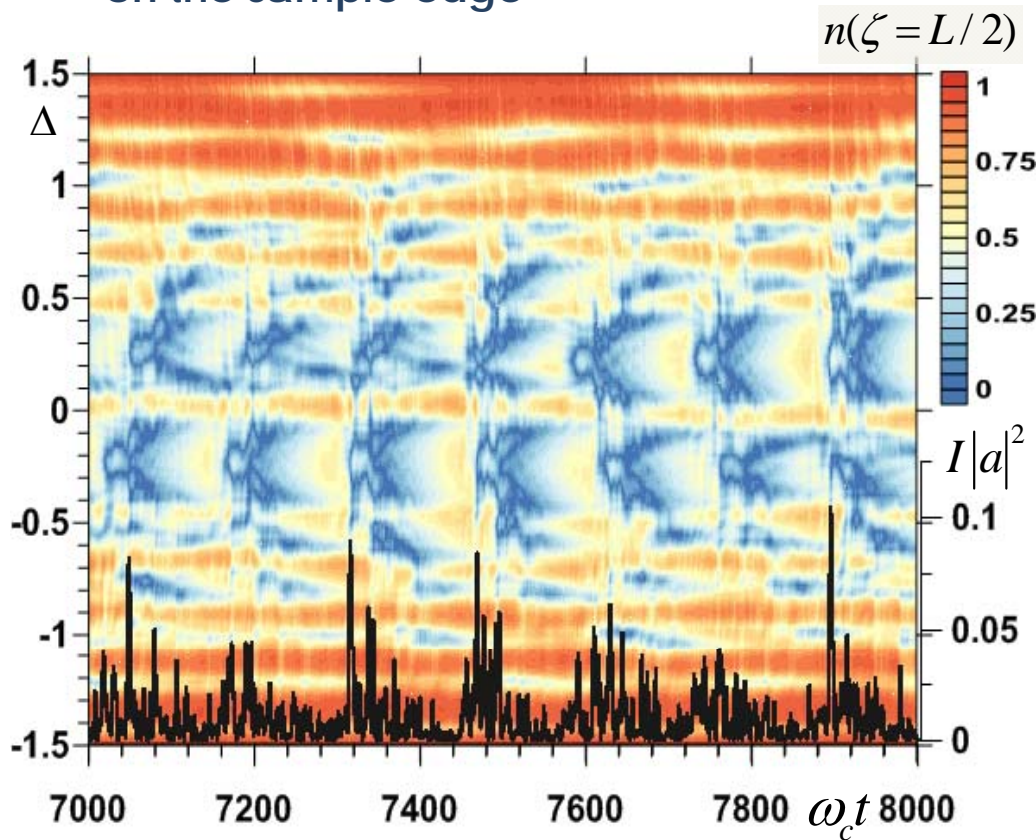
Band gap width is fixed. Q-factor is defined by a parameter



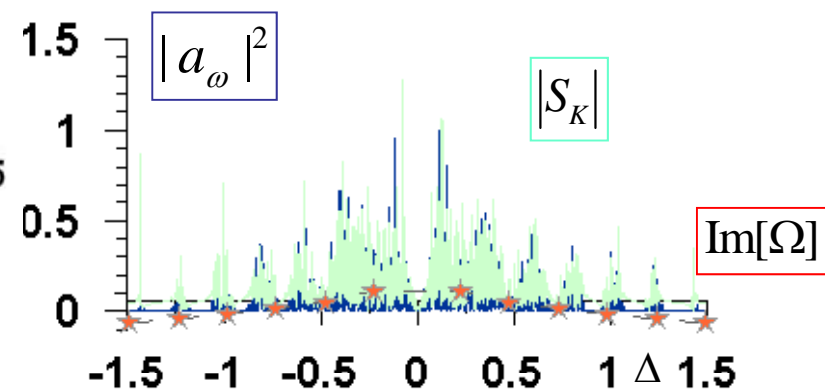
Spectral features of quasi-chaotic generation in the class D laser

correlation function

Dynamical spectrum of inversion on the sample edge



Normalized spectral power and spectrum of correlation function



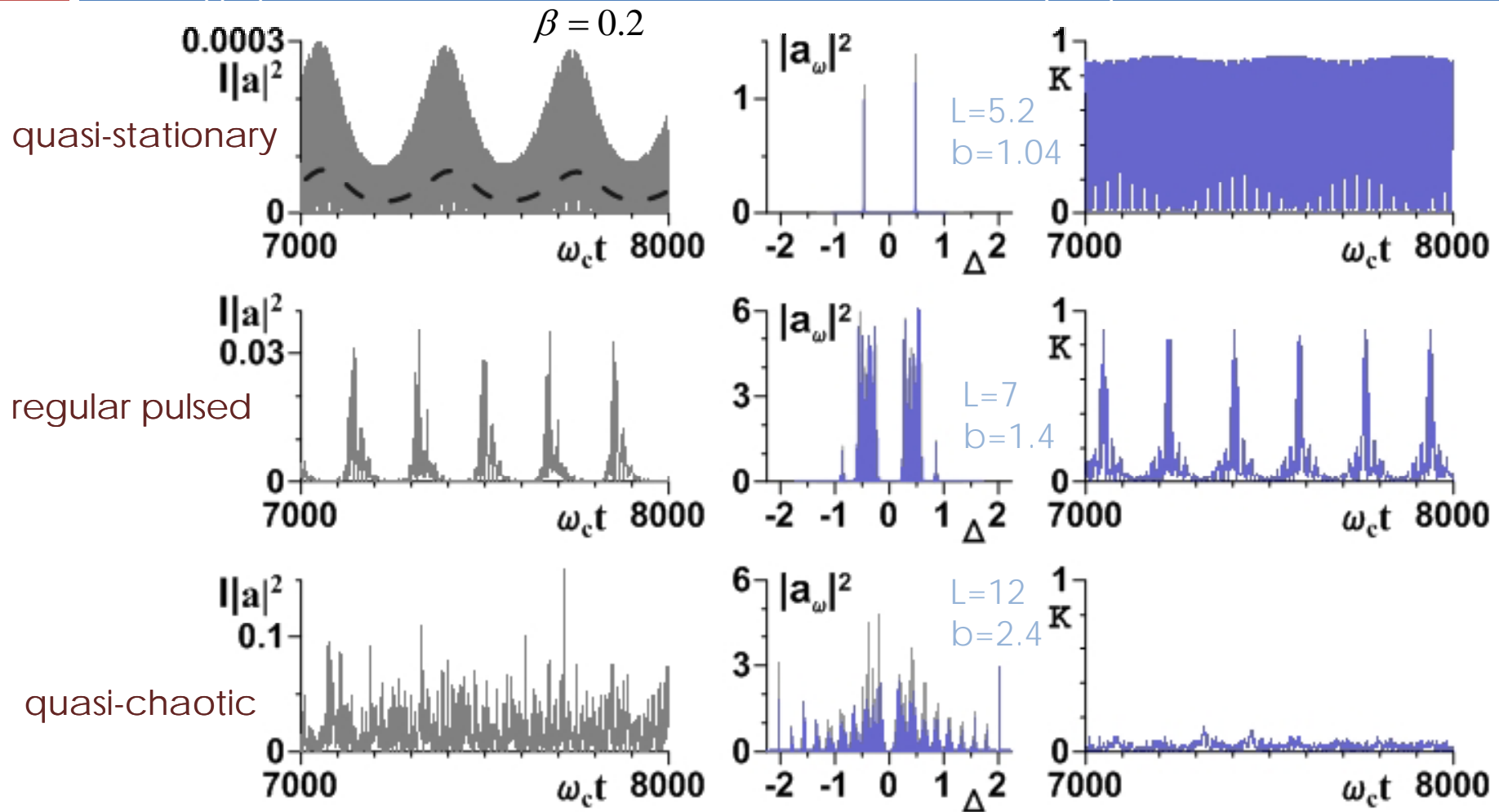
$$\Gamma_1 = 0.01, \Gamma_2 = 0.02, \Delta_0 = 4, I = 25 \cdot 10^{-6}$$

$$\beta = 0.1, L = 12, b = \beta L = 1.2$$

Typical regimes of generation and correlation properties of emission for the class D laser

$$b = \beta L$$

Band gap width is fixed. Q-factor is defined by a parameter



$$\Gamma_1 = 0.01, \Gamma_2 = 0.03, \Delta_0 = 4$$

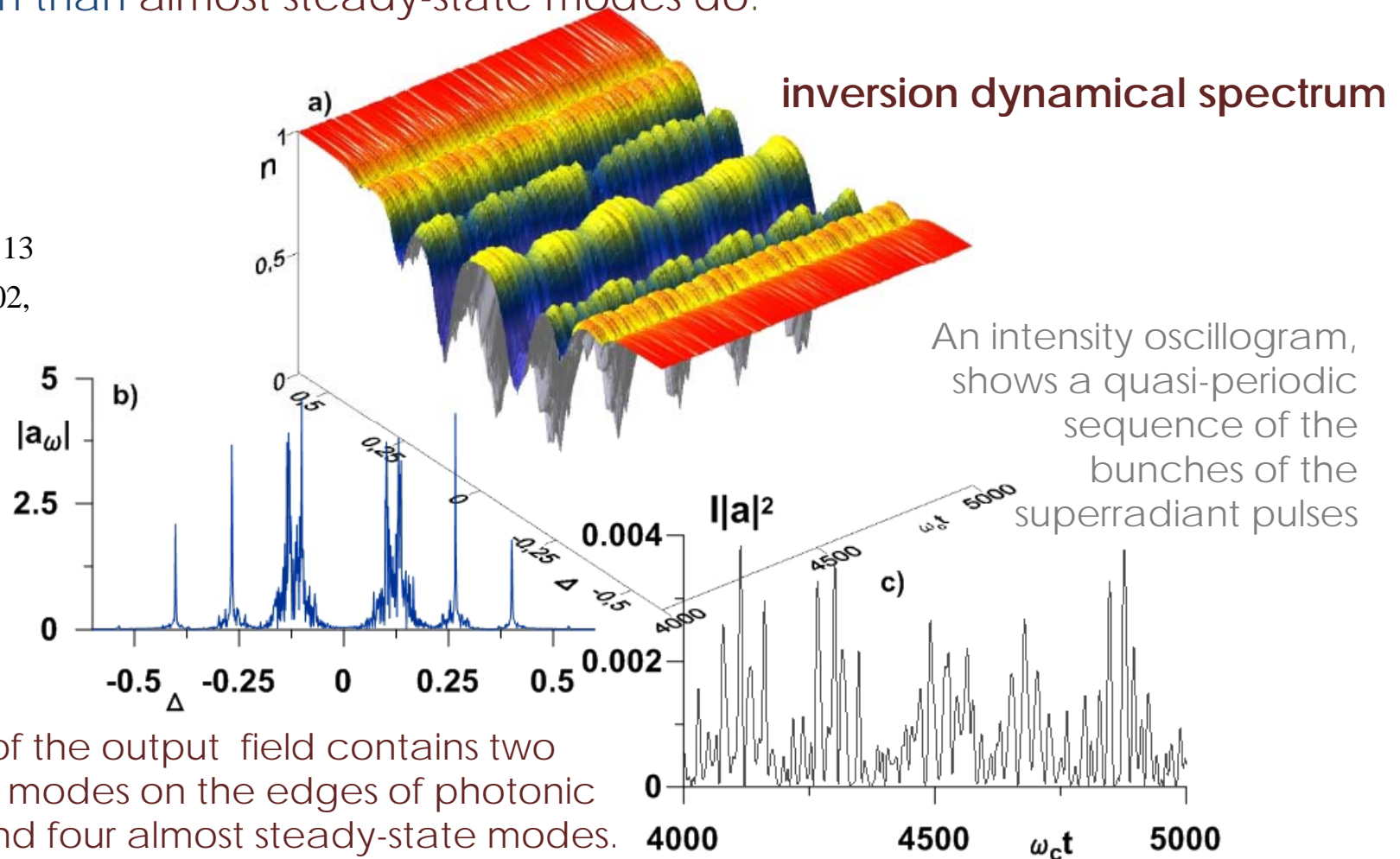
Multimode superradiant lasing in a combined distributed feedback Fabry-Perot cavity in the case of an active medium with a strong inhomogeneous broadening

Superradiant modes make much deeper holes in a dynamical spectrum of the inversion than almost steady-state modes do.

$$L = 20, b = 1, \Delta_0 = 13$$

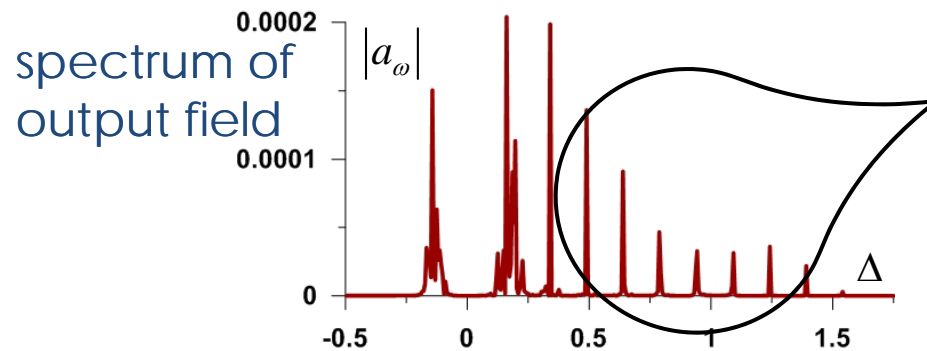
$$\Gamma_1 = 0.01, \Gamma_2 = 0.02,$$

$$R = 0.1e^{i\pi/2}$$



A spectrum of the output field contains two superradiant modes on the edges of photonic band gap and four almost steady-state modes.

Spontaneous self-mode-locking in a superradiant laser with a strong inhomogeneous broadening of an active medium in a combined distributed feedback Fabry-Perot cavity



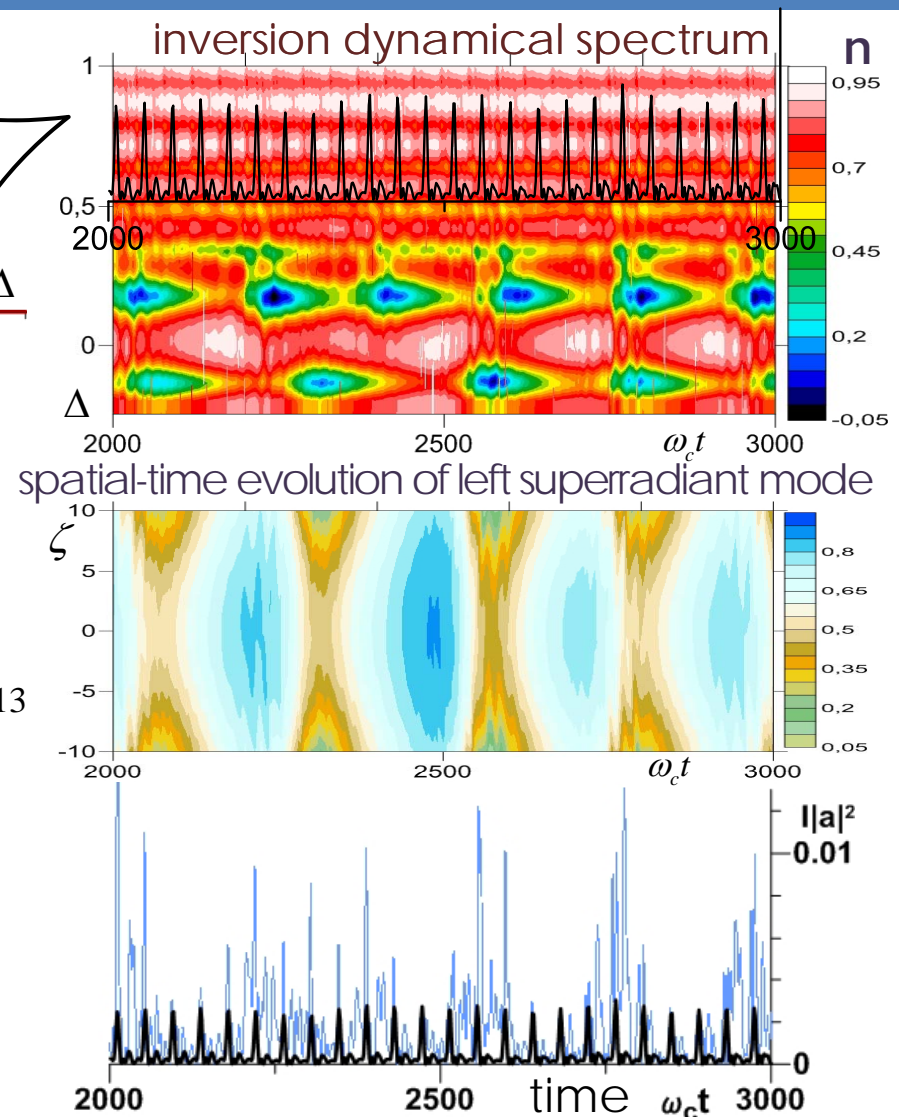
The deep holes of the bottom part of an inversion dynamical spectrum, caused by the superradiant pulses of the main two modes, play part of a saturable absorber and ensure synchronization of the other several steady-state modes.

$$L = 20, b = \sqrt{3}, \Delta_0 = 13$$

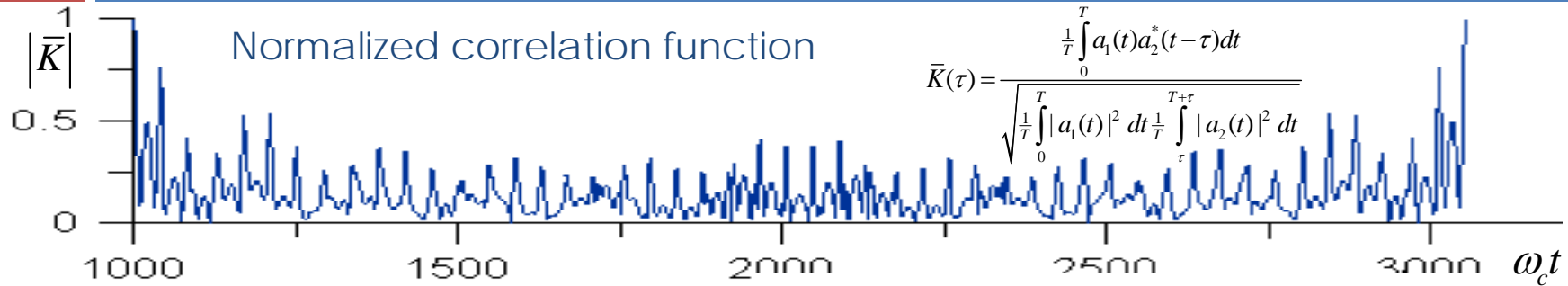
$$\Gamma_1 = 0.01, \Gamma_2 = 0.03,$$

$$R = 0.1$$

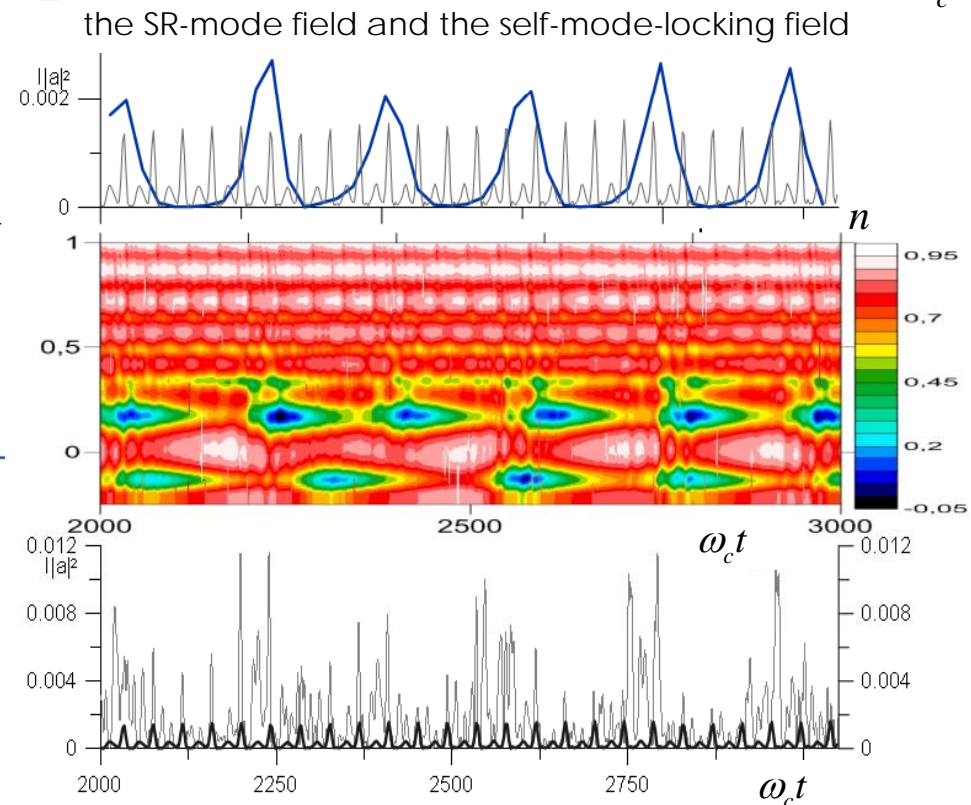
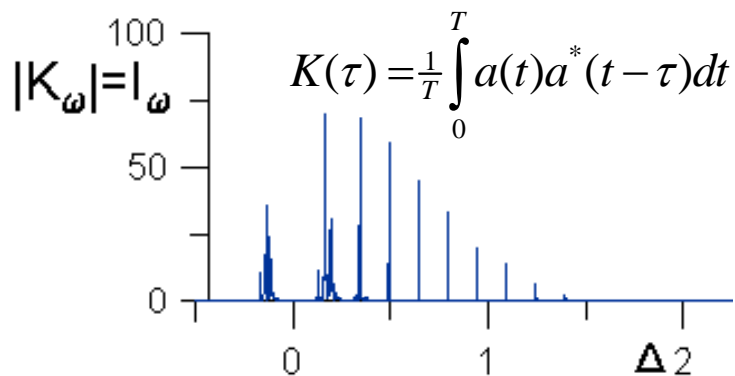
The synchronization of the several steady-state modes. produce an output field which is periodic and responsible for about 30% of the laser output power, according to an oscillogram .



Partial self-mode-locking. Correlation properties of superradiant lasing



Spectrum of the correlation function



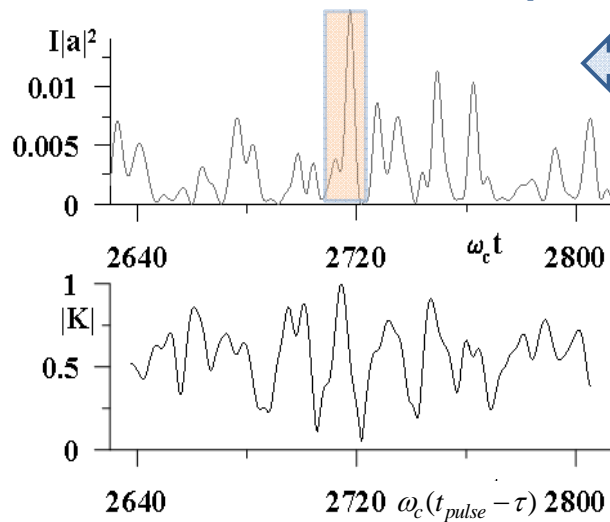
$$L = 20, b = \sqrt{3}, \Delta_0 = 13$$

$$\Gamma_1 = 0.01, \Gamma_2 = 0.03, R = 0.1$$

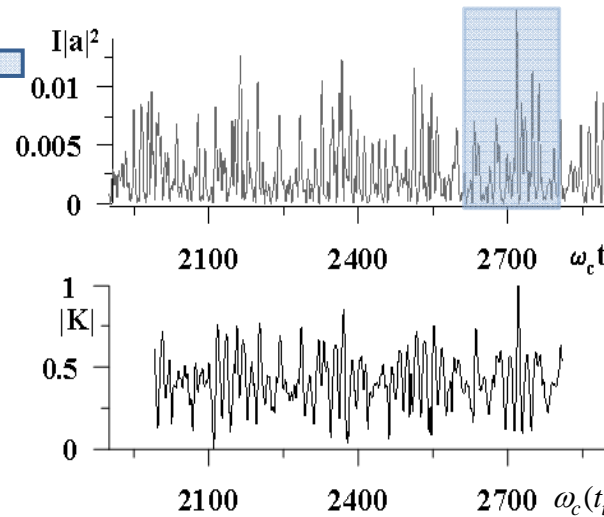
Correlation properties of superradiant lasing

$$K(\tau) = \frac{\int_0^T a_1(t) a_2^*(t-\tau) dt}{\sqrt{\int_0^T |a_1(t)|^2 dt \int_0^{T+\tau} |a_2(t)|^2 dt}}$$

Correlations with a SR-pulse

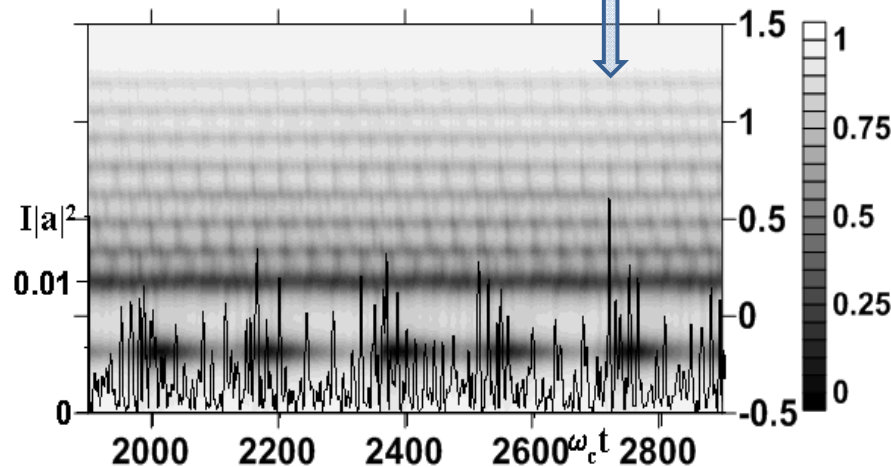


Correlations with a train of SR-pulses



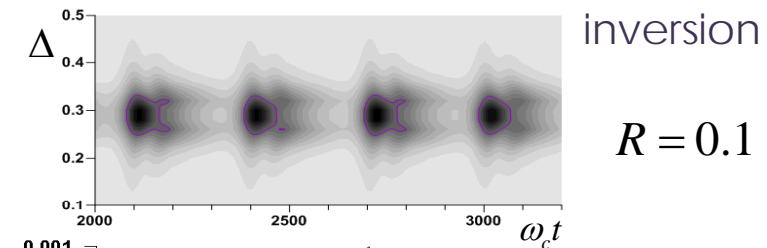
$$L=20, R=0.1, \Delta_0=13, b=\beta L=2.5$$

$$\Gamma_1=0.01, \Gamma_2=0.03, I=2.3 \cdot 10^{-6}$$

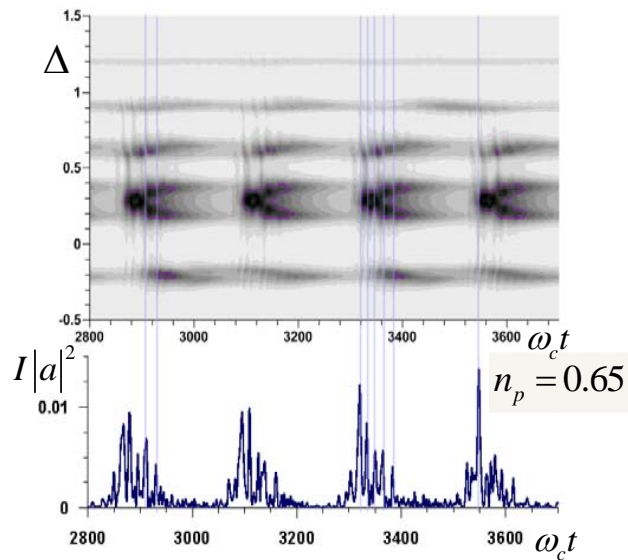
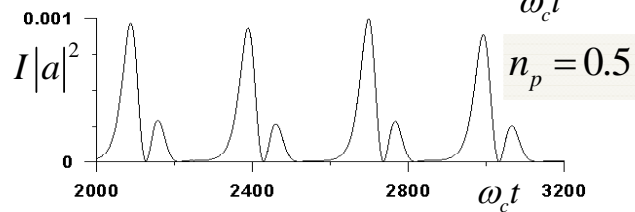


Partial self-mode-locking due to superradiance of active medium in a combined DFB Fabry-Perot cavity

Management of spectral-dynamical features of generation



$R = 0.1$

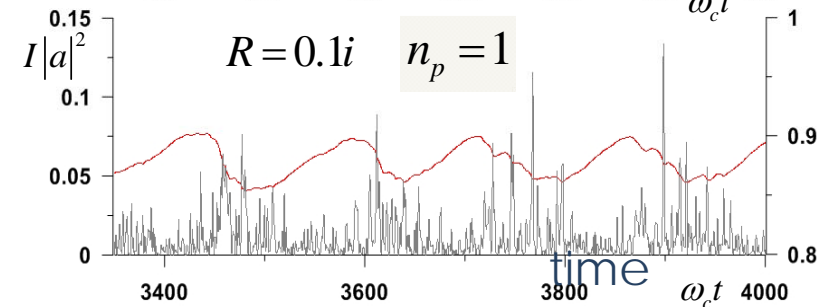
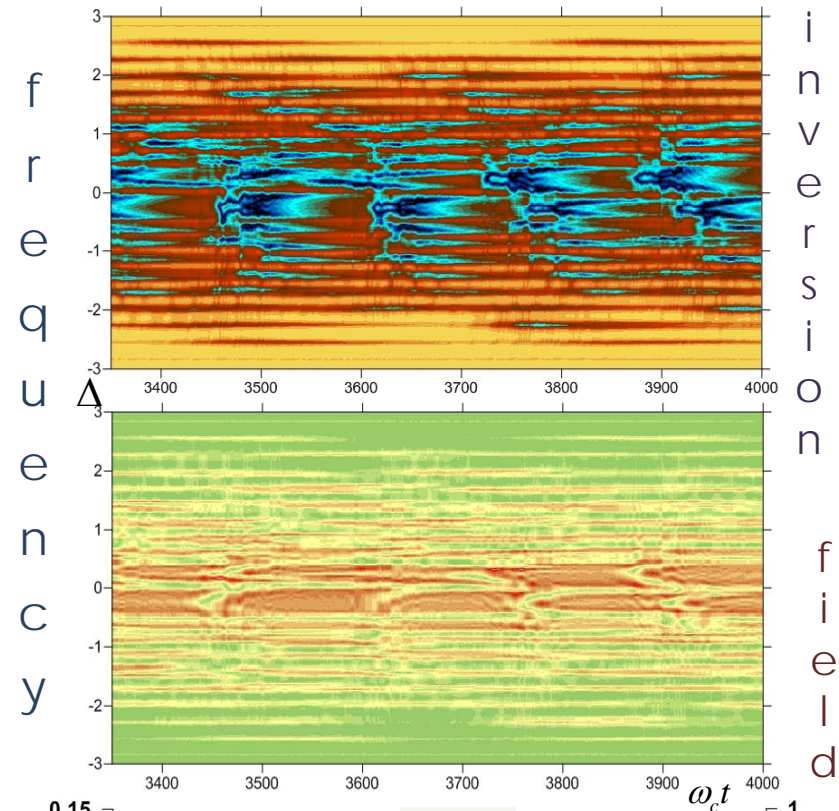


$n_p = 0.65$

time

$L=10, \Delta_0=4, b=\beta L=1.$

$\Gamma_1=0.01, \Gamma_2=0.03, I=25 \cdot 10^{-6}$



Conclusions

Variety of generation regimes and dynamical spectra of the DFB lasers with inhomogeneous broadening ($T_2^* \ll T_2, T_E$) are strongly enriched if the inequality $T_2 \ll T_E$ (the class B lasers) is changed to the opposite one, $T_2 \gg T_E$ (the class D lasers). We show that efficient mode selection near photonic band edges makes coherent superradiant lasing possible even in the case of strong inhomogeneous broadening, e.g., in the DFB sub-monolayer quantum-dot heterolasers. The transition from class B to class D lasers opens the door to the independent dynamical evolution of the active centers with different frequencies of working 2-level transitions and, hence, makes it possible complicated coherent phenomena like Dicke superradiance and Rabi oscillations under CW pumping.

In a typical regime of the class B lasers, there are two quasi-monochromatic modes (with utmost Q-factors) which burn deep wide holes in the inversion spectral profile and dominate over other modes. As a result, a multimode non-stationary (self-modulated) lasing becomes possible only under very strong pumping or in the case of very long cavity. The laser pulsations originate from a subtle nonlinear mode coupling, and the inversion never becomes negative or strongly modulated, even if the level of pumping greatly exceeds the laser threshold.

On the contrary, for the class D lasers (DFB ones in particular case) the steady-state lasing is almost impossible due to the superradiance phenomenon and Rabi oscillations. Typically, several modes are excited and demonstrate simultaneous or independent pulsed generation even in the case of short laser cavity at the pumping level on the order of the laser threshold value. In this case the hole burning proceeds in the pulsed regime also, the width of that holes may be rather narrow as compared to the intermode spectral spacing, and the inversion inside the holes can make deep jumps, oscillate strongly and reach negative values during some periods of time. The field dynamical spectra shows specific order of mode switching on and switching off which is responsible for the frequency shift (regular or not) in the consecutive mode superradiant pulses. The latter appear in bunches usually, which follow quasi-periodically with a typical period on the order of the inversion timescale given by the pumping.

We describe main regimes of the DFB class D lasers, find optimal conditions of the superradiant pulsed operation, and give qualitative explanations of the major features in the dynamical spectra of the output field and the inversion of an active medium. The effects of facet reflections are also described.

Summary



Dense ensembles of active centers capable of superradiant lasing are promising for both fundamental and applied research, e.g., for an ultrafast information processing in an optical system of strongly interacting particles. The dynamics of such class D lasers, especially with inhomogeneous broadening of an active medium, becomes extremely rich and results in complicated, though quite regular dynamical spectra of emission.

One can smoothly change the dynamical spectra and correlation features of emission by adapting a proper coherent composition of the "hot" lasing modes via managing the parameters of pumping, an active sample, and a low-Q cavity. On the other hand, one can get information on the transitions between cooperative states in a many-particle system (including phase transitions) by tracing the changes in its "hot" mode composition and dynamical spectra of emission. Thus, it seems that the superradiant lasing and other nonlinear phenomena in the class D lasers will enter soon the modern technologies of information optics and diagnostics of many-particle states.