Dynamics of two-dimensional (2D) dark solitons in a smoothly inhomogeneous flow of Bose-Einstein condensate (BEC)

Scattering of a vortex-antivortex pair by a single quantum vortex

L. A. Smirnov, *V. A. Mironov and A. I. Smirnov*

Institute of Applied Physics of the Russian Academy of Sciences
Nizhny Novgorod, Russia

Solitons Collapses and Turbulence – 2014

Mean-field approximation. Basic equation of BEC dynamics

Gross-Pitaevskii (GP) equation:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m_b}\Delta\Psi + g\left|\Psi\right|^2\Psi + V_{ext}(\mathbf{r},t)\,\Psi$$

Here, $\Psi(\mathbf{r},t) = \left|\Psi(\mathbf{r},t)\right| \exp\left(i\vartheta(\mathbf{r},t)\right)$ is the classical wave function;

$n(\mathbf{r},t) = \left|\Psi(\mathbf{r},t)\right|^2$ is the density of condensate atoms;

$\mathbf{v}(\mathbf{r},t) = \hbar/m\nabla\vartheta(\mathbf{r},t)$ is the velocity field in the BEC;

$g = 4\pi\hbar^2 a_s/m_b$ is the interaction coupling constant;

a_s is the s-wave scattering length; m_b is the atomic mass.

External-force potential:

$$V_{ext}(\mathbf{r},t) = V_{trap}(\mathbf{r}) + V_{pb}(\mathbf{r},t),$$

where $V_{trap}(\mathbf{r}) = m_b\left(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2\right)/2$ is the confining trap potential,

$V_{pb}(\mathbf{r},t)$ is the nonstationary potential barrier.

Characteristic scales for the BEC wave function:

$$a_x = \sqrt{\hbar/m_b\omega_x}, \quad a_y = \sqrt{\hbar/m_b\omega_y}, \quad a_z = \sqrt{\hbar/m_b\omega_z}.$$

- $a_s > 0$ for repulsive interactions between atoms.

$$a_z \ll a_h = \sqrt{\hbar^2/m_b g n_0} \ll a_x, a_y. \Rightarrow \phi_{ho,0}(z) = \left(\pi a_z^2\right)^{-1/4} \exp\left(-z^2/2a_z^2\right).$$

- We restricted our consideration to the 2D problem: $\mathbf{r} = (x, y)$.

Dimensionless variables: $\mathbf{r}' = \mathbf{r}/r_0, \quad t' = t/t_0, \quad \mathbf{v}' = \mathbf{v}/c_s.$

$$\Psi' = \Psi \exp(-it')/\sqrt{n_0}, \qquad V'_{ext} = V_{ext}/g_{2d} n_0.$$

$r_0 = \hbar/\sqrt{m g_{2d} n_0}$ is the healing length; $t_0 = r_0/c_s = \hbar/g_{2d} n_0$;

$c_s = \sqrt{g_{2d} n_0/m}$ is the sound velocity; $g_{2d} = g/\sqrt{2\pi} a_z$.

NONLINEAR SCHRÖDINGER (NLS) EQUATION:

$$i\frac{\partial \Psi}{\partial t} + \frac{1}{2}\Delta\Psi + \left(1 - |\Psi|^2\right)\Psi = V_{ext}(\mathbf{r}, t)\,\Psi.$$

Madelung transform: $\Psi(\mathbf{r}, t) = \psi(\mathbf{r}, t)\exp\left(i\theta(\mathbf{r}, t)\right).$

System of hydrodynamic equations for a compressible inviscid liquid:

$$\frac{\partial \psi^2}{\partial t} + \operatorname{div}\left(\psi^2 \nabla\theta\right) = 0, \qquad \frac{\partial\,\theta}{\partial t} + \frac{1}{2}\left(\nabla\theta\right)^2 = 1 - \psi^2 + \frac{\Delta\psi}{2\psi} + V_{ext}(\mathbf{r}, t)\,.$$

2D dark solitons in a homogeneous BEC

The GP equation without $V_{ext}(\mathbf{r},t)$ has a single-parameter family of solutions in the form of 2D dark solitons moving at a constant velocity $\bar{v}$.

$$V_{ext}(\mathbf{r},t)=0, \quad \Psi_s=\Psi_s(\xi,y,\bar{v}), \quad \Psi_s\left(\sqrt{\xi^2+y^2}\to\infty\right)\to 1, \quad \xi=x-\bar{v}t, \quad \bar{v}=\text{const}.$$

$$-i\bar{v}\frac{\partial\Psi_s}{\partial\xi}+\frac{1}{2}\frac{\partial^2\Psi_s}{\partial\xi^2}+\frac{1}{2}\frac{\partial^2\Psi_s}{\partial y^2}+\left(1-\left|\Psi_s\right|^2\right)\Psi_s=0.$$

The properties of the 2D dark solitons were studied in detail in the references:

[1] *Jones C. A., Roberts H.* Motions in a Bose condensate. IV. Axisymmetric solitary waves. // J. Phys. A: Math. Gen. 1982. V. 15, no. 8. Pp. 2599 – 2619.

[2] *Jones C. A., Putterman S., Roberts H.* Motions in a Bose condensate. V. Stability of solitary wave solutions of nonlinear Schrödinger equations in two and three dimensions. // J. Phys. A: Math. Gen. 1986. V. 19, no. 15. Pp. 2991 – 3011.

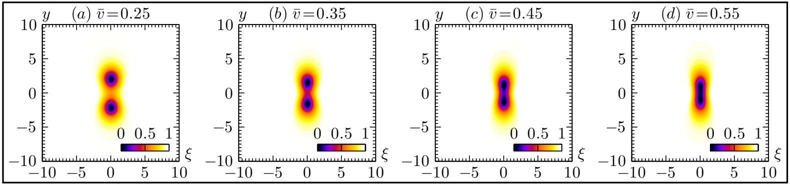
[3] *Kuznetsov E. A., Rasmussen J. J.* Instability of two-dimensional solitons and vortices in defocusing media. // Phys. Rev. E. 1995. V. 51, no. 5. Pp. 4479 – 4484.

[4] *Berloff N. G.* Padé approximations of solitary wave solutions of the Gross-Pitaevskii equation. // J. Phys. A: Math. Gen. 2004. V. 37, no. 5. Pp. 1617 – 1632.

[5] *Tsuchiya S., Dalfovo F., Pitaevskii L. P.* Solitons in two-dimensional Bose-Einstein condensates. // Phys. Rev. A. 2008. V. 77, no. 4. Pp. 045601 (4).

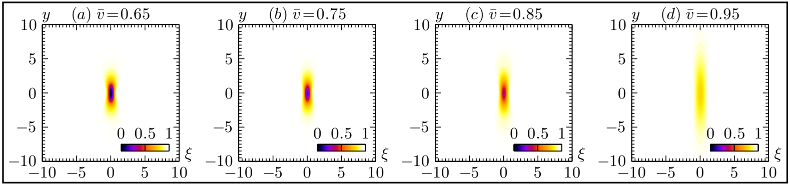
The velocity $\bar{v}$ plays the role of the problem parameter. Depending on its value, 2D dark solitons can be vortex and vortex-free.

Vortex 2D dark solitons ($\bar{v} < \bar{v}_* = 0.61$)



In the limit of small velocities $\bar{v}$ ($|\bar{v}| \ll 1$) a soliton is a vortex pair, whose momentum $\bar{\mathcal{P}}$ are related with its energy $\bar{\mathcal{E}}$ as for a pair of point vortices. $\bar{\mathcal{P}} \approx 2\pi \exp(\bar{\mathcal{E}}/2\pi)$.

Vortex-free 2D dark solitons ($\bar{v}_* = 0.61 < \bar{v} \leq 1$)



In the limit of transonic velocities $\bar{v}$ ($1 - |\bar{v}| \ll 1$) a 2D dark soliton coincides with known solution of the Kadomtsev-Petviashvili (KP) equation. $\bar{\mathcal{P}} \approx \bar{\mathcal{E}} - 3\bar{\mathcal{E}}^3/128\pi^2$.

- **Variation problem:** $\delta\left(\mathcal{H} - \bar{v}\mathcal{P}_x\right) = 0.$

$$\mathcal{H} = \frac{1}{2}\int_{-\infty}^{+\infty} dy \int_{-\infty}^{+\infty} d\xi \left[\left|\nabla\Psi\right|^2 + \left(1 - \left|\Psi\right|^2\right)^2 \right],$$

$$\mathbf{P} = \frac{i}{2}\int_{-\infty}^{+\infty} dy \int_{-\infty}^{+\infty} d\xi \left(\Psi\nabla\Psi^* - \Psi^*\nabla\Psi \right).$$

- **Integral relations for the energy $\bar{\mathcal{E}}$ and the momentum $\bar{\mathcal{P}}$:**

$$\bar{\mathcal{E}} = \int_{-\infty}^{+\infty} dy \int_{-\infty}^{+\infty} d\xi \left| \frac{\partial \Psi_s}{\partial \xi} \right|^2,$$

$$\bar{\mathcal{E}} - \bar{v}\bar{\mathcal{P}} = \int_{-\infty}^{+\infty} dy \int_{-\infty}^{+\infty} d\xi \left| \frac{\partial \Psi_s}{\partial y} \right|^2,$$

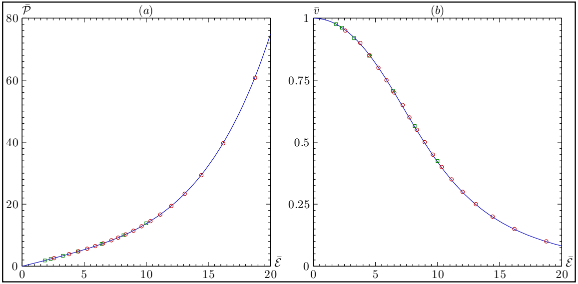
$$\bar{v}\bar{\mathcal{P}} = \int_{-\infty}^{+\infty} dy \int_{-\infty}^{+\infty} d\xi \left(1 - \left|\Psi_s\right|^2 \right)^2.$$

- **Velocity of a 2D dark soliton:** $\bar{v} = d\bar{\mathcal{E}}/d\bar{\mathcal{P}} < \bar{\mathcal{E}}/\bar{\mathcal{P}}.$

Approximation for the dependence of $\bar{\mathcal{P}}$ on $\bar{\mathcal{E}}$:

$$\bar{\mathcal{P}}(\bar{\mathcal{E}}) = \alpha(\bar{\mathcal{E}})\,\mathrm{sh}\left[\frac{\bar{\mathcal{E}}}{\alpha(\bar{\mathcal{E}})}\right],$$

$$\alpha(\bar{\mathcal{E}}) = 2\pi + \frac{2\pi}{3}\exp\left[-\frac{\bar{\mathcal{E}}^2}{9.8^2}\right].$$



Asymptotic description of the dynamics of 2D dark solitons in a smoothly inhomogeneous BEC

$$\Psi(\mathbf{r},t)=\Psi_b(\mathbf{r})\,F(\mathbf{r},t)\,.$$

$\Psi_b(\mathbf{r})=\psi_b(\mathbf{r})\exp\left[\theta_b(\mathbf{r})\right]$ is the undisturbed part of the wave function, whose structure is considered known and which satisfies the stationary GP equation.

$$\frac{1}{2}\Delta\Psi_b+\left(1-\left|\Psi_b\right|^2\right)\Psi_b-V_{ext}(\mathbf{r})\,\Psi_b=0.$$

Here, $n_b(\mathbf{r})=|\psi_b(\mathbf{r})|^2$ and $\mathbf{v}_b(\mathbf{r})=\nabla\theta_b(\mathbf{r})$ are the density distribution and the flow velocity of background BEC, for example, formed under the action of $V_{ext}(\mathbf{r})$.

Λ_b is a characteristic scale of the function $\Psi_b(\mathbf{r})$ (medium inhomogeneity).

$F(\mathbf{r},t)$ is the part of the wave function describing the behavior of the disturbances in an initially inhomogeneous BEC.

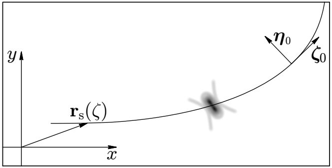
$$i\frac{\partial F}{\partial t}+\frac{1}{2}\Delta F+n_b\left(1-\left|F\right|^2\right)F=$$
$$=-\nabla\ln\sqrt{n_b}\cdot\nabla F-i\mathbf{v}_b\cdot\nabla F.$$

Λ_s is a characteristic size of the localization region of the considered disturbance initially specified in the form of a 2D dark soliton.

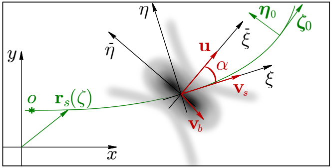
Small parameter: $\mu=\Lambda_s/\Lambda_b\ll 1.$

Schematic representation of propagation of a 2D dark soliton moving along the trajectory $\mathbf{r}_s(\zeta)$ in a smoothly inhomogeneous BEC.

Here, $\boldsymbol{\zeta}_0$ and $\boldsymbol{\eta}_0$ are the unit vectors of the tangent and the normal to the propagation path $\mathbf{r}_s(\zeta)$.



$\mathbf{r} = \mathbf{r}_s(\zeta) + \eta\,\boldsymbol{\eta}_0(\zeta)$. $\xi = \zeta - \zeta_s(t)$. $s_s(t) = \int_0^t v_s(t)\mathrm{d}t$.



Here, ζ is the arc length; η is the distance along the normal dropped on the curve $\mathbf{r}_s(\zeta)$; ξ is the coordinate, which accompanies the localized solitonic formation moving along the trajectory $\mathbf{r}_s(\zeta)$; $s_s(t)$ is the position of the center of the soliton on the curve $\mathbf{r}_s(\zeta)$; $v_s(t)$ is the velocity of the 2D dark soliton.

▶ We passed to the orthogonal curvilinear coordinates ξ, η.

Lamé coefficients: $h_\xi = (1 - \kappa\eta)$, $h_\eta = 1$, where $\kappa(s)$ is the curvature of the line $\mathbf{r}_s(\zeta)$.

▶ We used the assumption that the characteristic variation scale Λ_b of the function $\Psi_b(\mathbf{r})$ significantly exceeds the sizes Λ_s of the localization region of the solitonic formation. We expanded the functions $n_b(\mathbf{r})$, $v_{b\xi}(\mathbf{r})$ and $v_{b\eta}(\mathbf{r})$ near the center of the soliton formation into the Taylor series.

▶ The solution of the equation for the function $F\big(\xi, \eta, t\big)$ near the curve $\mathbf{r}_s(\zeta)$ can be represented as an asymptotic series of the small parameter μ:

$$F\big(\xi, \eta, t\big) = \mu^0 F_0\big(\xi, \eta, v_s(\mu t)\big) + \mu^1 F_1\big(\xi, \eta, \mu t\big) + \dots .$$

▶ We rotated the coordinates ξ, η by $\alpha(\mu t) = \arctan\big[-v_{b\eta,0}/\big(v_s(\mu t) - v_{b\xi,0}\big)\big]$ and multiplied them by $\sqrt{n_{b,0}}$: $\bar{\xi} = \sqrt{n_{b,0}}\big(\xi\cos\alpha + \eta\sin\alpha\big)$, $\bar{\eta} = \sqrt{n_{b,0}}\big(\eta\cos\alpha - \xi\sin\alpha\big)$.

▸ In the zero order of smallness we obtained the equation for the function F_0, which is reduced to a stationary GP equation with a solution in the form of 2D dark soliton.

- μ^0:

$$-i\bar{u}\frac{\partial F_0}{\partial \bar{\xi}} + \frac{1}{2}\frac{\partial^2 F_0}{\partial \bar{\xi}^2} + \frac{1}{2}\frac{\partial^2 F_0}{\partial \bar{\eta}^2} + \left(1-\left|F_0\right|^2\right)F_0 = 0.$$

$$\mathbf{u} = \mathbf{v}_s - \mathbf{v}_b, \quad \bar{v} = |\mathbf{u}|/\sqrt{n_{b,0}}, \quad F_0(\bar{\xi}, \bar{\eta}, \bar{v}) \equiv \Psi_s(\bar{\xi}, \bar{\eta}, \bar{v}).$$

▸ In the first order of smallness in μ, for the function F_1 we obtained the linear inhomogeneous differential equation in terms of the new variables $\bar{\xi}$ and $\bar{\eta}$.

- μ^1:

$$-i\bar{v}\frac{\partial F_1}{\partial \bar{\xi}} + \frac{1}{2}\frac{\partial^2 F_1}{\partial \bar{\xi}^2} + \frac{1}{2}\frac{\partial^2 F_1}{\partial \bar{\eta}^2} + \left(1-2\left|F_0\right|^2\right)F_1 - F_0^2 F_1^* = \mathcal{R}.$$

The conditions for the existence of localized solutions in the equation for the function F_1 are the fulfillment of the following equalities:

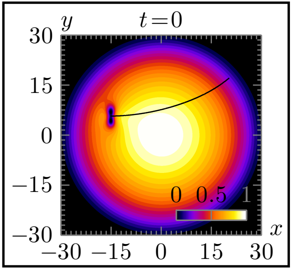
$$\operatorname{Re}\left[\int_{-\infty}^{+\infty} d\bar{\eta} \int_{-\infty}^{+\infty} d\bar{\xi}\, \mathcal{R}\, \frac{\partial F_0^*}{\partial \chi_{1(2)}}\right] = 0, \qquad \chi_1 \equiv \bar{\xi}, \qquad \chi_2 \equiv \bar{\eta}.$$

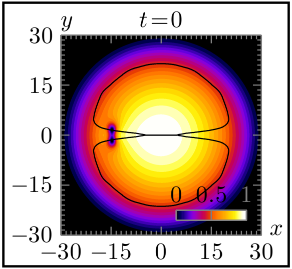
These relations for the complex equation for the function F_1 essentially counterparts of the Fredholm theorem on alternative.

Propagation of vortex pairs in an inhomogeneous BEC formed under the action of a trap potential

[1] *Smirnov L. A., Mironov V. A.* JETP Letters. 2012. V. 95, no. 11. Pp. 549 – 554.
[2] *Smirnov L. A., Mironov V. A.* Phys. Rev. A. 2012. V. 85, no. 5. Pp. 053620 (12).

- We developed the method of calculation the trajectories, along which 2D dark solitons move in a smoothly inhomogeneous BEC without flow.
- We described and explained the structural changes in the 2D dark solitons.





The comparisons of the results of numerical simulation of the dynamics of 2D dark solitons directly within GP equation with the results obtained by using the developed asymptotic theory confirmed the validity of the proposed method of describing the behavior of such solitons in a smoothly inhomogeneous BEC.

Geometric optics for the scattering process of vortex-antivortex pairs by a single quantum vortex

► We assumed, the background velocity field $\mathbf{v}_b(\mathbf{r})$ is equal the velocity in a homogeneous BEC around a single stationary quantum vortex:

$$\mathbf{v}_b = (1/r)\boldsymbol{\varphi}_0.$$

► When $r_s \gg \Lambda_s$, the 2d dark soliton velocity $v_s(\mu t)$ is significantly larger than local values of components of the flow created by isolated topological defect.

$$\left|v_{b\xi,0}/v_s\right| \sim \mu, \qquad \left|v_{b\eta,0}/v_s\right| \sim \mu.$$

► We took into account the smallness of $\alpha \approx v_{b\eta,0}/v_s \sim \mu$ and $\kappa \sim \mu^2$.

► At sufficiently large distances $r \gg 1$ from the core of a quantum vortex the flow velocity $\mathbf{v}_b(\mathbf{r})$ satisfies conditions $\mathrm{div}(\mathbf{v}_b) = 0$ and $\mathrm{rot}(\mathbf{v}_b) = 0$, and the density $n_b(\mathbf{r})$ of the background condensate is determined by the following expressions:

$$n_v \approx 1 - \left(v_{v\xi}^2 + v_{v\eta}^2 \right)/2.$$

► We introduced the normalized momentum $\bar{\mathcal{P}}(\bar{v})$ and the normalized energy $\bar{\mathcal{E}}(\bar{u})$ of a 2D dark soliton.

(*) The quantities $\bar{\mathcal{P}} = \bar{\mathcal{P}}(\bar{v})$ and $\bar{\mathcal{E}} = \bar{\mathcal{E}}(\bar{v})$ at each moment of time t are the corresponding characteristics of a 2D dark soliton propagating with the velocity $\bar{v}(t)$ for a given value of $\bar{v}$ in a homogeneous condensate of unit density.

Dependence of the normalized energy of a 2D dark soliton on its position along the propagation path

$$\mathrm{Re}\left[\int_{-\infty}^{+\infty}\mathrm{d}\bar{\eta}\int_{-\infty}^{+\infty}\mathrm{d}\bar{\xi}\,\mathcal{R}\frac{\partial F_0^*}{\partial\bar{\xi}}\right]=0. \quad \Rightarrow \quad \boxed{\bar{\mathcal{E}}(\zeta_s)=\bar{\mathcal{E}}_o+\Big[v_{b\,\xi}(\mathbf{r}_s(\zeta_s))-v_{b\,\xi}(\mathbf{r}_s(0))\Big]\bar{\mathcal{P}}_o.}$$

Here, $\bar{\mathcal{E}}_o$, $\bar{\mathcal{P}}_o$ and $\bar{v}_o$ are the initial values of the normalized energy, normalized energy and normalized velocity of the 2D dark soliton, respectively.

Moreover, the normalized momentum $\bar{\mathcal{P}}$ and the normalized velocity $\bar{v}$ are single-valued functions of the normalized energy $\bar{\mathcal{E}}$, and for the dependence $\bar{\mathcal{P}}\big(\bar{\mathcal{E}}\big)$ we have an analytical approximation.

Relationship for the curvature of the 2D dark soliton trajectory

$$\mathrm{Re}\left[\int_{-\infty}^{+\infty}\mathrm{d}\bar{\eta}\int_{-\infty}^{+\infty}\mathrm{d}\bar{\xi}\,\mathcal{R}\frac{\partial F_0^*}{\partial\bar{\eta}}\right]=0. \quad \Rightarrow \quad \boxed{\kappa(\zeta_s)=\frac{1}{2}\frac{\partial}{\partial\eta}\left(\mathcal{C}_1\frac{v_{b\,\xi}^2}{\bar{v}_o^2}+\mathcal{C}_2\frac{v_{b\,\eta}^2}{\bar{v}_o^2}\right)\bigg|_{\mathbf{r}=\mathbf{r}_s(\zeta_s)}.}$$

Coefficients $\mathcal{C}_1$ and $\mathcal{C}_2$ are constant along the 2d dark soliton trajectory:

$$\boxed{\mathcal{C}_1=1+\frac{\bar{\mathcal{E}}_o\bar{v}_o}{\bar{\mathcal{P}}_o}-\frac{\bar{v}_o^2}{2}, \qquad \mathcal{C}_2=1+\mathcal{C}_1-\bar{\mathcal{P}}_o\frac{\mathrm{d}\bar{v}}{\mathrm{d}\bar{\mathcal{E}}}\bigg|_{\bar{\mathcal{E}}=\bar{\mathcal{E}}_o}.}$$

Equation of the 2D dark soliton trajectory

The expression for the curvature $\kappa(\zeta)$ uniquely specify on the plane x, y the trajectory $\mathbf{r}_s(\zeta)$, along which the 2D dark soliton moves.

$$\frac{d\mathbf{r}_s}{d\zeta} = \boldsymbol{\zeta}_0(\zeta), \quad \frac{d\boldsymbol{\zeta}_0}{d\zeta} = \kappa(\zeta)\, \boldsymbol{\eta}_0(\zeta).$$

We introduced, instead of the arc length ζ, a new variable τ, and, instead of the unit vectors $\boldsymbol{\zeta}_0(s)$ of the tangent to the curve $\mathbf{r}_s(\zeta)$, a new vector $\mathbf{p}$.

$$d\tau = d\zeta / \nu(\mathbf{r}_s(\zeta)).$$

$$\mathbf{p} = \nu(\mathbf{r}_s)\, \boldsymbol{\zeta}_0.$$

Refractive index:

$$\nu^2(\mathbf{r}) = \exp\left[\mathcal{C}_1\left(\frac{\mathbf{p}\cdot\mathbf{v}_b(\mathbf{r})}{|\mathbf{p}|}\right)^2 + \mathcal{C}_2\left(\frac{[\,\mathbf{z}_0\times\mathbf{p}\,]\cdot\mathbf{v}_b(\mathbf{r})}{|\mathbf{p}|}\right)^2\right].$$

Equation of the propagation path:

$$\frac{d\mathbf{r}_s}{d\tau} = \mathbf{p}, \qquad \frac{d\mathbf{p}}{d\tau} = \frac{1}{2}\nabla\left[\nu^2(\mathbf{r})\right]\Big|_{\mathbf{r}=\mathbf{r}_s}.$$

$$\mathbf{r}_s(\tau=0) = \mathbf{r}_s(0),$$

$$\mathbf{p}(\tau=0) = \nu(\mathbf{r}_s(0))\,\zeta_0(0).$$

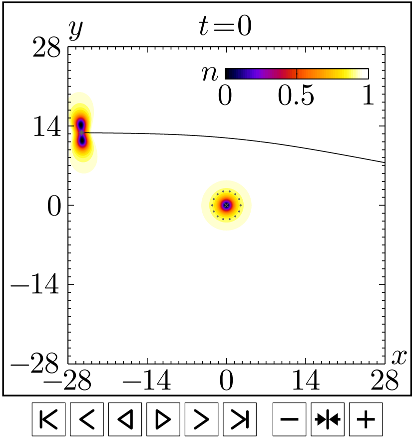
We assumed that at $t=0$ a 2D dark soliton is situated in the point with coordinates x_o, y_o and has velocity parallel x-axis.

$$\frac{d^2 y_s}{dx_s^2} = -\frac{y_s\left((2\mathcal{C}_2 - \mathcal{C}_1)\,x_s^2 + \mathcal{C}_1 y_s^2\right)}{\bar{v}_o^2\,(x_s^2 + y_s^2)^3}.$$

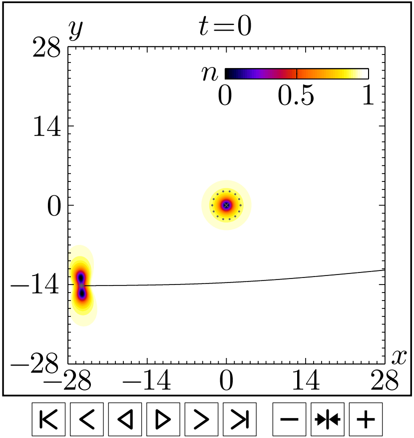
$$\beta_R = -\frac{\pi(\mathcal{C}_1 + \mathcal{C}_2)}{4\bar{v}_o^2 y_o^2}.$$

$$\sigma_R = \frac{\sqrt{\pi(\mathcal{C}_1 + \mathcal{C}_2)}}{4\bar{v}_o \beta_R^{3/2}}.$$

$x_o = -25.6$, $y_o = 12.8$, $\bar{v}_o = 0.4$.

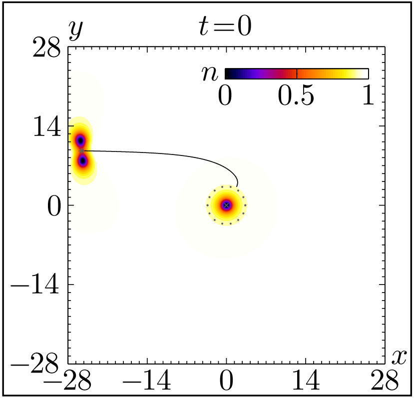


$x_o = -25.6$, $y_o = -14.2$, $\bar{v}_o = 0.4$.

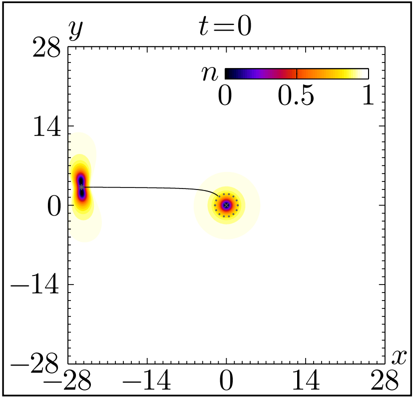


The movies clearly demonstrate that the vortex pairs move along the paths, which are calculated using the geometro-optical equation for the trajectory of a 2d dark soliton. Besides, the vortex-antivortex pairs turn with respect to the propagation path by the small angle $\alpha(\mu t)$ in a good agreement with the developed theory.

$x_o = -25.6,\ y_o = 9.6,\ \bar{v}_o = 0.3.$

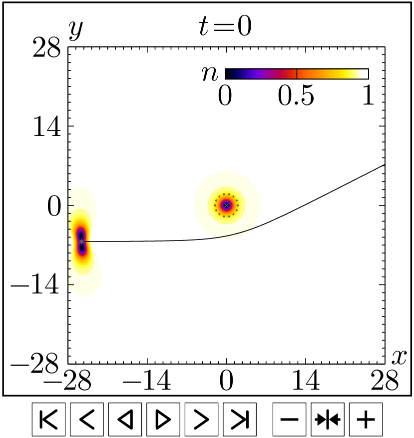


$x_o = -25.6,\ y_o = 3.2,\ \bar{v}_o = 0.5.$

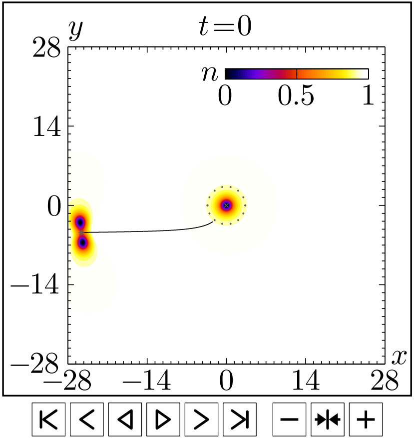


At the chosen parameters the vortex-antivortex pairs approache close to the point, where the initially isolated vortex was located, and this violates the applicability conditions for our theory. Then, a complicated dynamics of three topological defects is observed. As a result, the vortex center of the colliding vortex pair becomes the isolated phase singularity. Moreover, its antivortex and initially single vortex form a new 2D dark soliton.

$x_o = -25.6,\ y_o = -6.4,\ \bar{v}_o = 0.5.$



$x_o = -25.6,\ y_o = -4.8,\ \bar{v}_o = 0.3.$



At the chosen parameters the sound wave radiation breaks the validity of the developed asymptotic theory for the dynamics of a 2D dark soliton.

Conclusions

1) We have studied the scattering of a vortex-antivortex pairs by a single quantum vortex is investigated in a Bose-Einstein condensate with repulsive interaction between atoms.

2) We have developed an asymptotic theory, describing the dynamics of vortex pairs in an inhomogeneous flow created by a topological defect in an ultracold Bose gas.

3) In the limit of large impact parameters, we have found an analytical expression for the angle of scattering of a vortex-antivortex pair by a single phase singularity.

4) All theoretical constructs have been confirmed by numerical calculations performed directly within the framework of the Gross-Pitaevskii equation.

5) By numerical simulations, we have shown that in collisions, when the asymptotic theory breaks down, interaction of vortex pairs with the vortex can be accompanied with an exchange of phase singularities and sound waves radiation.