Nonlinear harmonic surface waves on a deep water. Experimental results in “Marintek” ocean basin (Trondheim, Norway)

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SCT-VII, Zakharov-75, 07 August 2014

Acoustic turbulence

Nonlinear waves (acoustic turbulence)

$$\frac{\partial \bar{v}}{\partial t} + (\bar{v}\nabla)\bar{v} = \frac{1}{\mathrm{Re}_a}\Delta\bar{v}$$

where dimensionless acoustic Reynolds's coefficient is

$$\mathrm{Re}_a = \frac{\varepsilon v_{m0}\lambda}{\pi \upsilon_{ef}}$$

ratio of nonlinear and viscous effects.

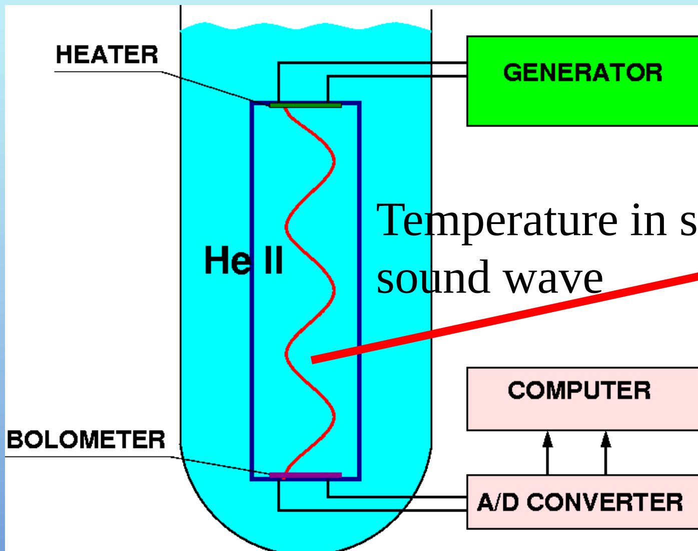
Second sound waves in SF helium – Burgers turbulence

$$\frac{\partial}{\partial t}\delta T + u_{20}(1 + \alpha\delta T)\frac{\partial}{\partial x}\delta T = \nu\frac{\partial^2}{\partial x^2}\delta T$$

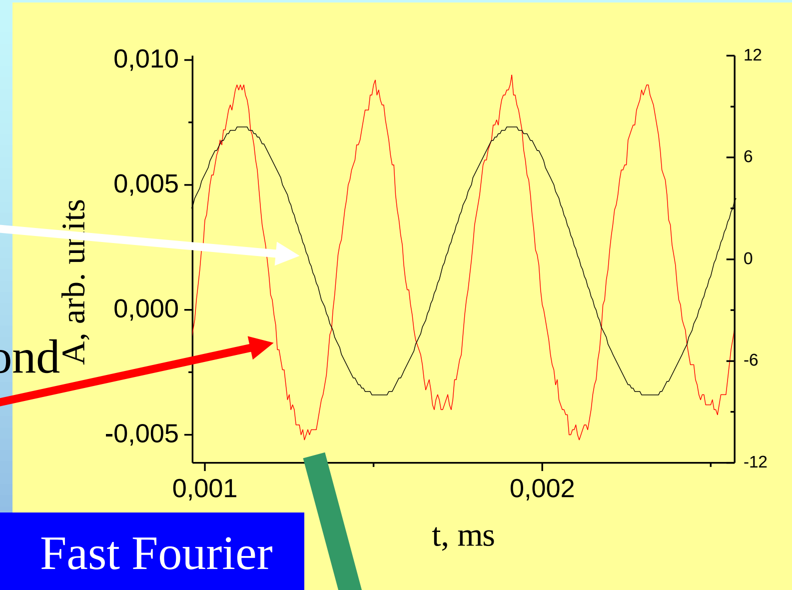
$\omega \sim k$

Experimental studies of one dimensional nonlinear second sound waves in the cylindrical cell

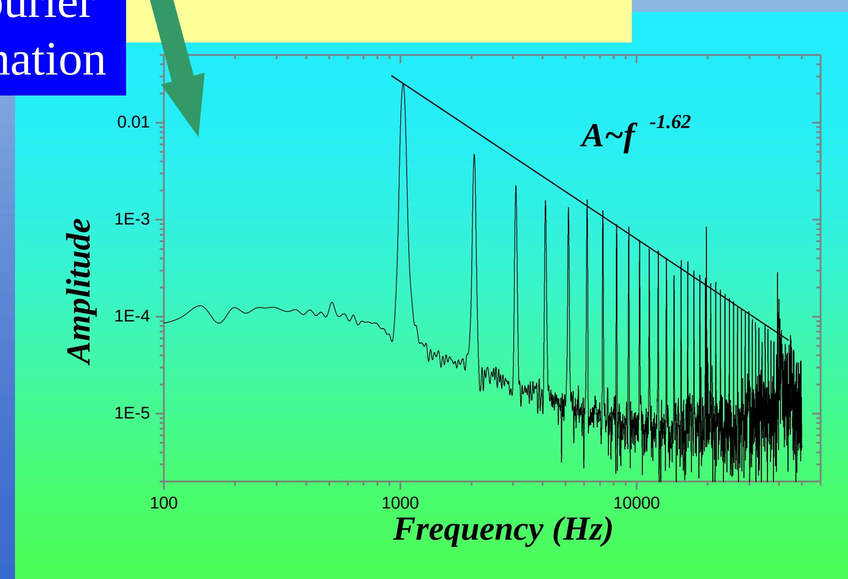
Applied signal $A=A_0*\sin(\omega t)$



Temperature in second sound wave

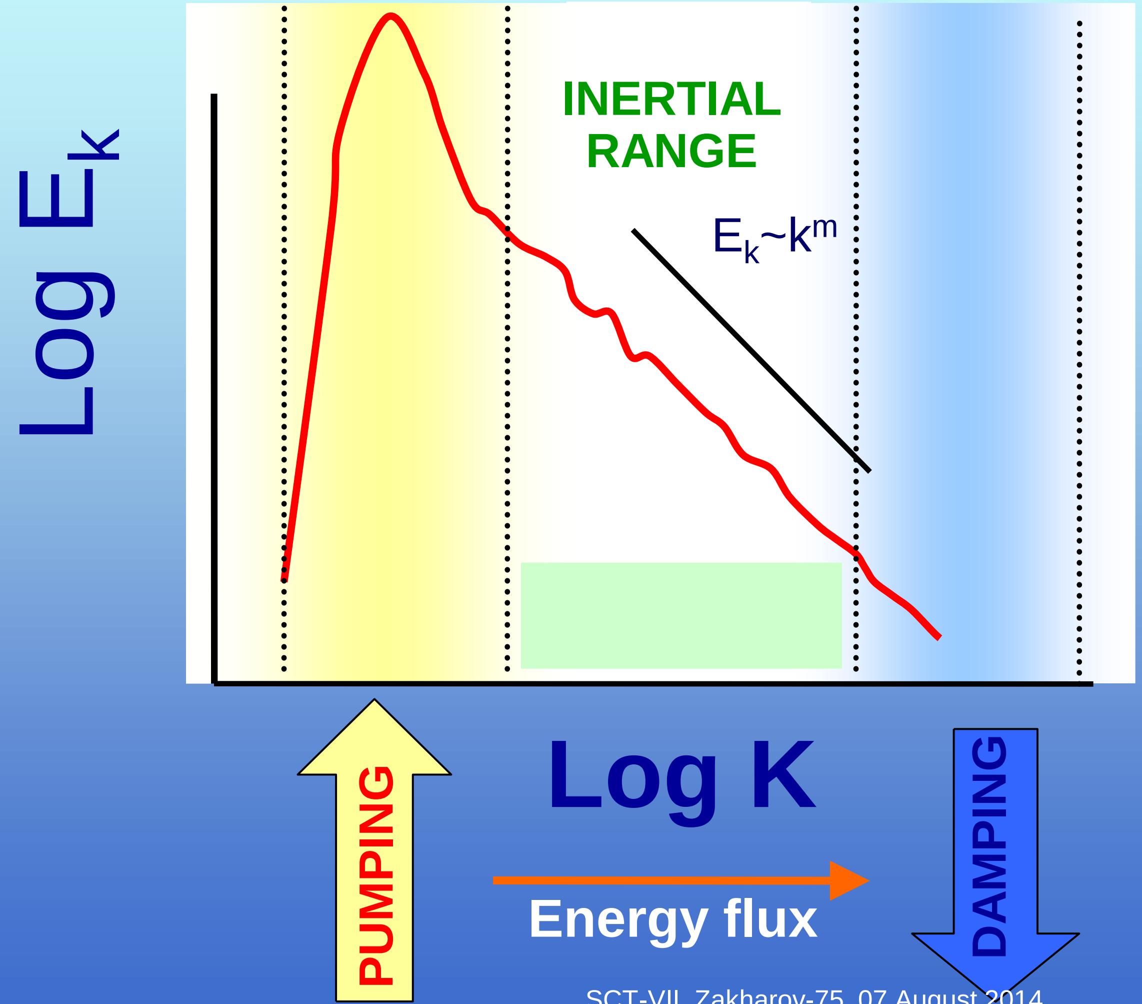


Fast Fourier Transformation



Spectrum of temperature oscillations of the nonlinear second sound waves in a resonator

Energy transfer in k-space over scales



Kolmogorov, Obukhov, 1941,

In the inertial range

$$E_k \sim k^m$$

For the turbulence of incompressible fluid

$m = -5/3$

Mixing of resonance frequencies

- We applied two harmonic frequencies corresponding two different resonances:

  R32 (5V) and R 11 (2V) (R11 signal didn't format cascade)
- We observed mixing respond of system:

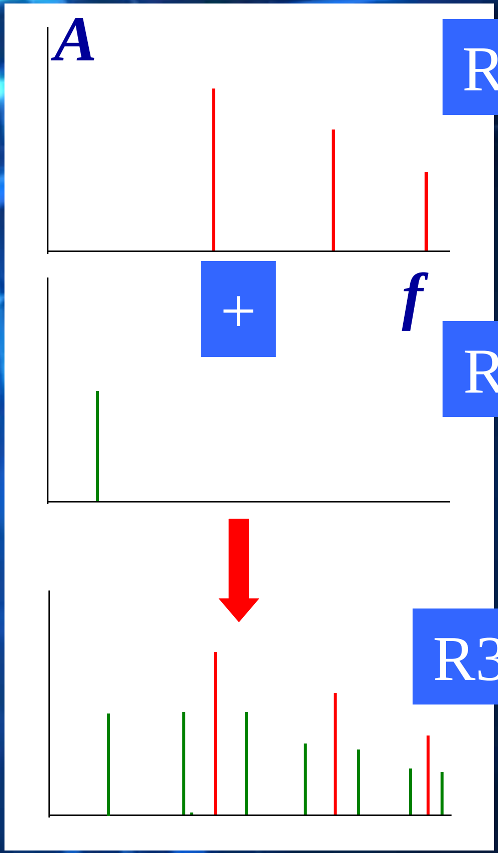
  R11

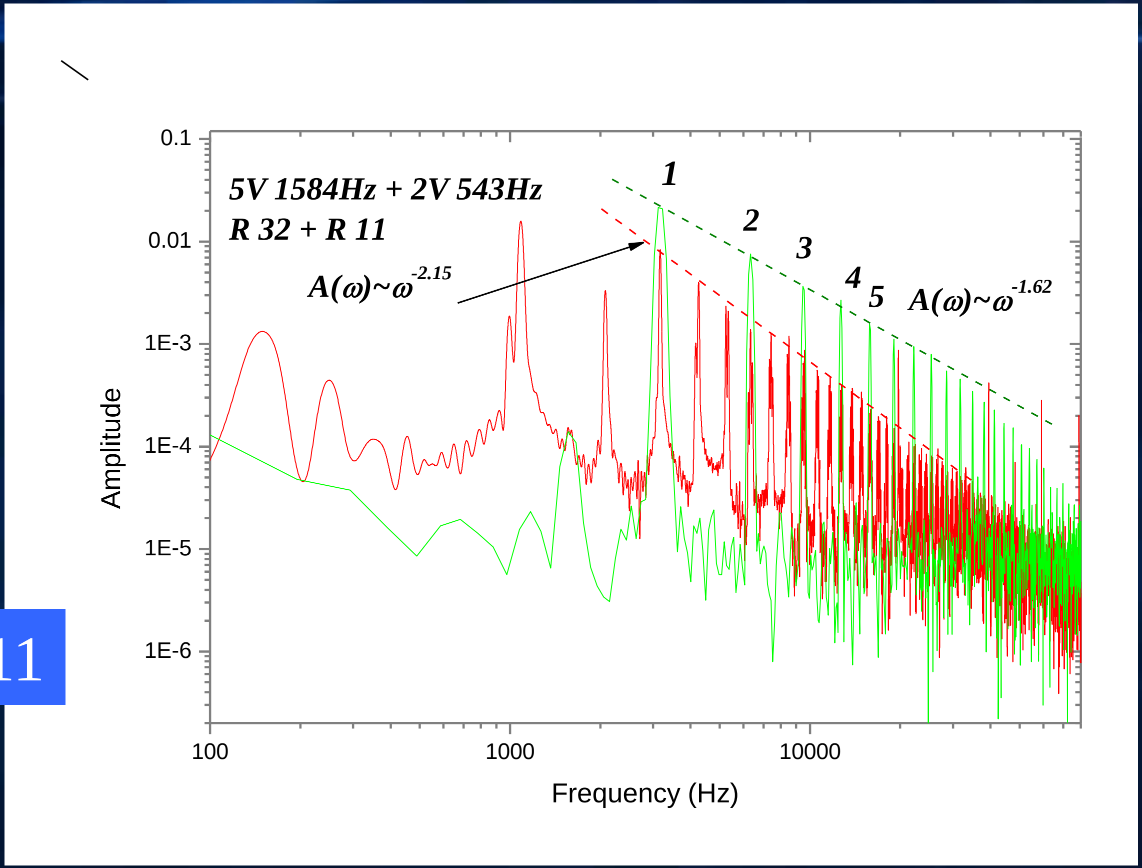
  R32-R11   R32   R32+R11

  2*R32-R11   2*R32   2*R32+R11

  ……
- The initial cascade suppressed by small second frequency.

Mixing of frequencies and suppression of cascade.





We applied f_D=3168 Hz (32 Resonance), and got the cascade $A(\omega)\sim\omega^{-1.62}$ (green line)
then we add second frequency f_D=1084 Hz (11 Resonance) and observed combinational frequencies and cascade suppression $A(\omega)\sim\omega^{-2.15}$ (red line)

Kinetic turbulence

- The Navier-Stokes equations are a set of equations that describe the motion of fluid substances with dissipation

$$\rho\left[\frac{\partial\bar{v}}{\partial t}+(\bar{v}\nabla)\bar{v}\right]=-\nabla P+\eta\Delta\bar{v}+(\xi+\frac{\eta}{3})grad\,div\bar{v}$$

- For incompressible fluid the equations change to

$$\frac{\partial\bar{v}}{\partial t}+(\bar{v}\nabla)\bar{v}=-\frac{1}{\rho}\nabla P+\upsilon\Delta\bar{v} \qquad div\,\bar{v}=0$$

- where $\upsilon=\eta/\rho$ coefficient of kinematical viscosity

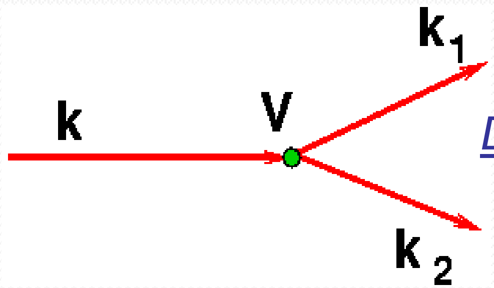
- Transformation the equations into dimensionless form we got Reynolds equations,

$$\frac{\partial\bar{v}}{\partial t}+(\bar{v}\nabla)\bar{v}=-\nabla P+\frac{1}{R}\Delta\bar{v}$$

where $\frac{VL}{\upsilon}$ is Reynolds number – measure of ratio of inertial/viscous forces

Wave interaction in dispersion medium

The dispersion relation $\omega \sim k^s$

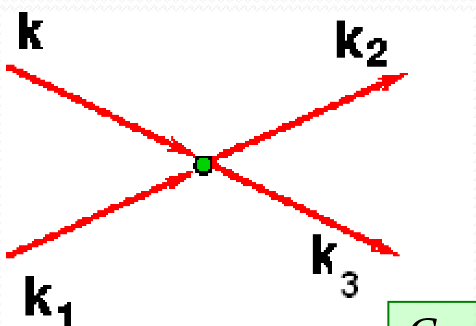


Decay (positive) dispersion (s>1)

$$\mathbf{k}=\mathbf{k}_1+\mathbf{k}_2$$
$$\omega=\omega_1+\omega_2$$

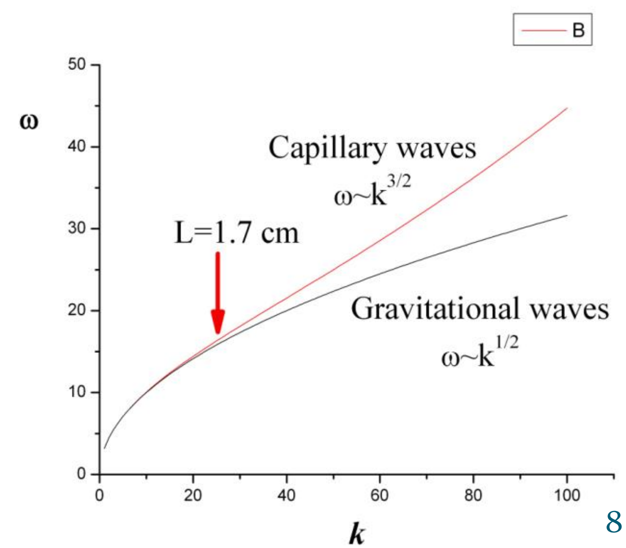
Capillary waves on the liquid surface $\omega \sim k^{3/2}$
$\lambda < 0.5$ cm

Non-decay (negative) dispersion (s<1)



$$\mathbf{k}+\mathbf{k}_1=\mathbf{k}_2+\mathbf{k}_3$$
$$\omega+\omega_1=\omega_2+\omega_3$$

Gravitational waves
$\omega \sim k^{1/2}$ $\lambda > 5$ cm



$$w^2=(gk+\frac{s}{r}k^3)\tanh(kh)$$

Dispersion ratios for gravitational surface waves

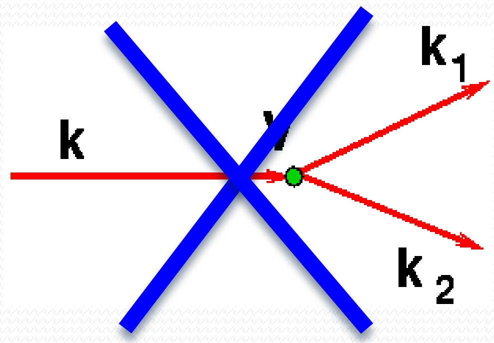
We launch flat harmonic wave with single frequency ω

$$w=\sqrt{gk}$$

Quasi-one dimensional situation

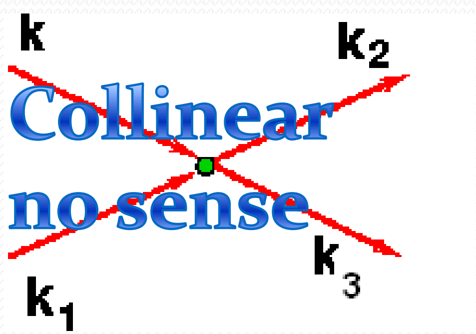
$$c_p=\sqrt{\frac{gl}{2p}}=\frac{g}{2p}t$$

For deep water



$$\mathbf{k}=\mathbf{k}_1+\mathbf{k}_2$$

$$\omega=\omega_1+\omega_2$$



$$\mathbf{k}+\mathbf{k}_1=\mathbf{k}_2+\mathbf{k}_3$$

$$\omega+\omega_1=\omega_2+\omega_3$$

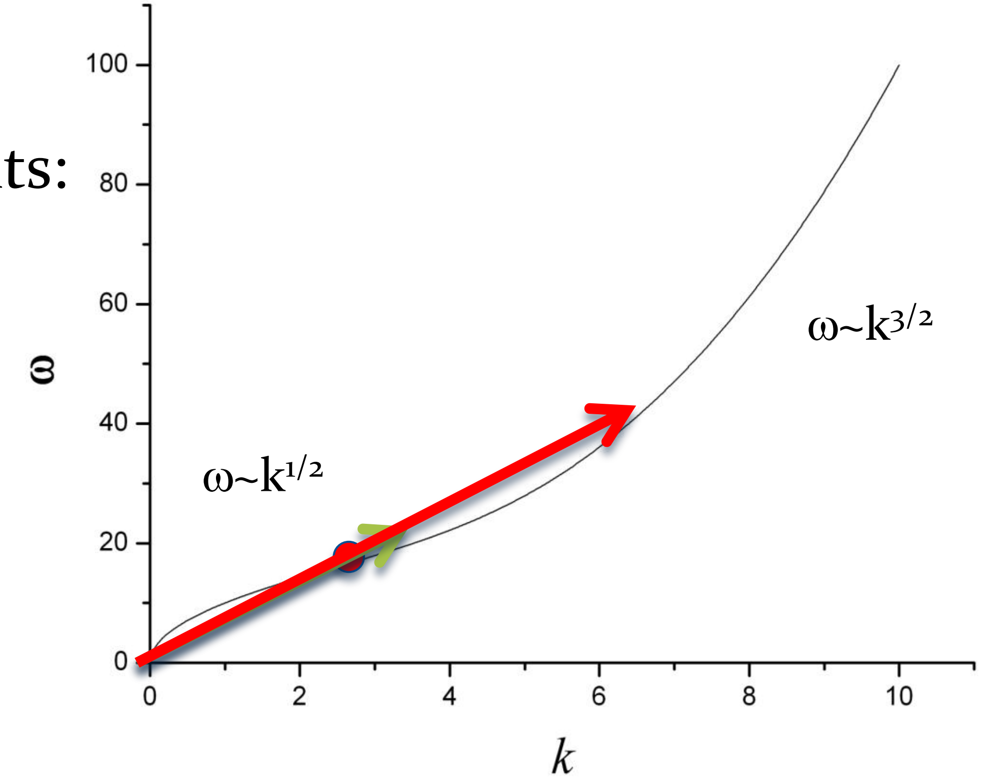
$$w^2=gk[1+(kA)^2]$$

$$w^2 = gk[1+(kA)^2]$$

There are some special points:

Change of concavity

The conservation law of energy and pulses are allowed 1-D three waves interaction

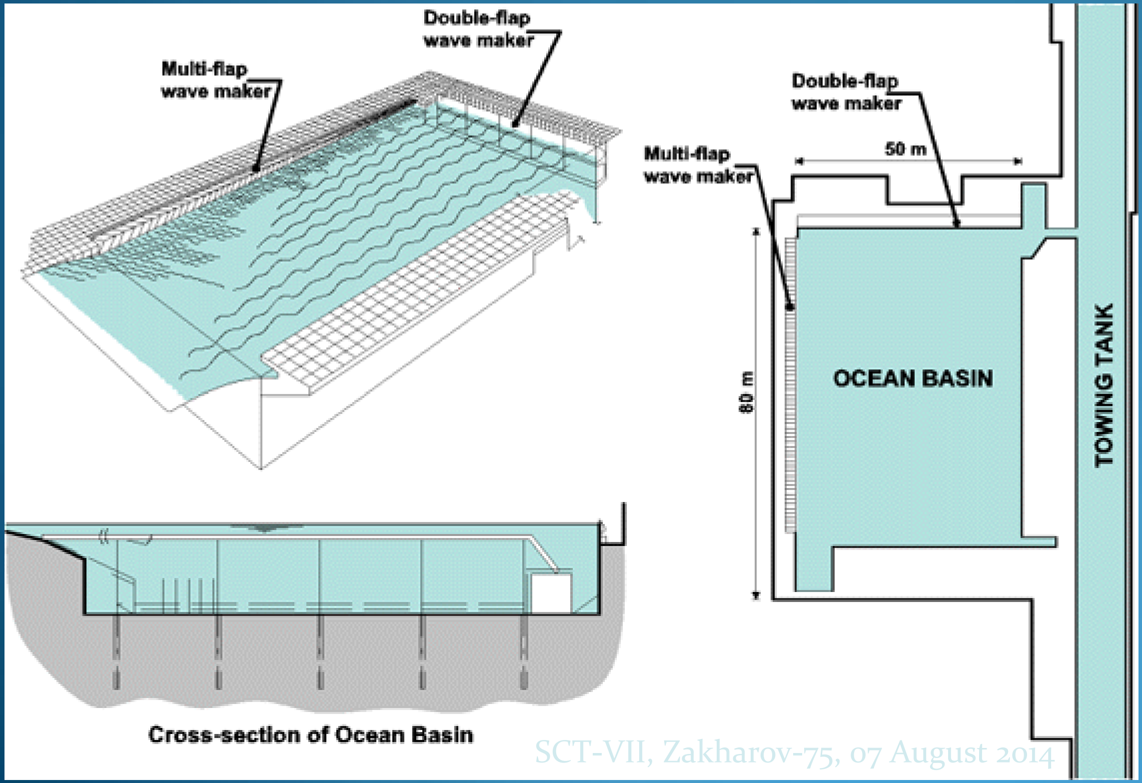


$d^2\omega/dk^2=0$ $A*k>0.2$ $A/\lambda>0.03$

$\omega=\omega_1+\omega_1$, → $\omega=2*\omega_1$ and $k=k_1+k_1$ → $k=2*k_1$

$A*k>1/2$ $A/\lambda>0.11$

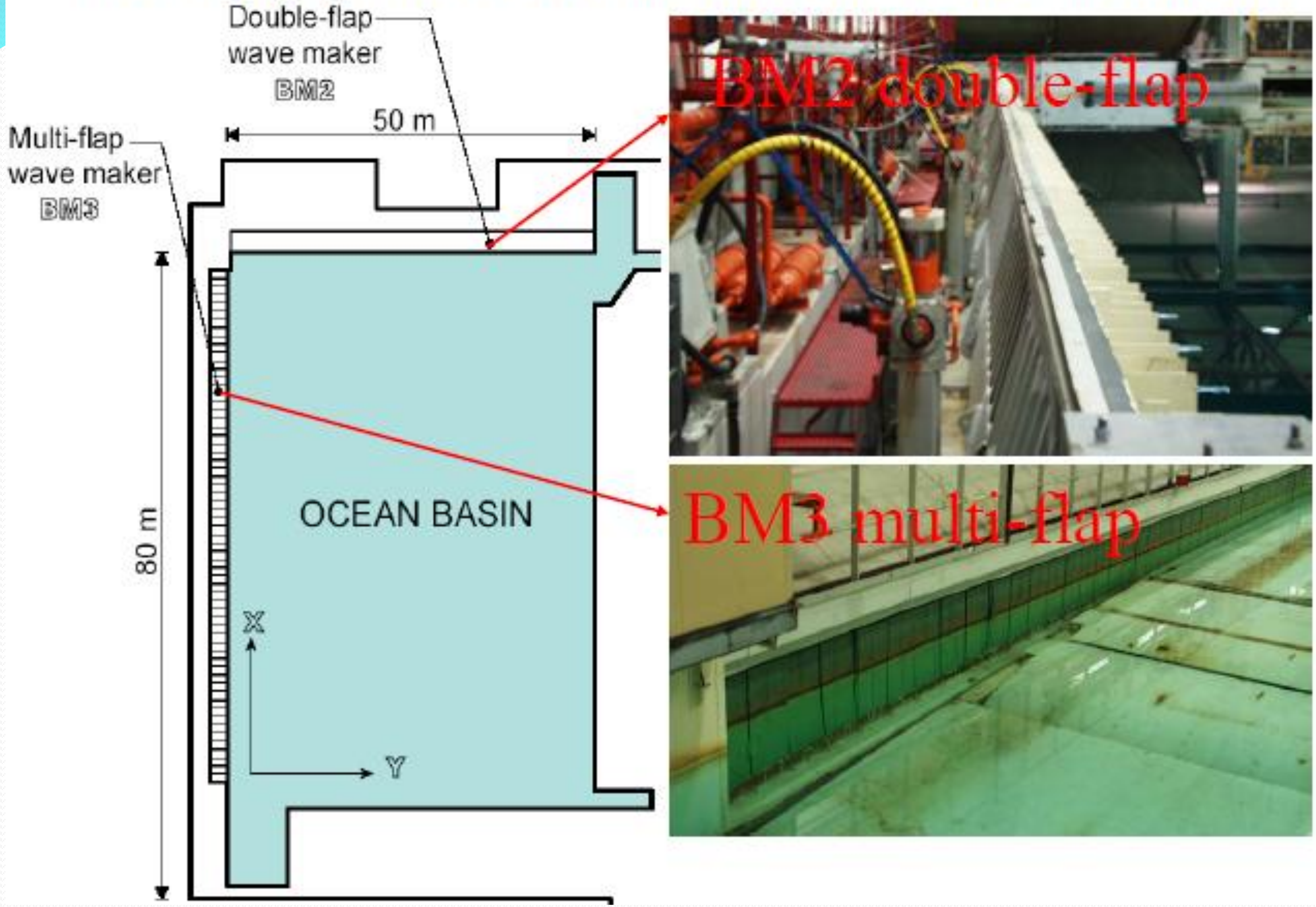
Marintek Ocean Basin



Sensors,

results and so on.

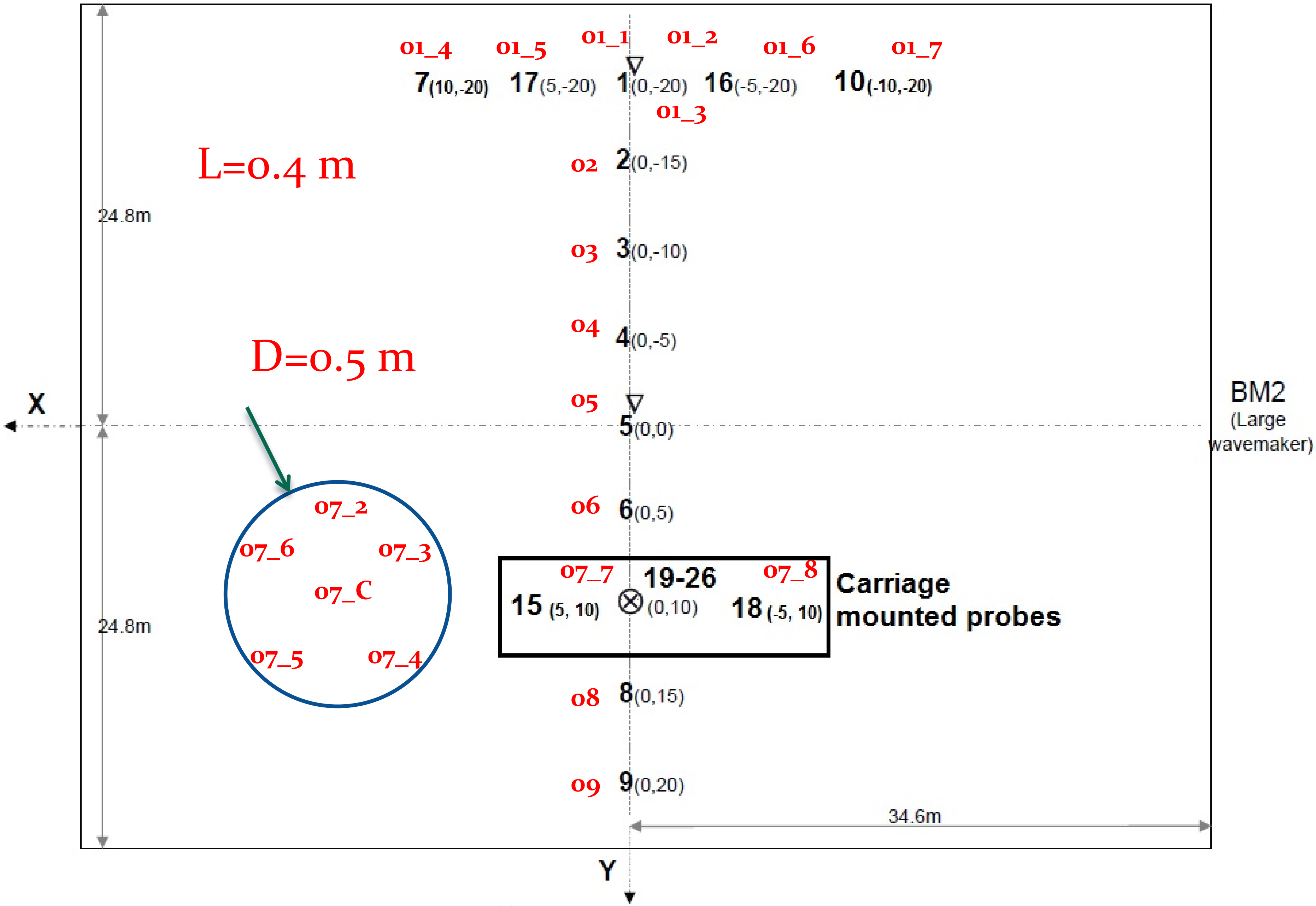
MARINTEK Ocean Basin Wave Makers



Experiments May-June 2012

Sensors and sensor

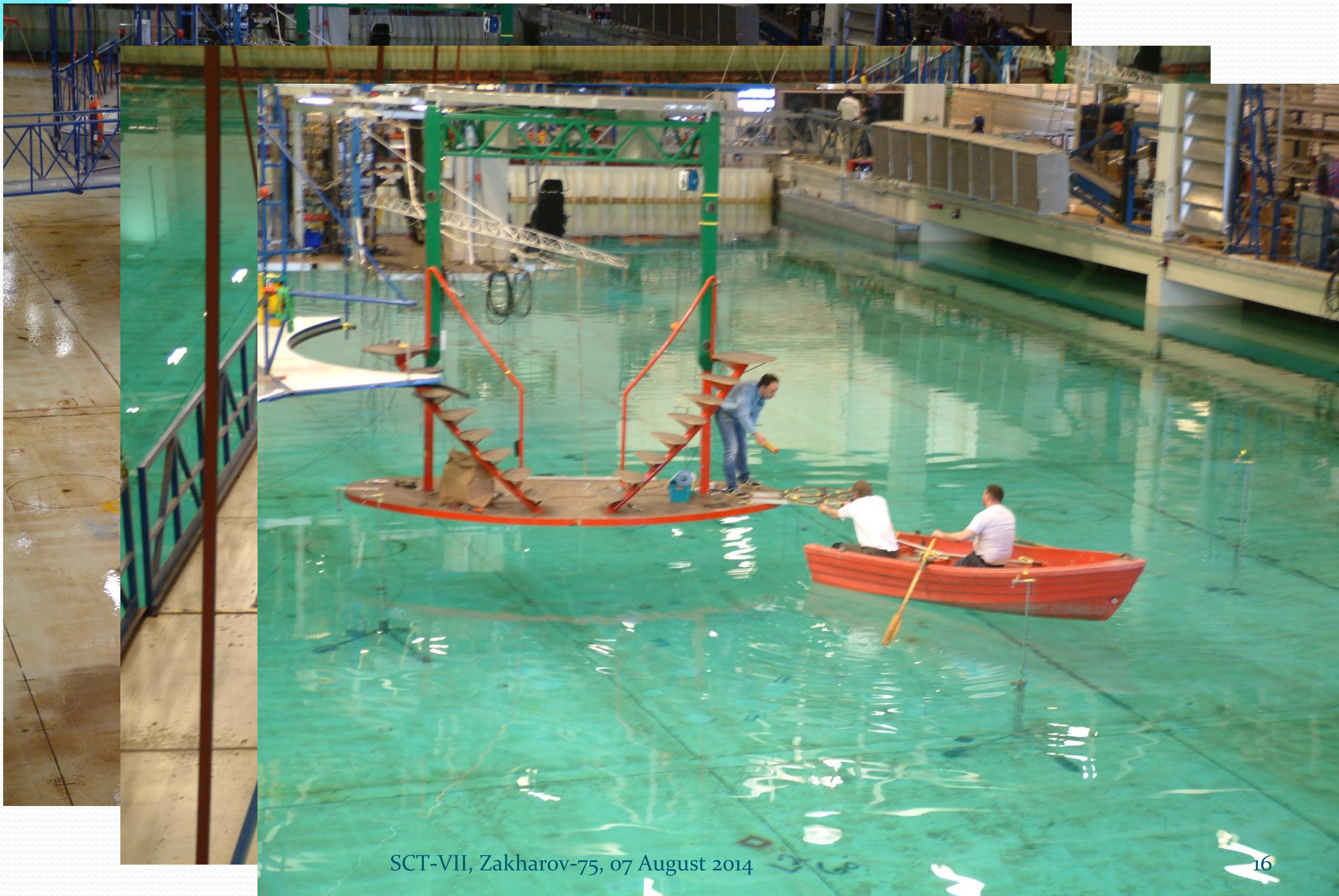
MULTIFLAP WAVEMAKER (BM3)



Distance from paddles (and from axe of basin)

- wave_01_1 (- 0.2 m), wave_01_2 (+ 0.2 m) L=4.7 m from paddles
- wave_01_4 (-10 m), wave_01_5 (-5 m), wave_01_6 (+5 m), wave_01_7 (+10 m) L=4.8 m
- wave_01_3 (0 m) L=5.0 m
- wave_02 (0 m) L=9.8 m
- wave_03 (0 m) L=14.8 m
- wave_04 (0 m) L=19.8 m
- wave_05_1 (-0.2 m) and wave_05_2 (+0.2 m) L=24.7 m
- wave_05_3 (0 m) L=25 m
- wave_06 (0 m) L=29.8 m
- wave_07_2 (0 m) L=34.3 m
- wave_07_6 (-0.4 m) and wave_07_3 (+0.4 m) L=34.6 m
- wave_07_7 (-5 m), wave_07_C (0 m), wave_07_8 (+5 m) L=34.8 m
- wave_07_5 (-0.3 m) and wave_07_4 (+0.3 m) L=35.2 m
- wave_08 (0 m) L=39.8 m
- wave_09 (0 m) L=44.8 m

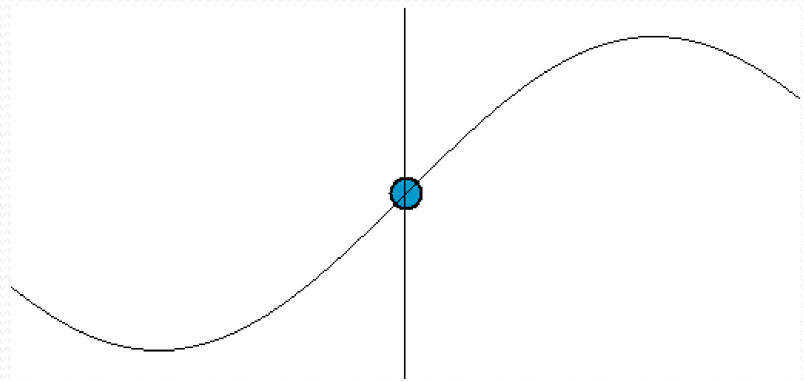
Filling of basin – bottom is sinking



Physical parameters of the sensors

- The sensor records phase motion of the waves.
- Deep water (λ<D) in our case λ<3 m, k=2π/λ>2 m^{-1}
- Phase velocity for deep water is

$$v_{Ph} = \frac{g}{w} = \frac{gT}{2p} = \sqrt{\frac{g}{k}}$$



- Wavelength must be λ>0.05 m

  L~2.5 cm, so λ>2*L and

  k < 120 m^{-1}

- Spectrum in our case is

$$f = \frac{w}{2p} = \frac{\sqrt{gk}}{2p}$$

and real frequency is

0.7Hz<f<5.5 Hz

max f_S of sensors = 200 Hz

f_S>> real f

$t = L/v_{Ph} = L*2\pi/(g*T)$ s

The set of regular wave experiments (quasi-one dimensional waves, angle =90 ^{o})

- Frequency dependence.

  $A=0.068m$, $\tau=1$ s, $\tau=1.05$ s, $\tau=1.1$ s

- Frequency and amplitude dependences. $A=0.2m$, $\tau=1$ s, $\tau=1.67$ s; $A=0.15$ m, $\tau=2.22$ s; $A=0.12m$, $\tau=2.5$ s
- Amplitude dependence. $A=0.068m$, $\tau=1$ s; $A=0.2$ m, $\tau=1.$ s
- Sum of two harmonic waves

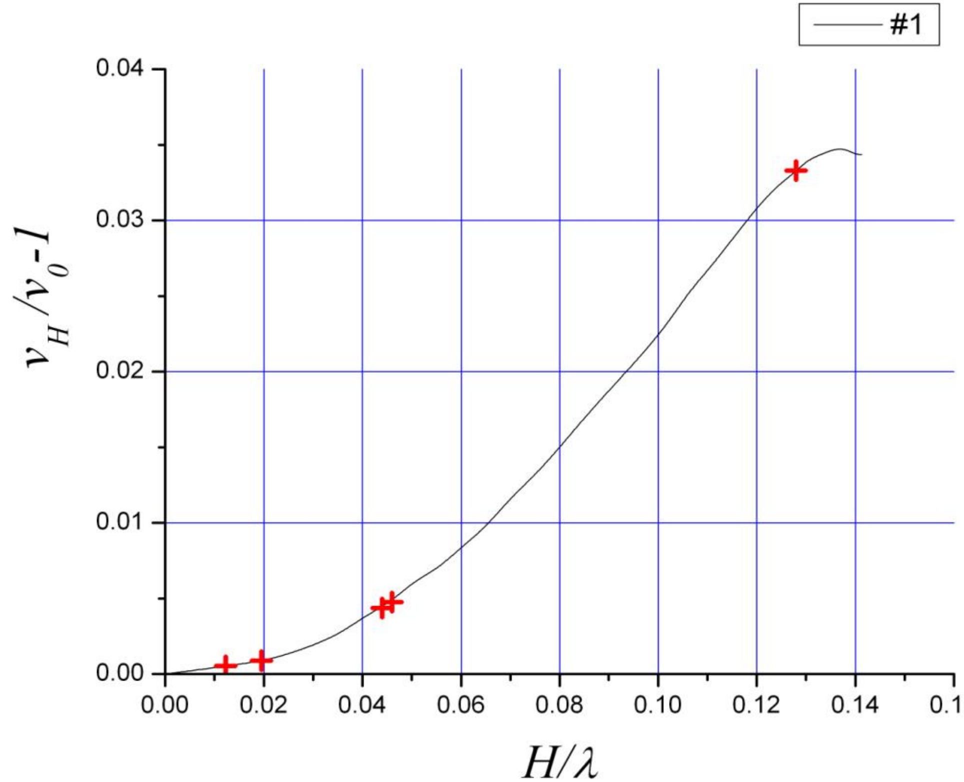
  $A=0.2$ m, $\tau=1.67$ s + $A=0.12$ m, $\tau=2.5$ s

- Sum of three harmonic waves.

  $A=0.2m$, $\tau=1.67$ s + $A=0.15$ m, $\tau=2.22$ s + $A=0.12m$, $\tau=2.5$ s

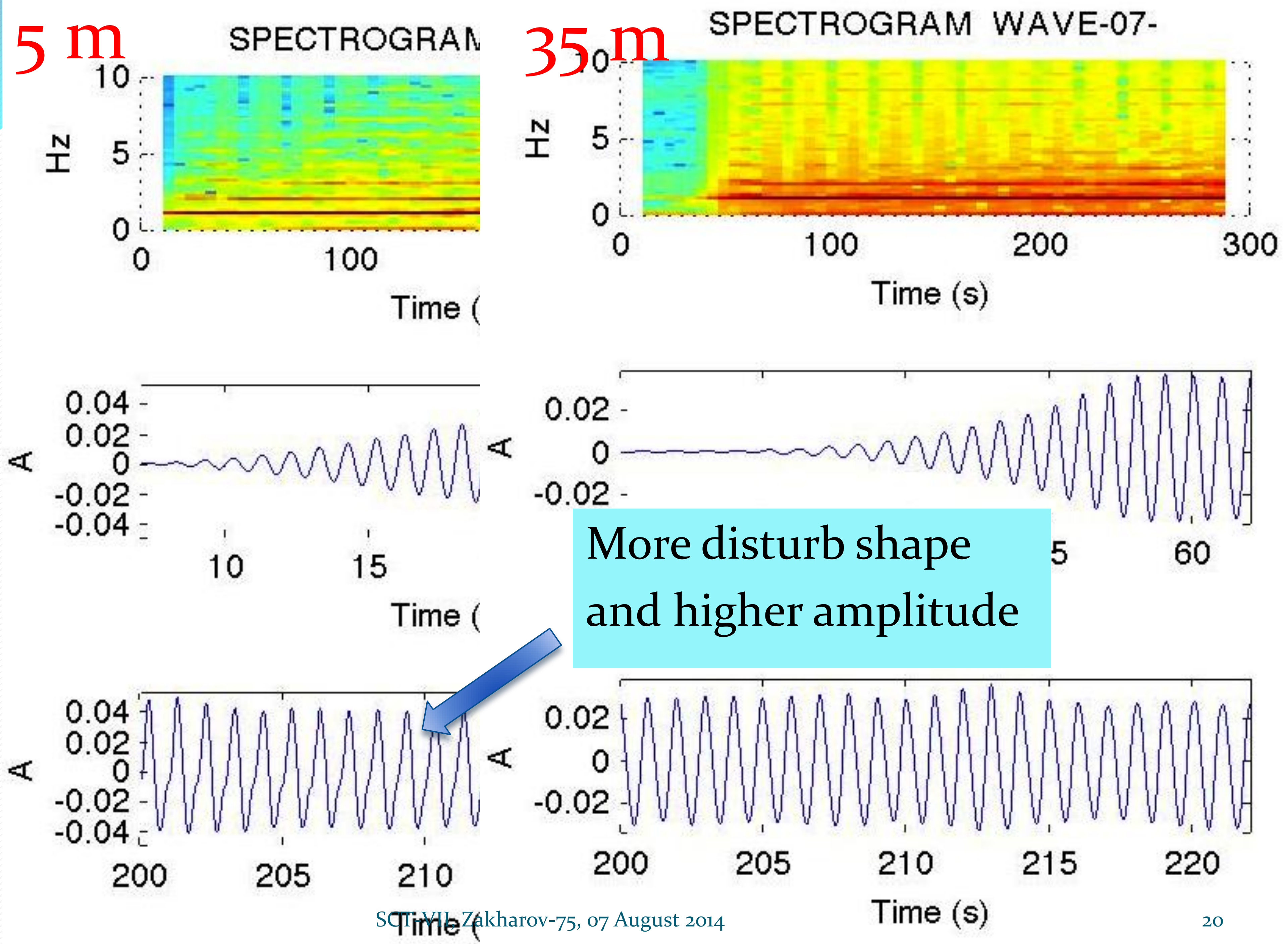
Nonlinear waves

L.W.Schwartz, Strong nonlinear waves, Ann. Rev. Fluid Mech., 14, 39-60, 1982

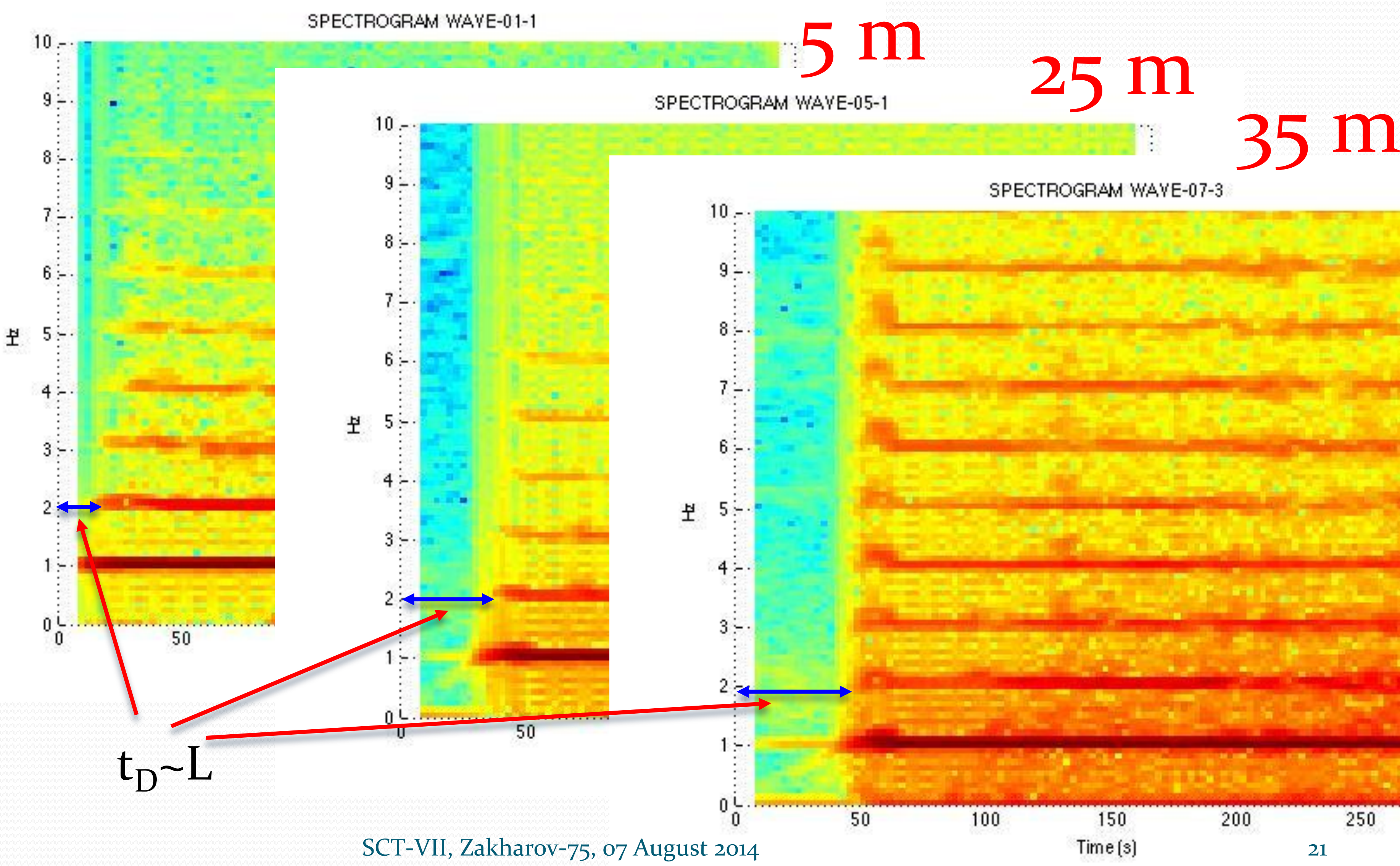


Set of our measurements was next:

Test	H, m	t, [illegible]	l, m	H/λ
6190	0.2	1	1.56	0.128
6200	0.2	1.67	4.35	0.046
6500	0.068	1	1.56	0.044
6210	0.15	2.22	7.69	0.0195
6220	0.12	2.5	9.75	0.0123

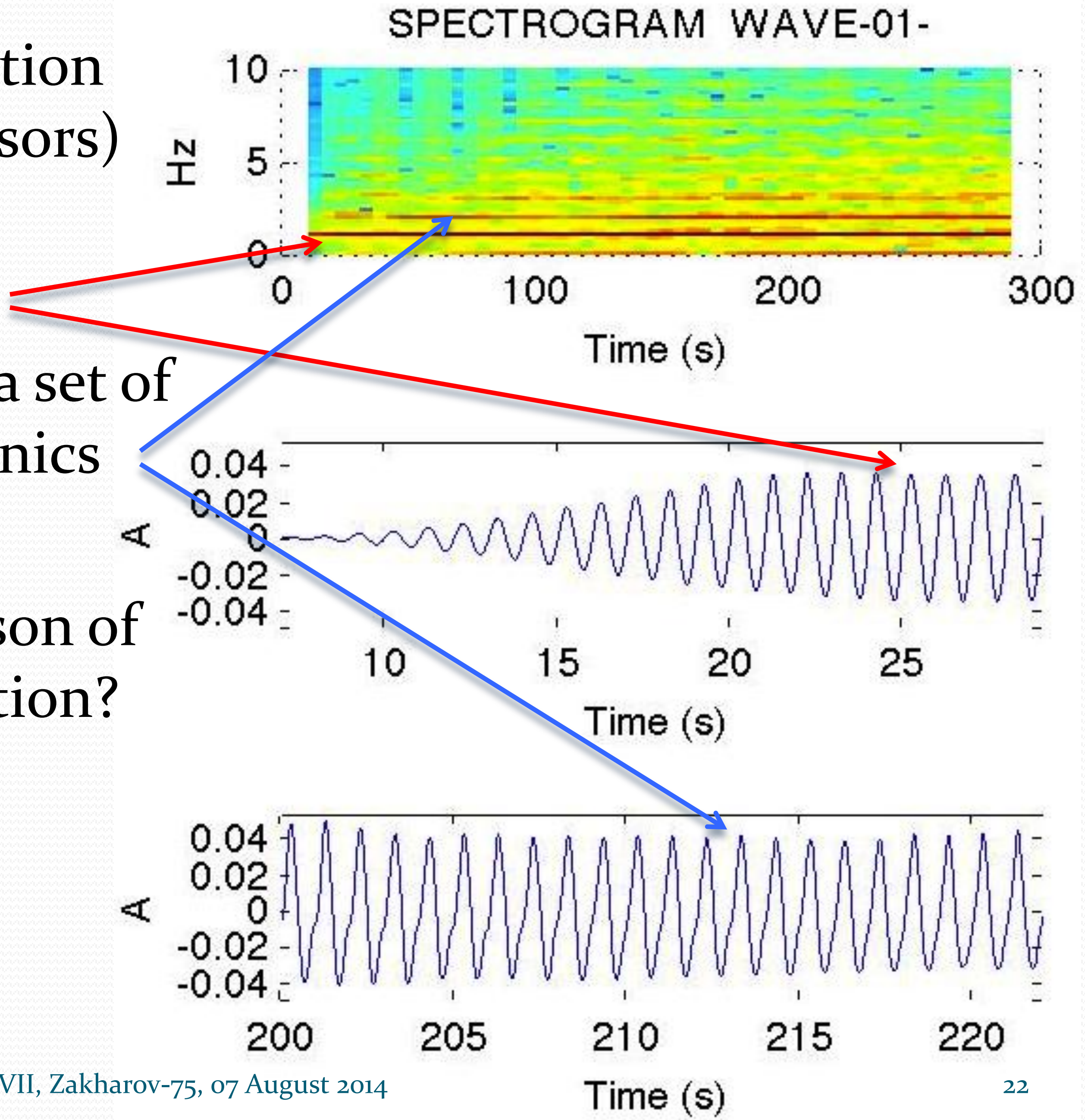

5 m
35 m
SPECTROGRAM
SPECTROGRAM WAVE-07-
Hz
Time (s)
A
More disturb shape
and higher amplitude

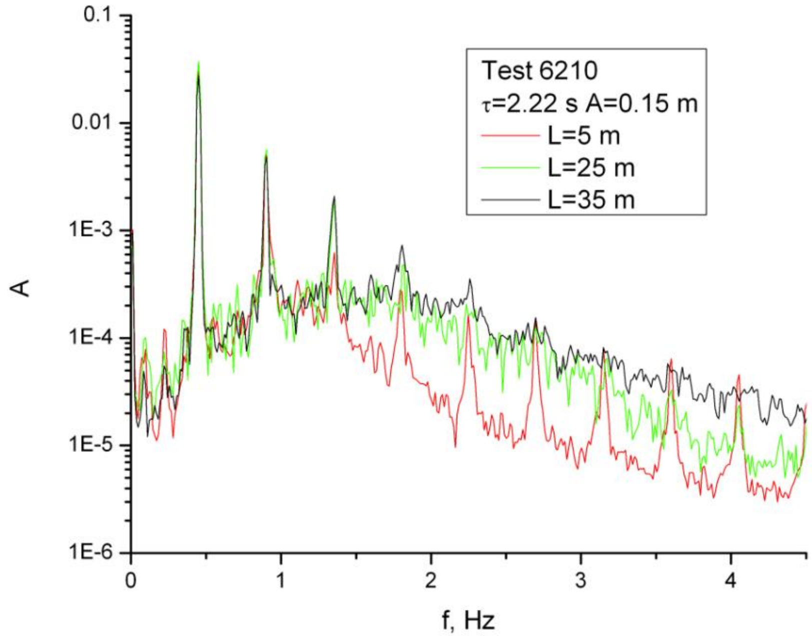
Transformation of the harmonic wave shape with distance

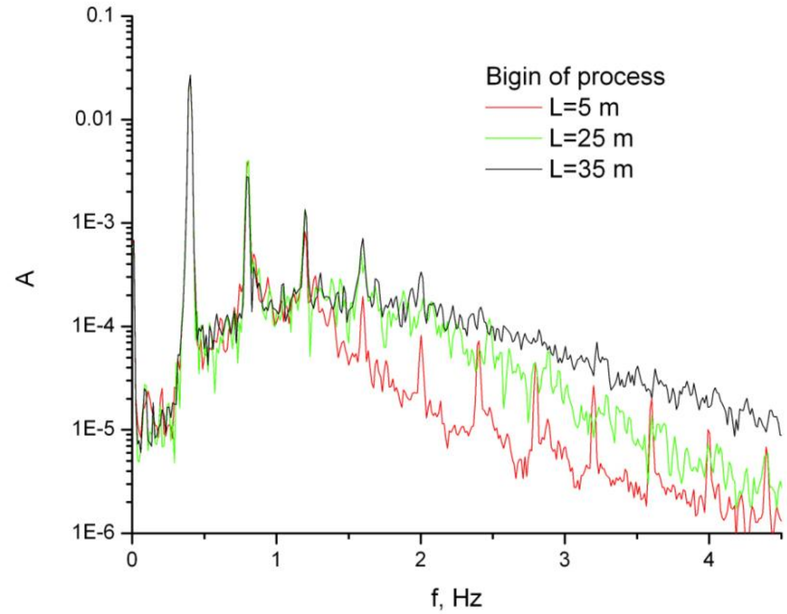


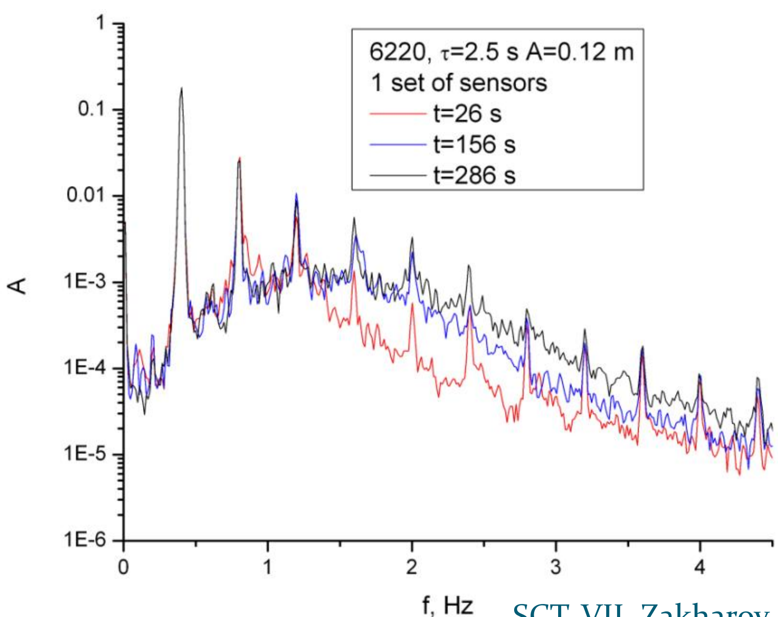
Change of harmonic wave with time

- The wave formation (first row of sensors)
- Initial we had harmonic wave
- Latter – appear a set of multiple harmonics
- What is the reason of this transformation?









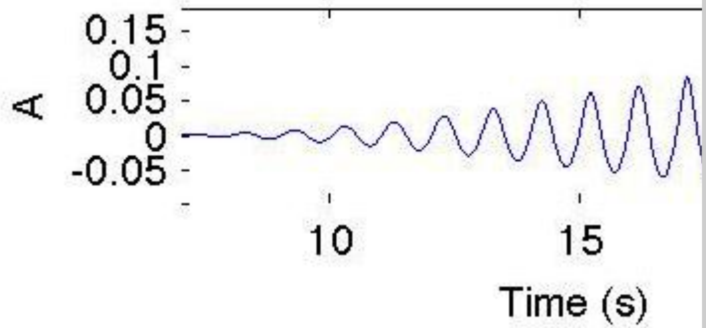
Test 6210 A=0.15 m, τ=2.22 s

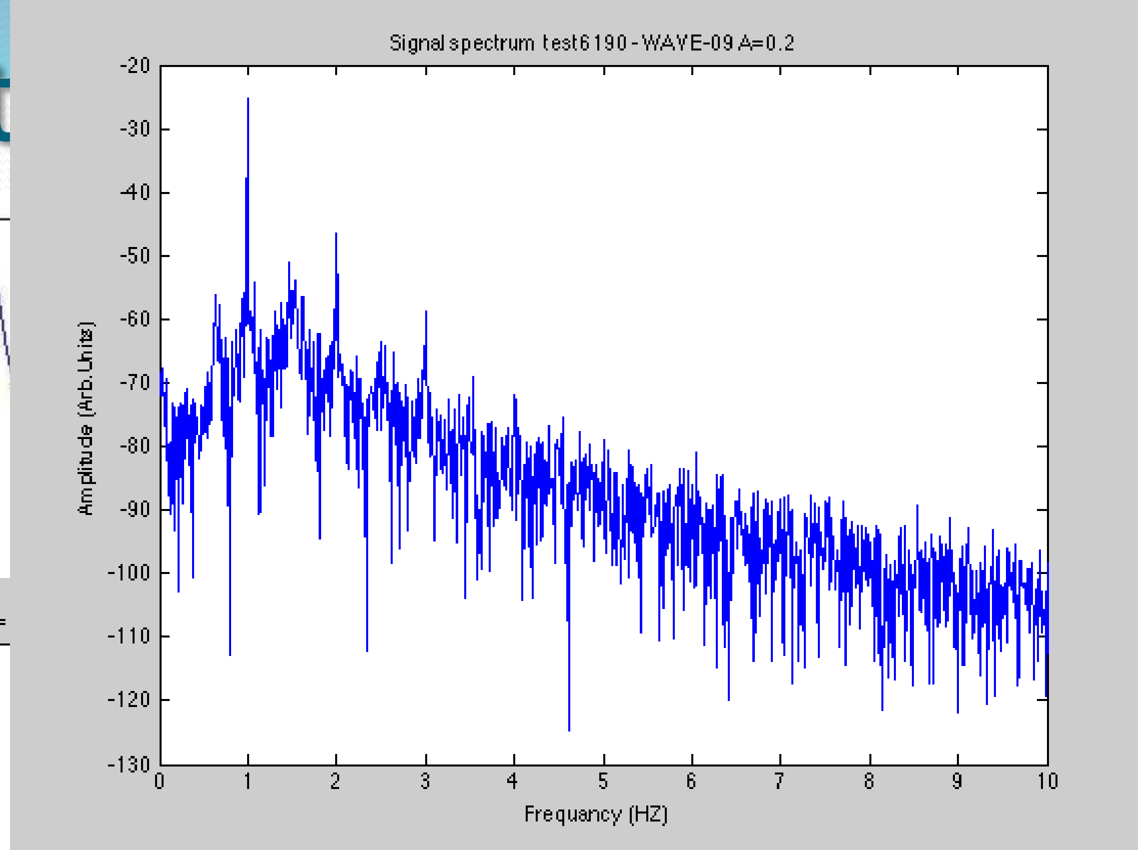
$A/\lambda=0.0195$

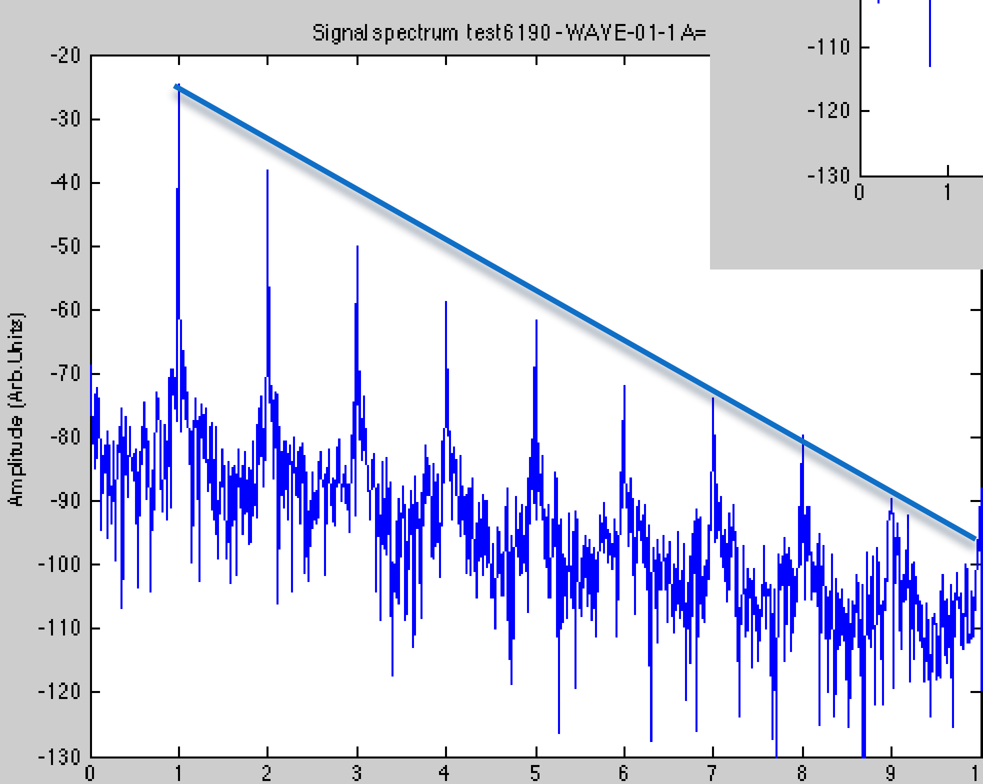
Test 6220 A=0.12 m, τ=2.5 s

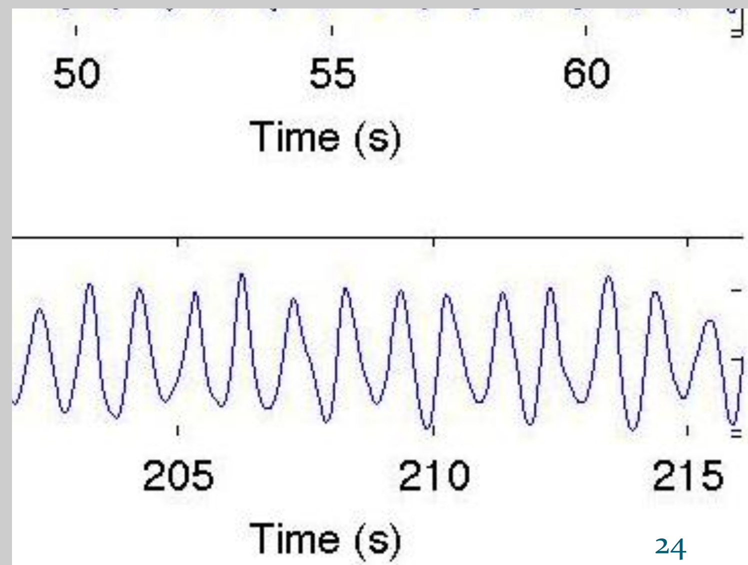
$A/\lambda=0.0123$

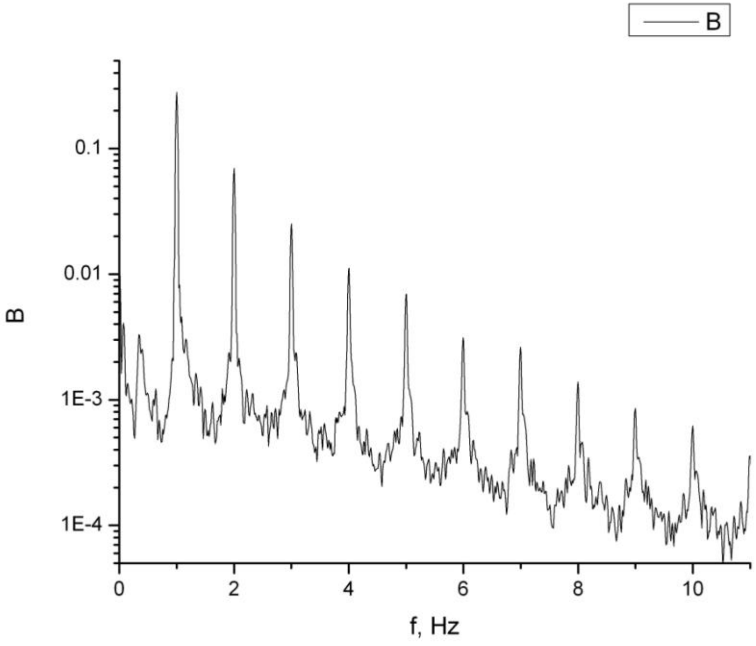
High intensit

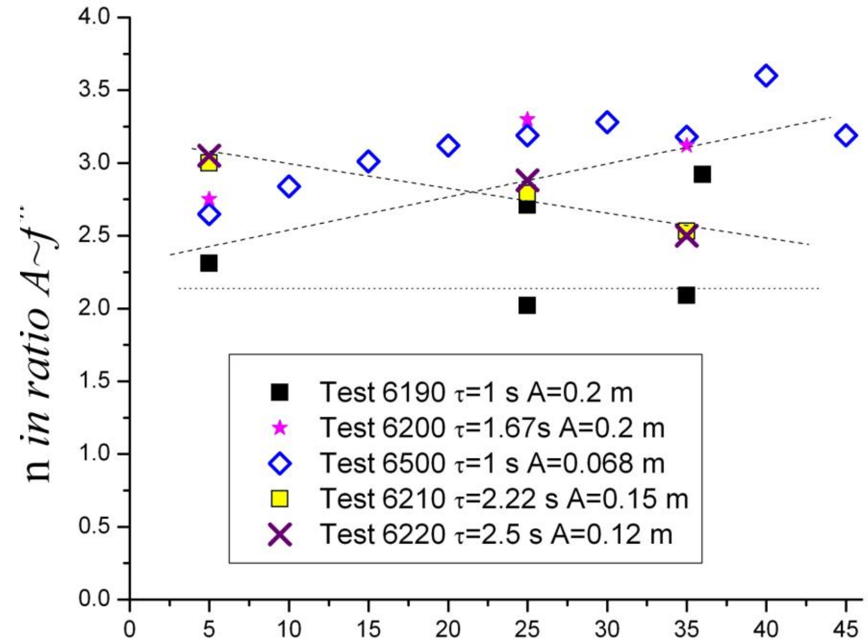


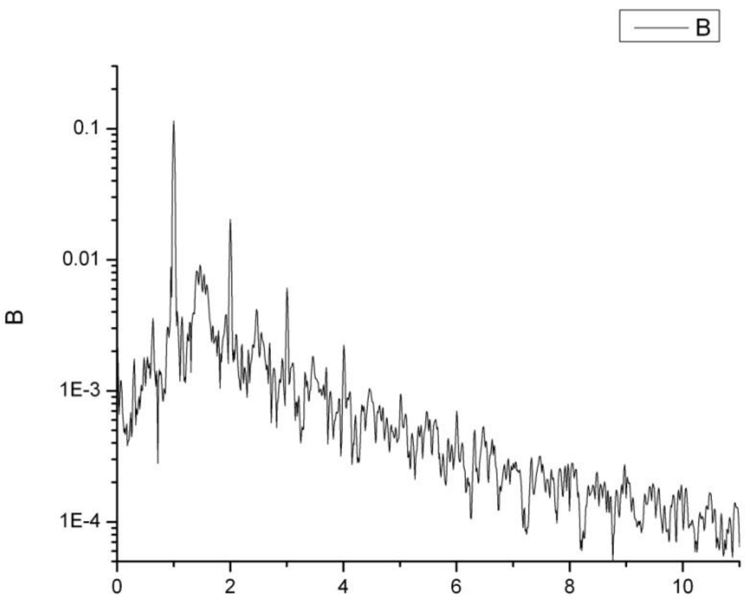


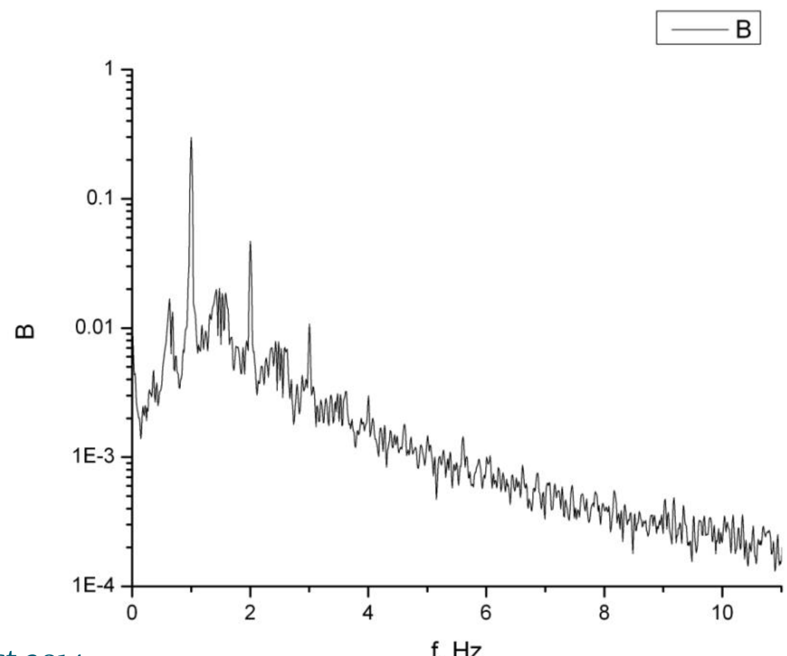




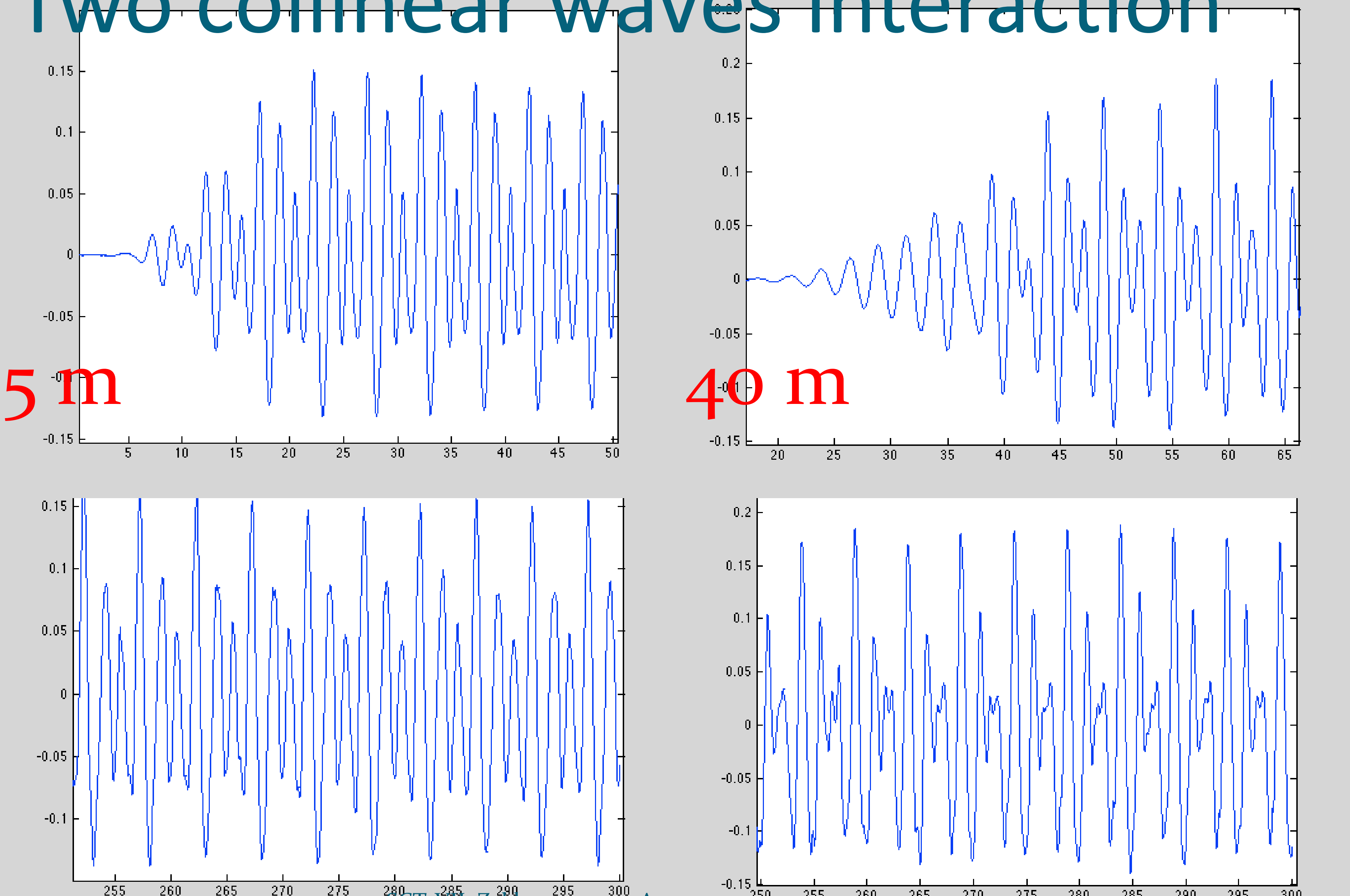

B
0.1
0.01
1E-3
1E-4
0
2
4
6
8
10
f, Hz


n in ratio A~f^-n
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
5
10
15
20
25
30
35
40
45
Test 6190 τ=1 s A=0.2 m
Test 6200 τ=1.67s A=0.2 m
Test 6500 τ=1 s A=0.068 m
Test 6210 τ=2.22 s A=0.15 m
Test 6220 τ=2.5 s A=0.12 m

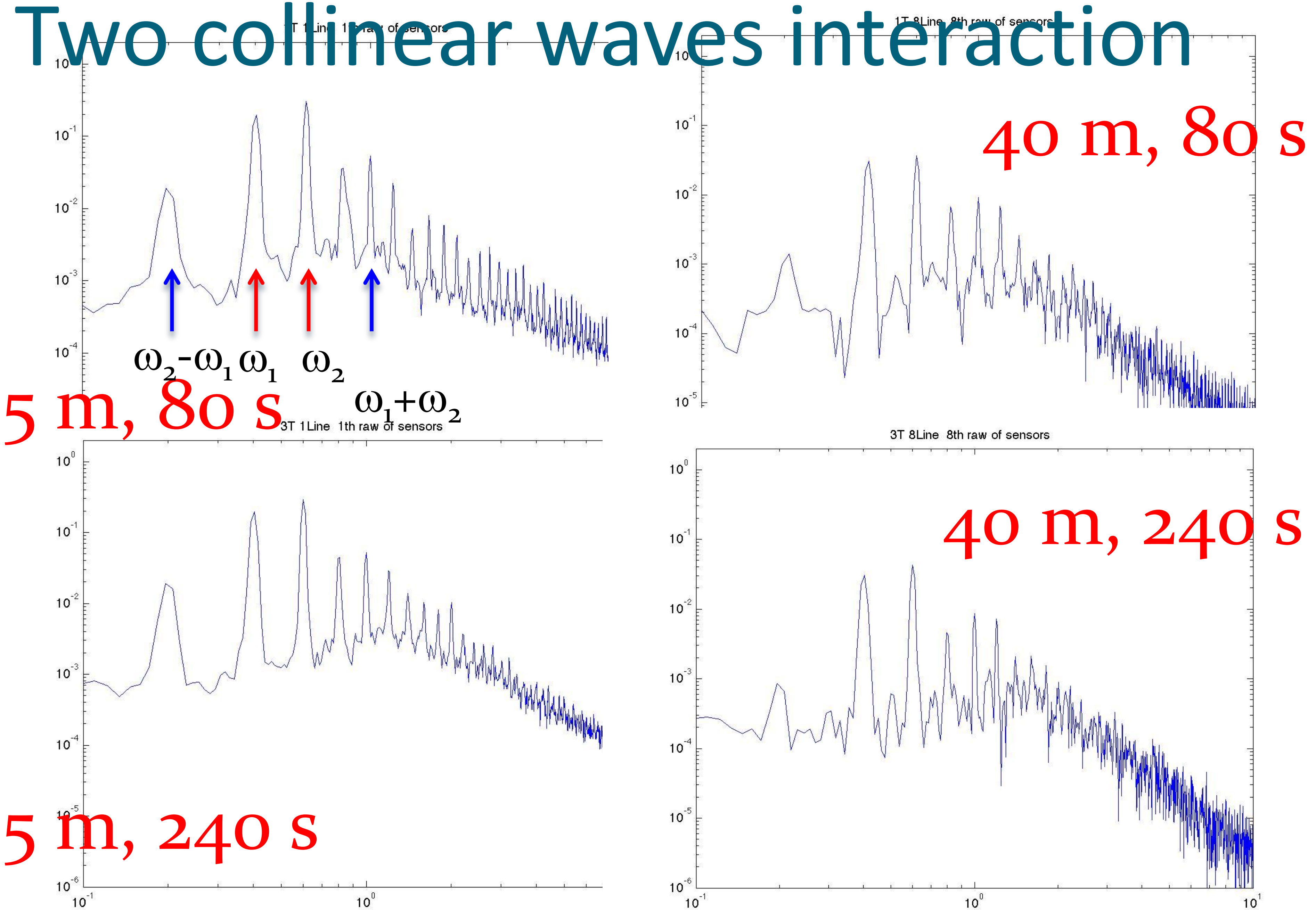

B
0.1
0.01
1E-3
1E-4
0
2
4
6
8
10
f, Hz


B
1
0.1
0.01
1E-3
1E-4
0
2
4
6
8
10
f, Hz

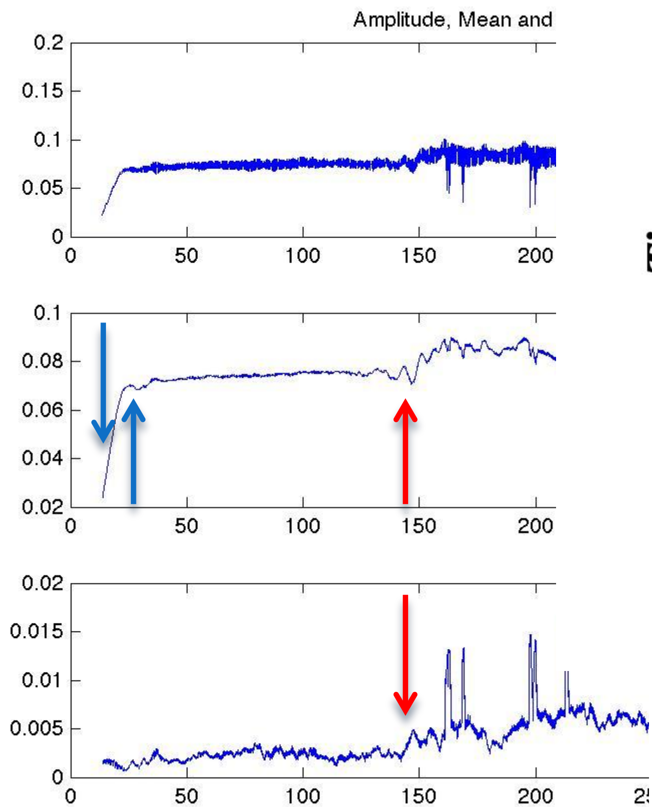
Two collinear waves interaction

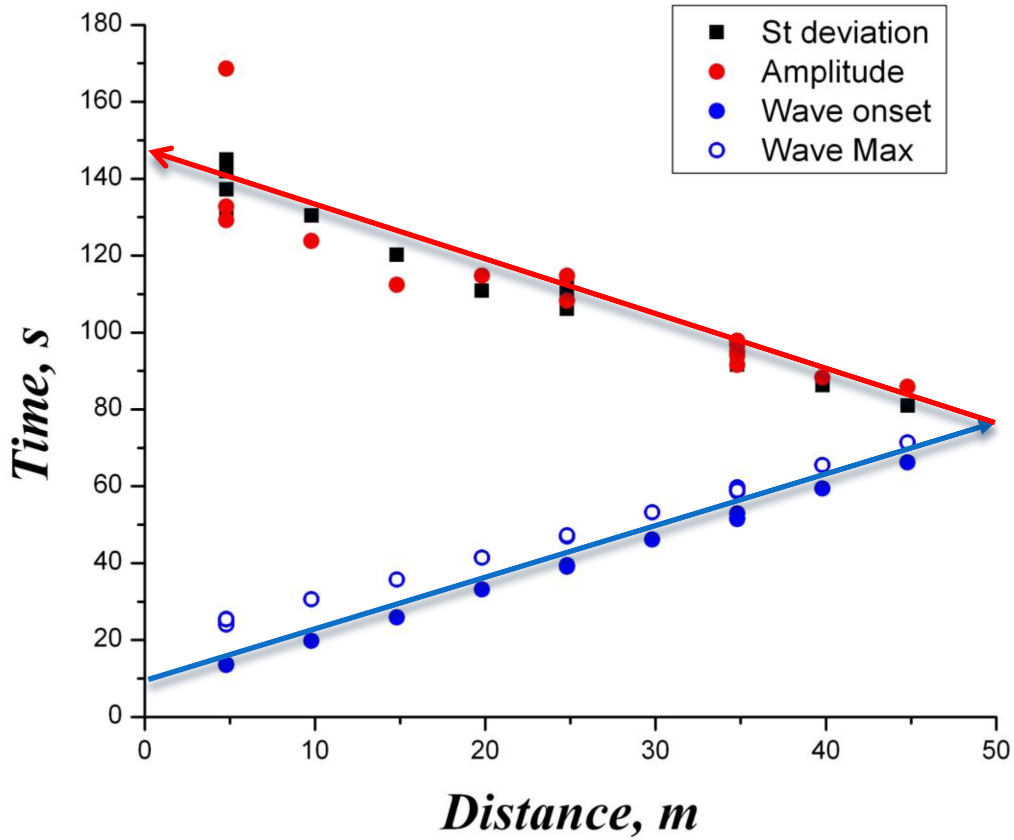


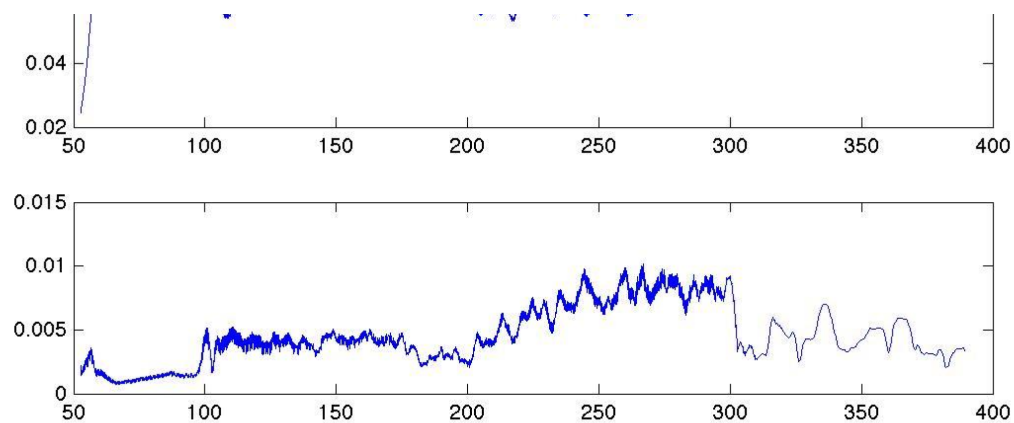
Two collinear waves interaction



Reflection



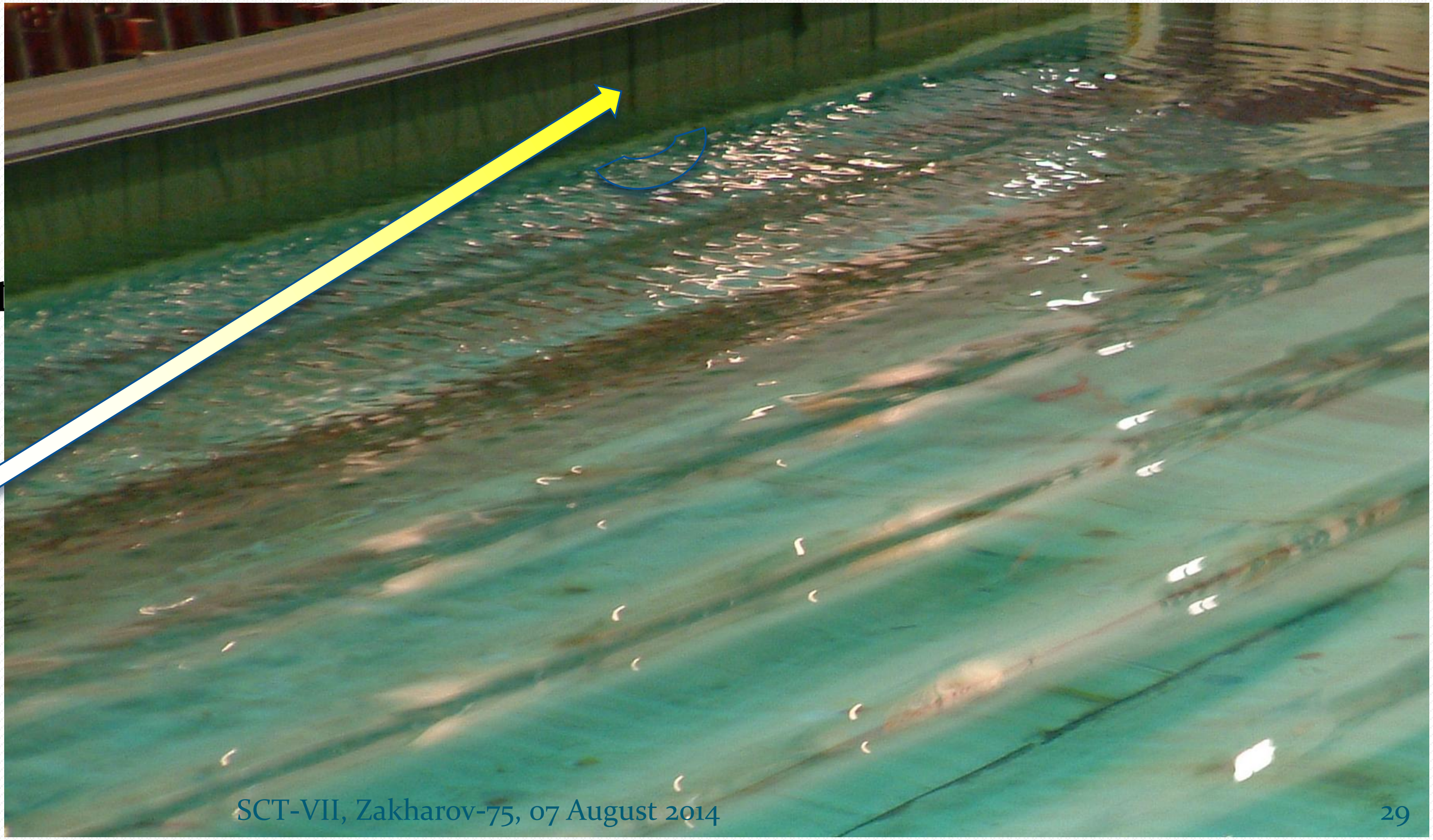




5 m

Why harmonic wave format a wide spectrum of frequencies?

- Nonlinear effects – different wave frequency and its velocity, which interact.
- Inte



Paddle split

Ripple propagation in the MARINTEK basin Trondheim, Norway

23 may 2012
f=1 Hz, A=0.068 cm
L=25 m, **regular wave**

t=8:15:23

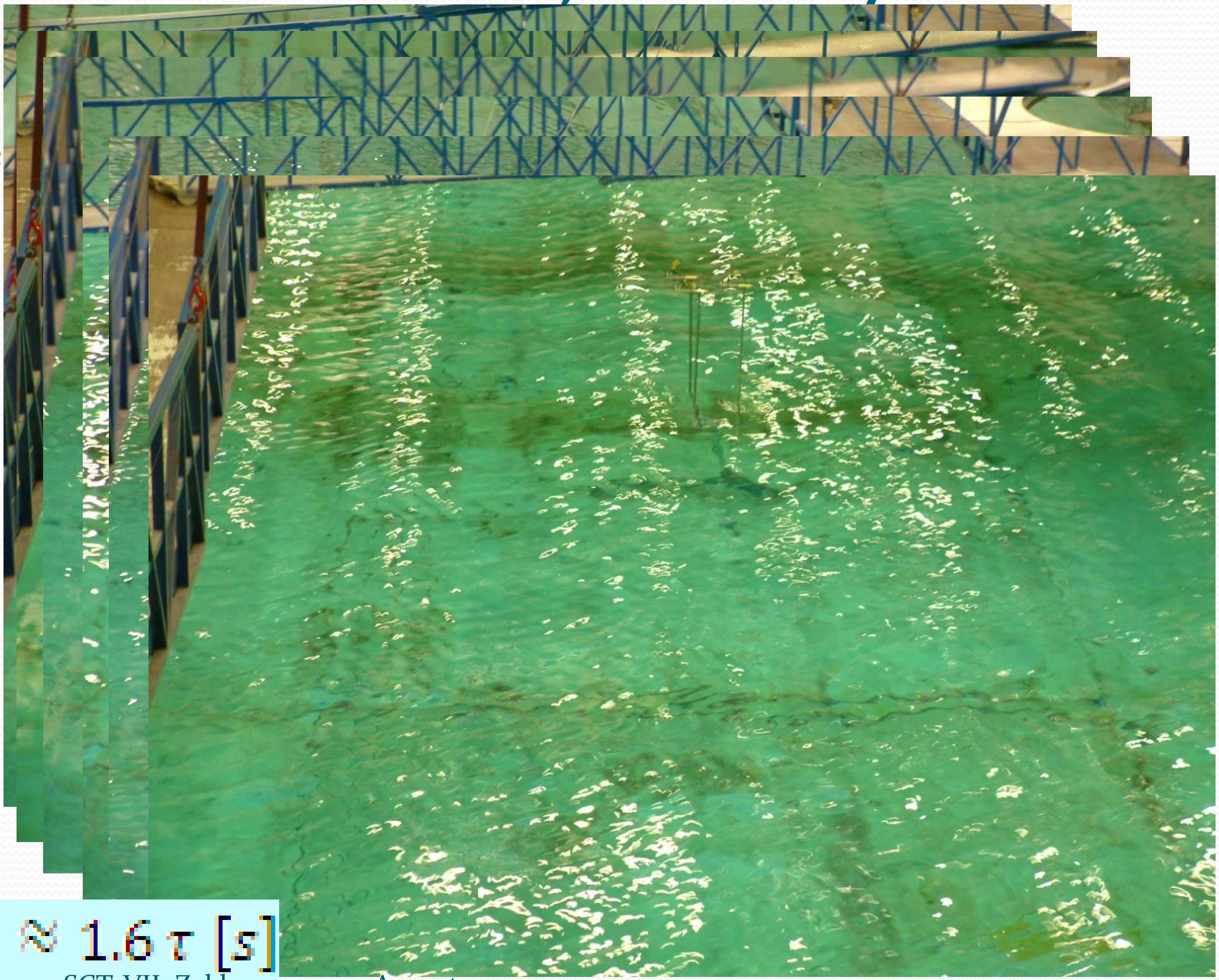
t=8:15:44

t=8:16:09

t=8:16:29

t=8:16:54

t=8:17:10



$$c \approx 1.25\sqrt{\lambda}[m] \approx 1.6\,\tau\ [s]$$

Experimental team

Lancaster University, Lancaster, UK

Marintek Ocean Wave Basin, Trondheim, Norway

- Peter McClintock
- Suzy Ilic
- James Luxmoor
- Carl Stansberg
- Ivar Nygaard
- Csaba Pâkozdi






Thank you for your attention!

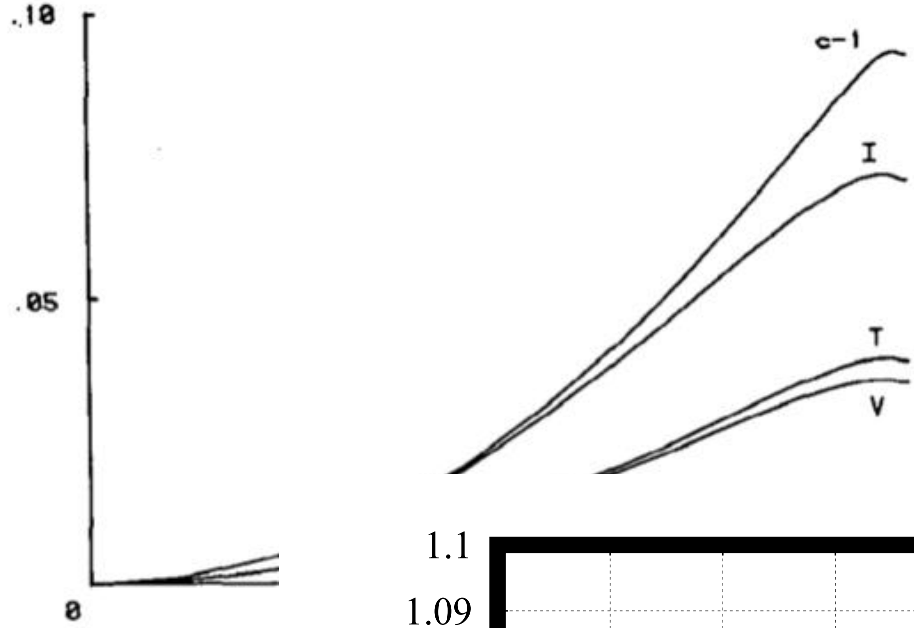
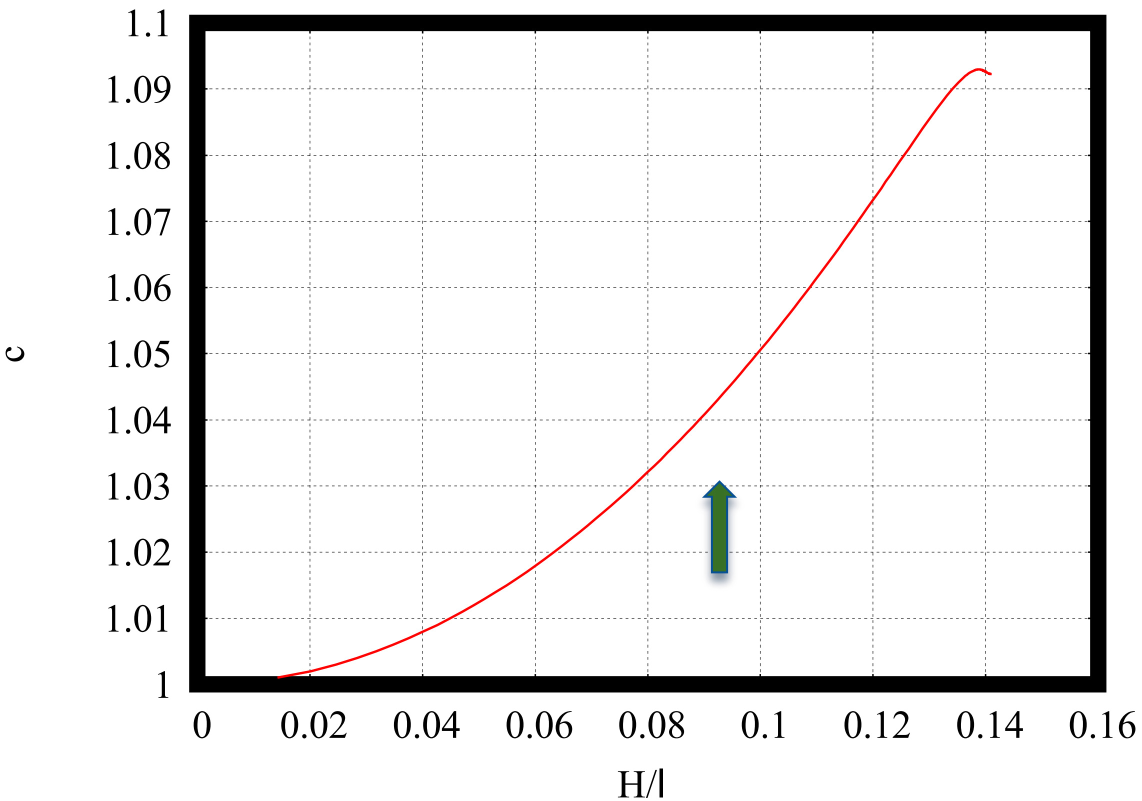


Figure 1 Dimensionless wave s
height for deep-water progressiv

STRONGLY NONLI
WAVES

L. W. Schwartz
Exxon Research and Engineering Company,

Ann. Rev. Fluid Mech.



Ripple propagation in the MARINTEK basin Trondheim, Norway

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t=8:15:23

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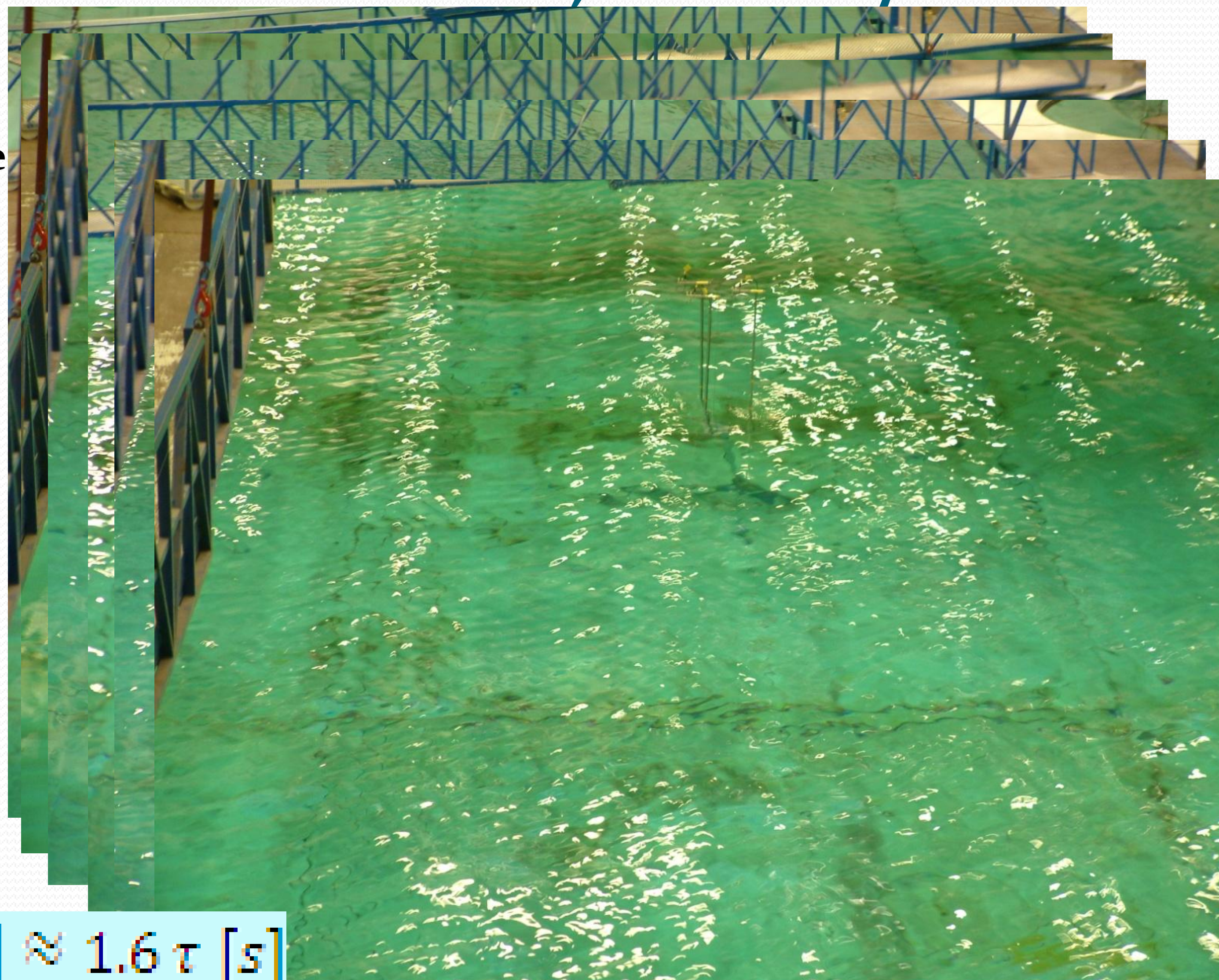
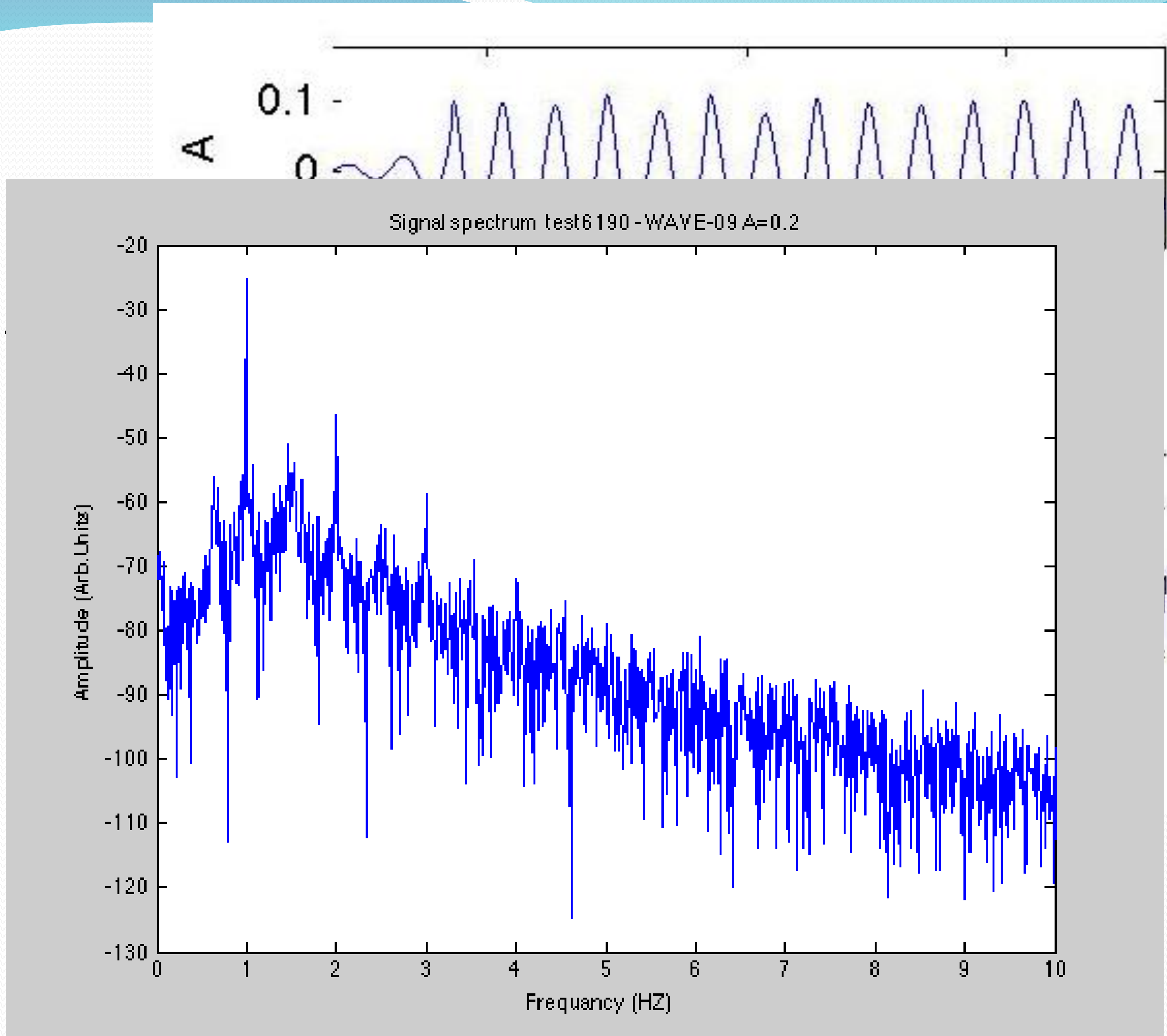
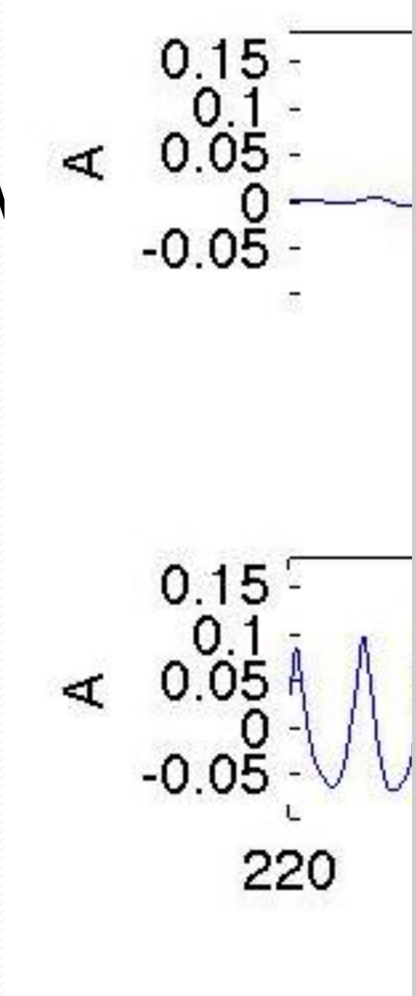


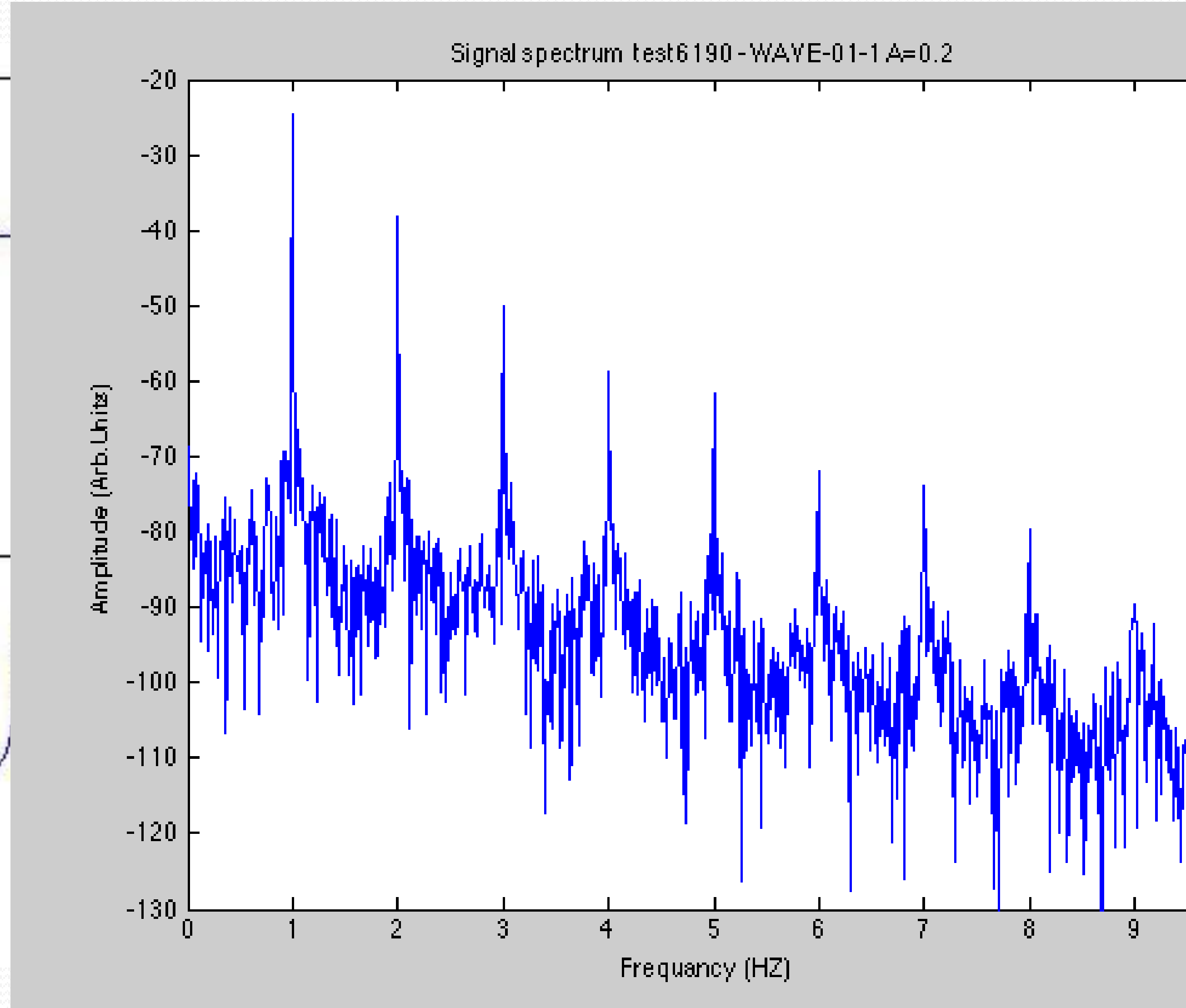
$$c \approx 1.25\sqrt{\lambda}[m] \approx 1.6\,\tau\ [s]$$

- WAVE-


A
0.1
0
Signal spectrum test6190 - WAVE-09 A=0.2
Amplitude (Arb.Units)
-20
-30
-40
-50
-60
-70
-80
-90
-100
-110
-120
-130
0
1
2
3
4
5
6
7
8
9
10
Frequancy (HZ)

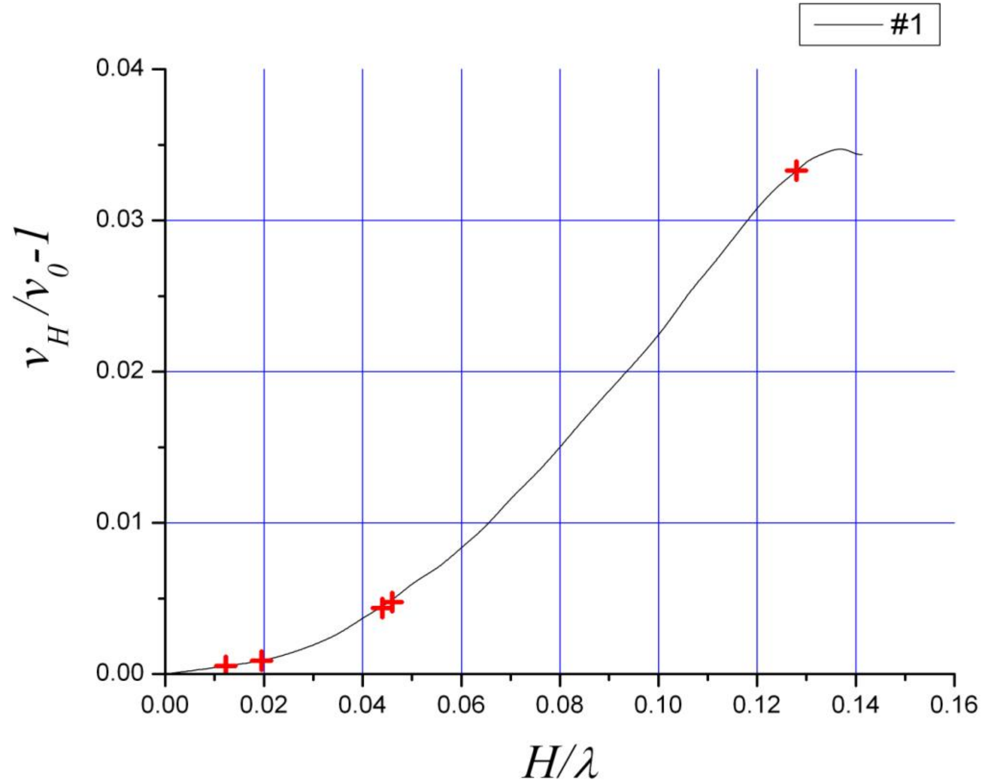
- W


A
0.15
0.1
0.05
0
-0.05
A
0.15
0.1
0.05
0
-0.05
220


Signal spectrum test6190-WAVE-01-1 A=0.2
Amplitude (Arb.Units)
-20
-30
-40
-50
-60
-70
-80
-90
-100
-110
-120
-130
0
1
2
3
4
5
6
7
8
9
Frequancy (HZ)

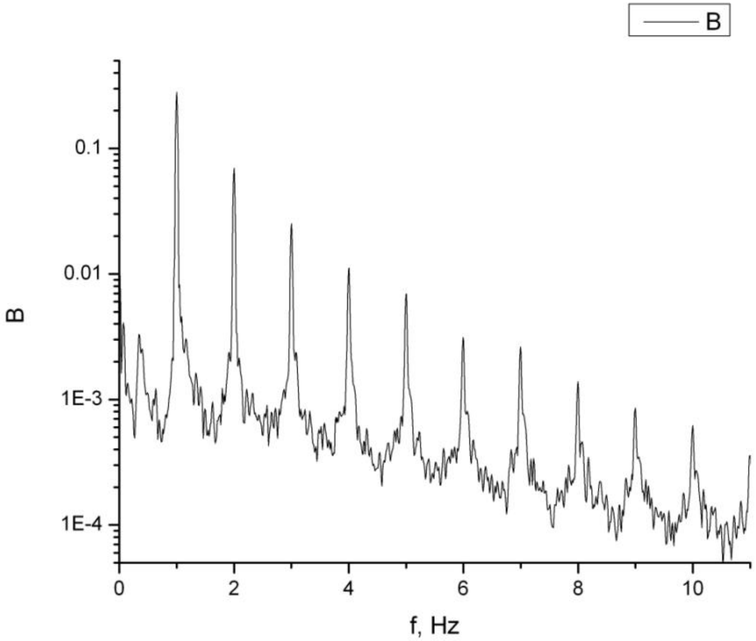
Nonlinear waves

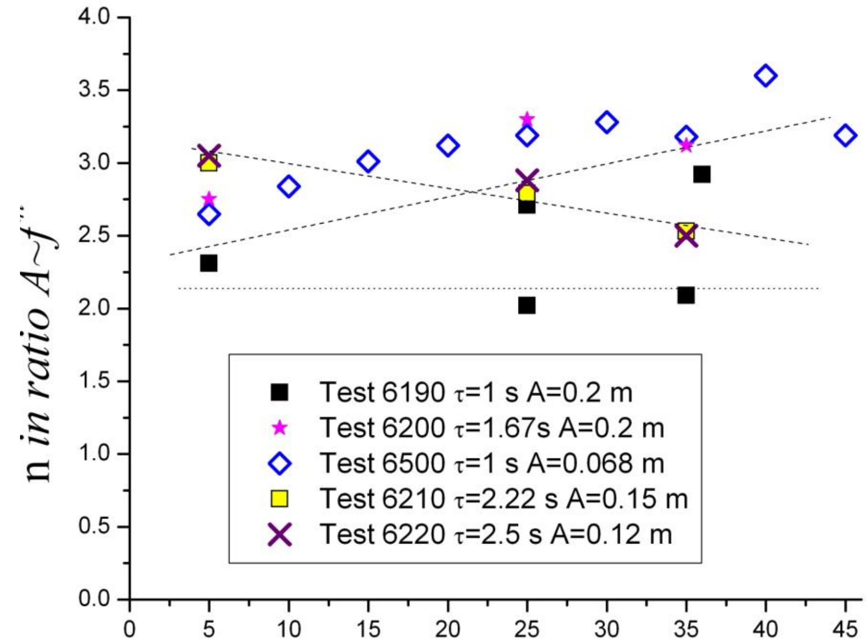
L.W.Schwartz, Strong nonlinear waves, Ann. Rev. Fluid Mech., 14, 39-60, 1982

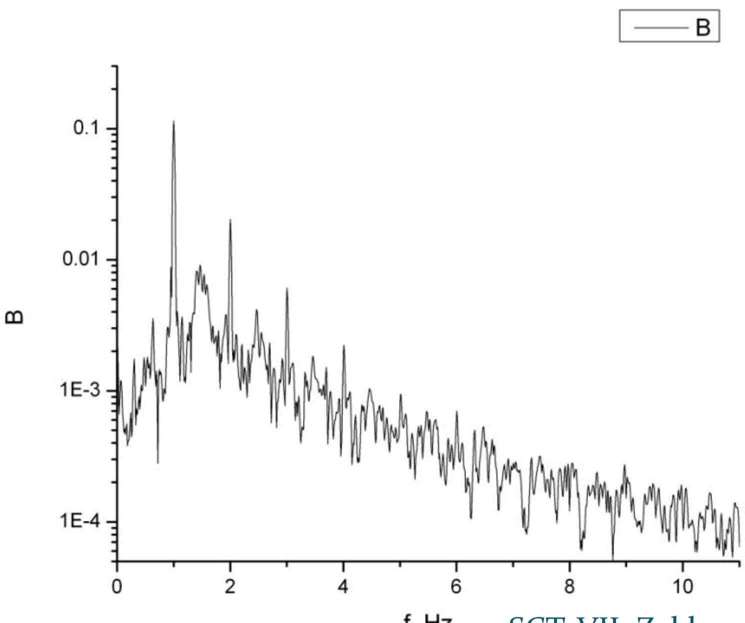


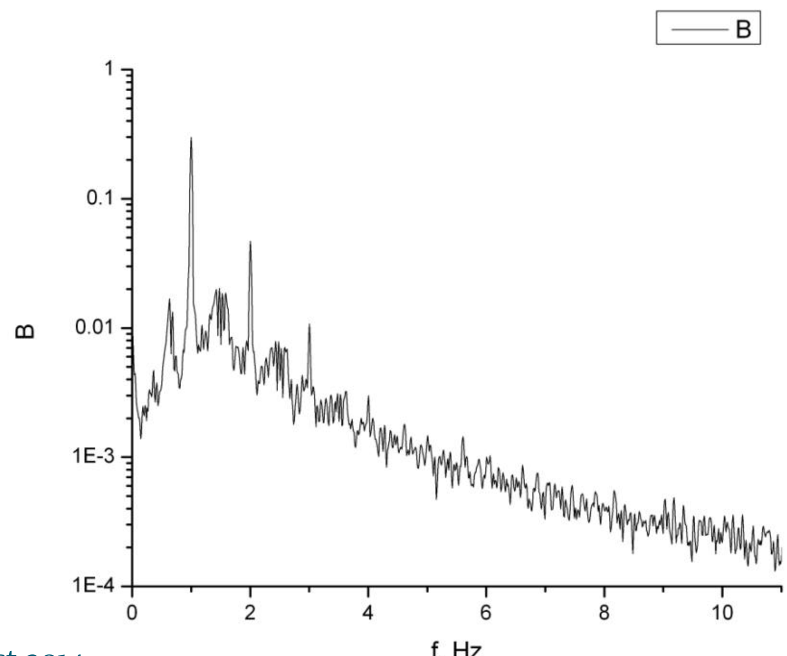
Set of our measurements was next:

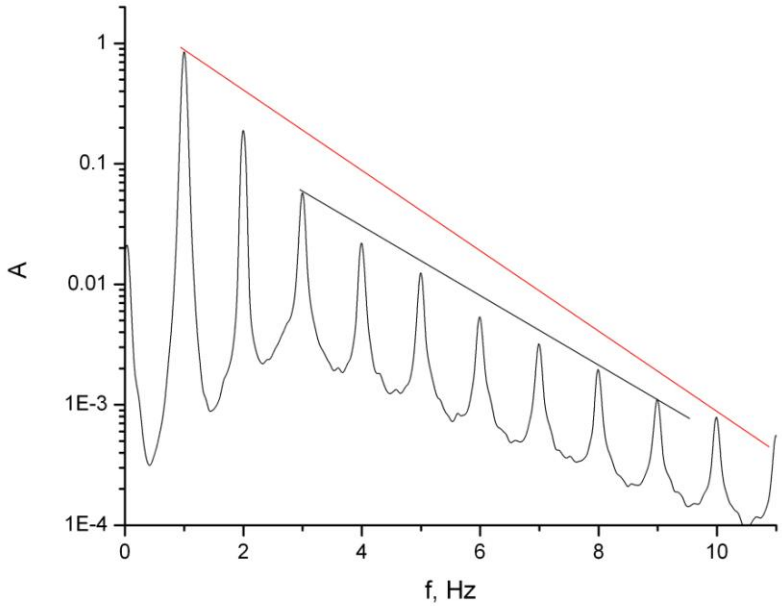
Test	H, m	t, [illegible]	l, m	H/λ
6190	0.2	1	1.56	0.128
6200	0.2	1.67	4.35	0.046
6500	0.068	1	1.56	0.044
6210	0.15	2.22	7.69	0.0195
6220	0.12	2.5	9.75	0.0123

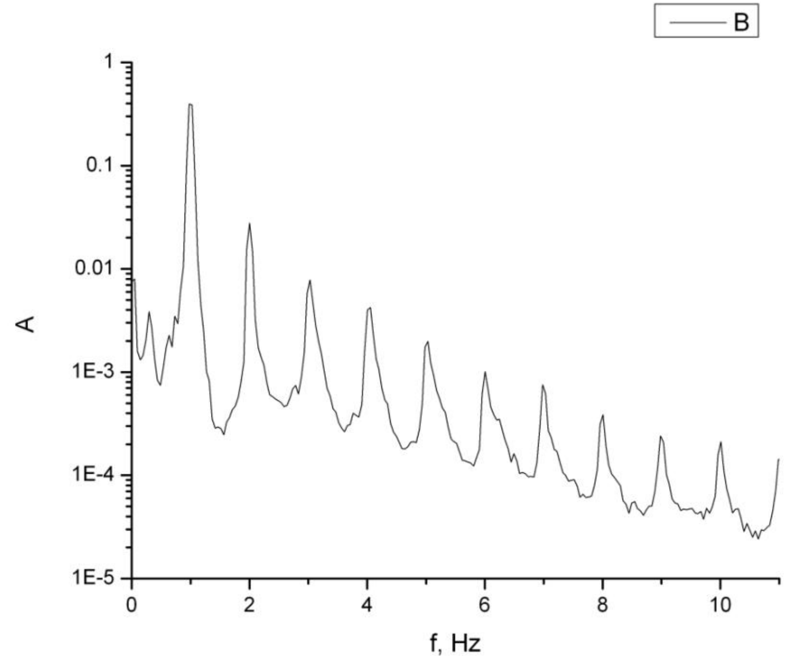

B
0.1
0.01
1E-3
1E-4
0
2
4
6
8
10
f, Hz


n in ratio A~f
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
5
10
15
20
25
30
35
40
45
Test 6190 τ=1 s A=0.2 m
Test 6200 τ=1.67s A=0.2 m
Test 6500 τ=1 s A=0.068 m
Test 6210 τ=2.22 s A=0.15 m
Test 6220 τ=2.5 s A=0.12 m


B
0.1
0.01
1E-3
1E-4
0
2
4
6
8
10
f, Hz

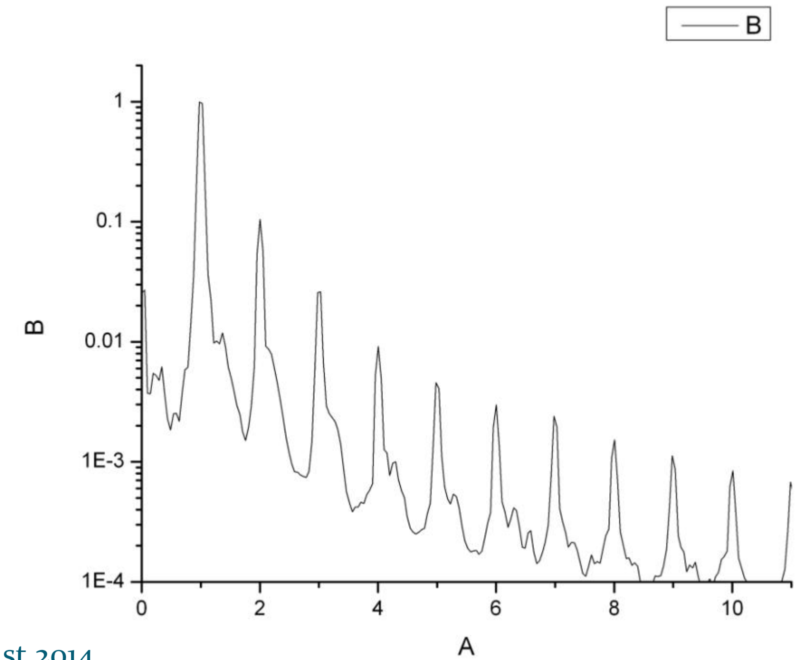

B
1
0.1
0.01
1E-3
1E-4
0
2
4
6
8
10
f, Hz





Test 6500 A=0.068 m, τ=1 s

A/λ=0.045



Complex singularity of a Stokes wave.
S.A. Dyachenko, P.M. Lushnikov, A.O. Korotkevich
20th of June, 2014, Scientific Council at Landau ITP

$c = v(H)/v_0$