Observations of low-frequency surface waves in a vertically oscillating elastic container

Looking for inverse cascade and finding the Dragon Wash

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Characteristic points in ω and k (1)

Dispersion relation: $\omega_k = \sqrt{gk + \frac{\sigma}{\rho}k^3}$, $g = 980\ \frac{cm}{s^2}$, $\sigma \approx 70\frac{dyn}{cm}$, $\rho = 1\frac{g}{cm^3}$

Centre of the capillary-gravity region: $k^* = \sqrt{\frac{g\rho}{\sigma}} \approx 3.7\ \text{cm}^{-1}$, $\lambda^* \approx 1.7\ \text{cm}$, $\omega^* \approx 13.6$ H

A point of zero dispersion: $\beta = \frac{d^2y}{dk^2}$=0, $k_d \approx 1.46\ \text{cm}$, $\lambda_d \approx 4.3\ \text{cm}$, $\omega_d \approx 6.5\ \text{cm}$

Gravity range: $\omega < \omega_d$, $k < k_d$

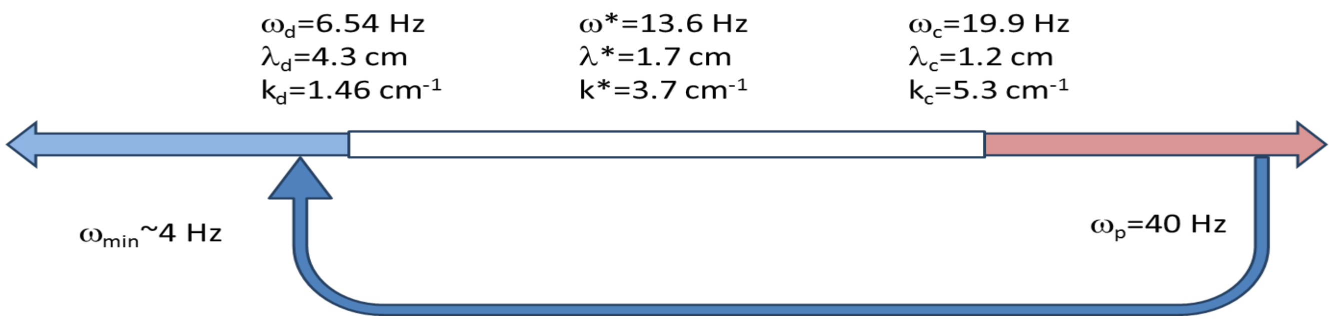
Threshold for decay instability: $\omega_c = 2\omega_{\frac{k}{2}}$, $k_c \approx 5.3\text{cm}^{-1}$, $\lambda_c \approx 1.2\text{cm}$, $\omega_c \approx 19.\ \text{Hz}$

Capillary range: $\omega > \omega_C$ (3-wave processes are allowed)

The lowest mode resulted from the 3-wave process can be found from

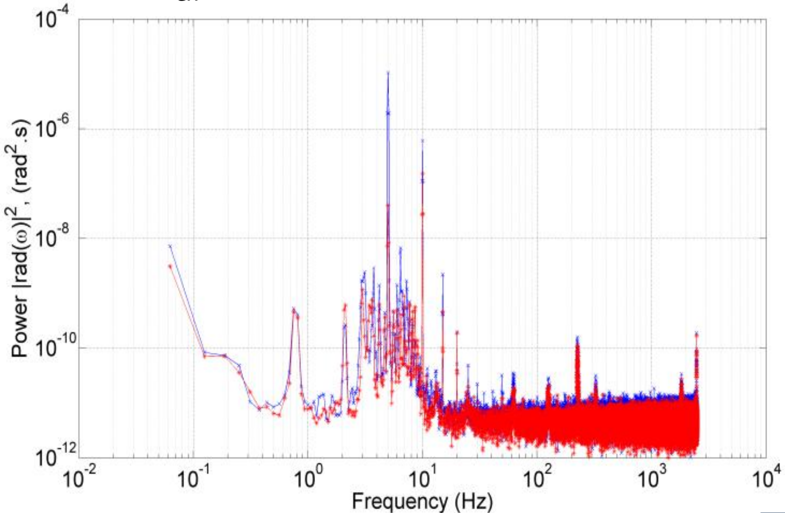
$$\omega_p = \omega_{k_1} + \omega_{k_2},\quad k_1 + k_2 \geq k_L$$

For example, for a parametric wave with ω_p=40 Hz min(ω) ~ 4 Hz, max(λ) ~ 8.5 cm

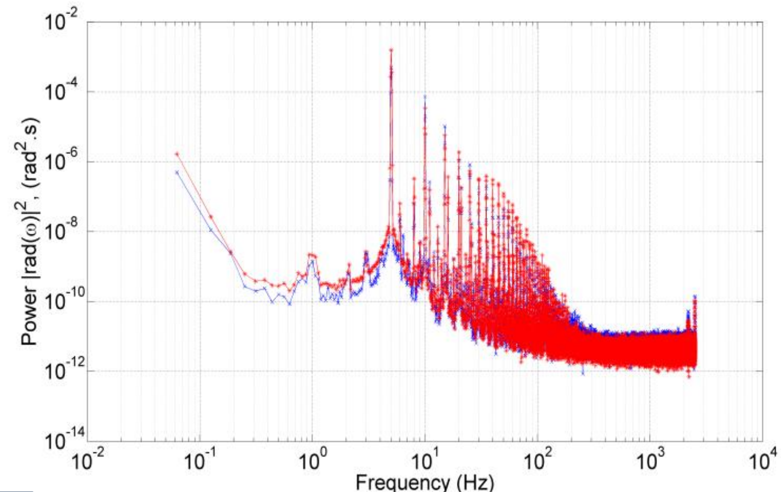


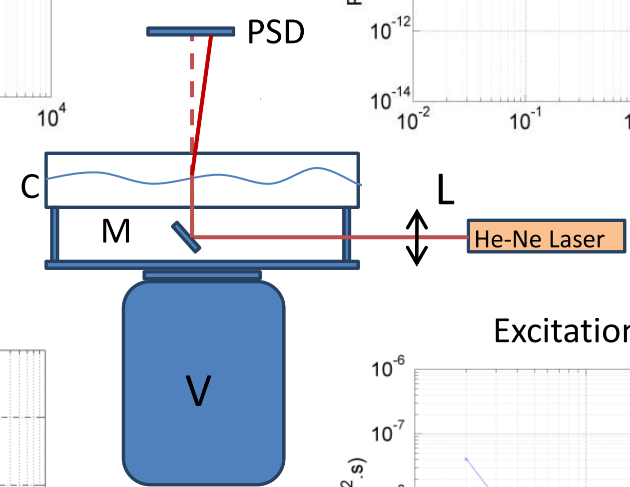
Attempts to observe the inverse cascade in parametrically excited waves

f_{ex}=10hz, small amplitude

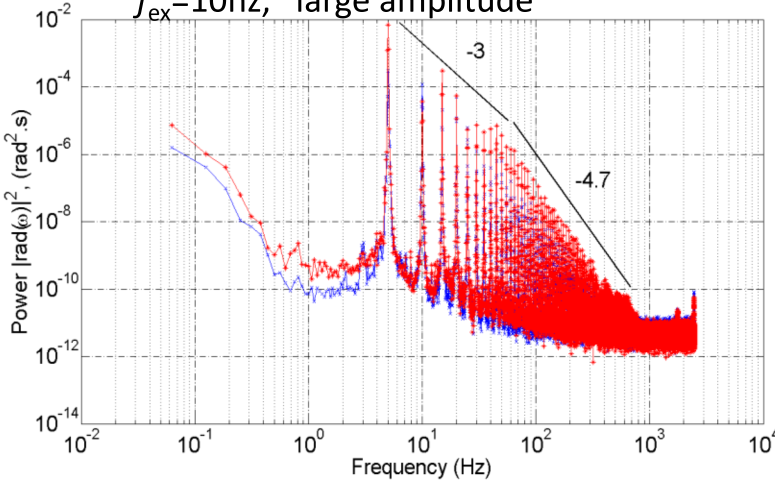


f_{ex}=(12+18)hz, large amplitude

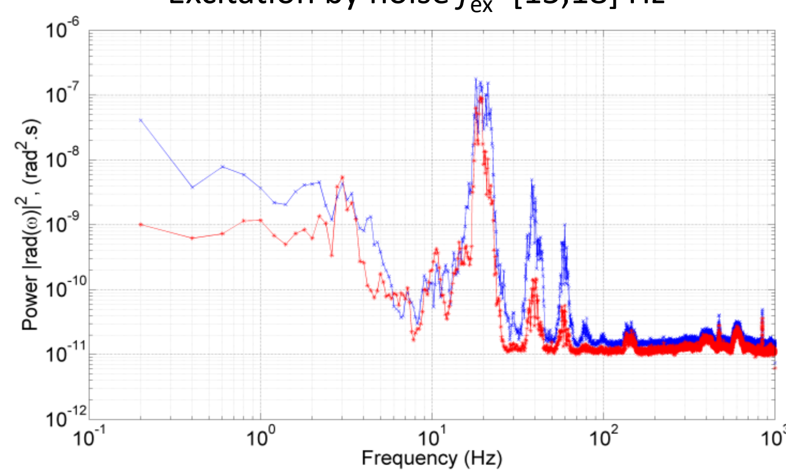




f_{ex}=10hz, large amplitude



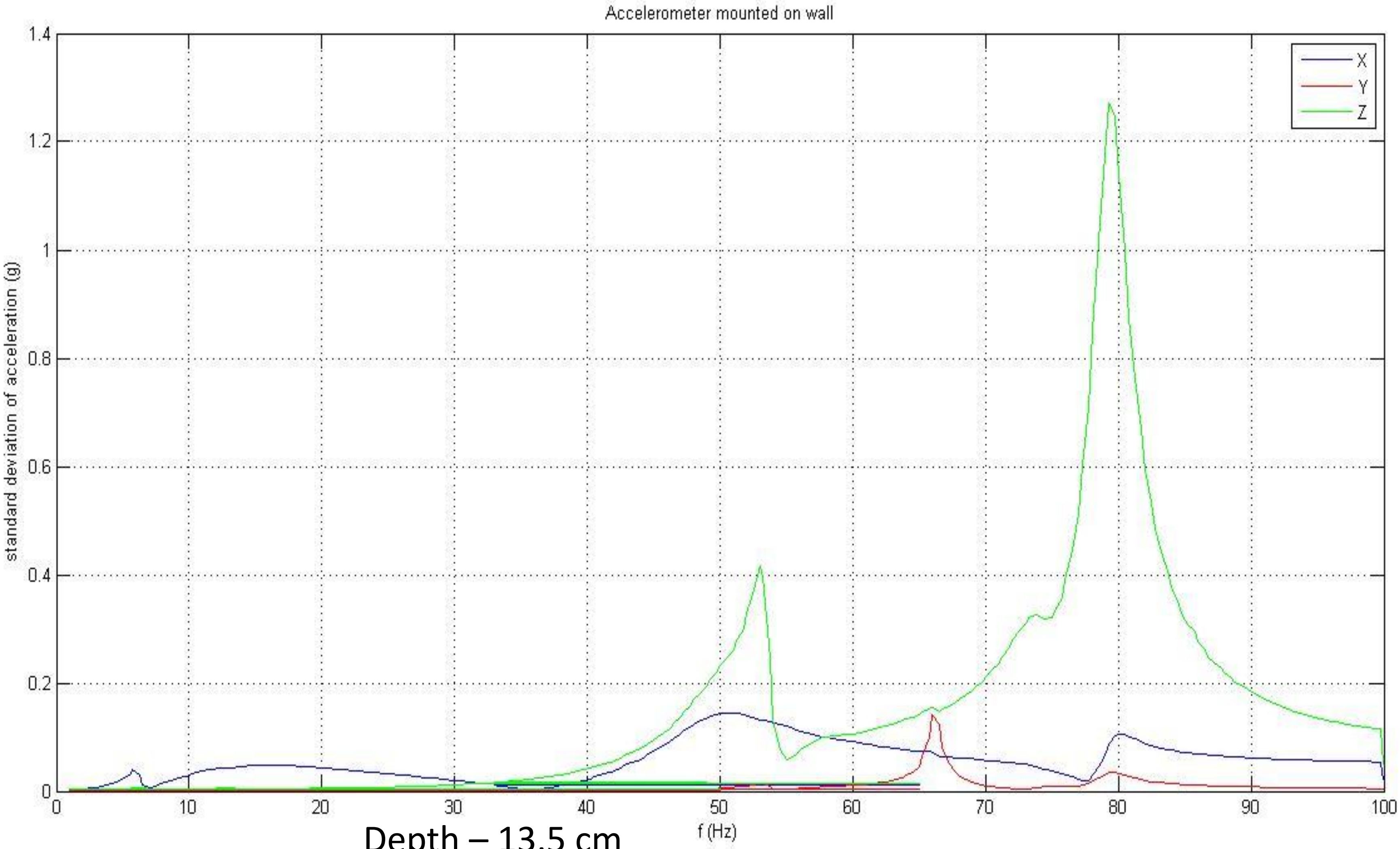
Excitation by noise f_{ex}=[15,18] Hz



Low frequency mode

- LF mode appears at F_{ex} ~ 80 Hz
- Frequency F_{LF} ~ 2.3 Hz, λ_{LF} ~42 cm (= the cell size)
- Amplitude ~ up to 1-2 cm
- Characteristic time of the mode growth ~ 10-20 min
- LF mode is a results of supercritical instability at vertical acceleration $a > a_{LF} > a_{par}$
- LF instability follows by development of cross waves at the cell boundaries

Vibration resonances of the empty cell



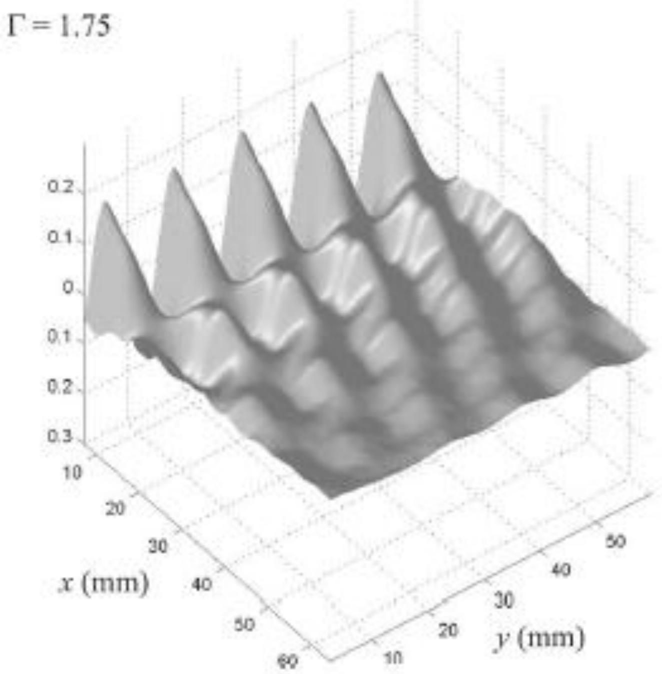
Depth – 13.5 cm

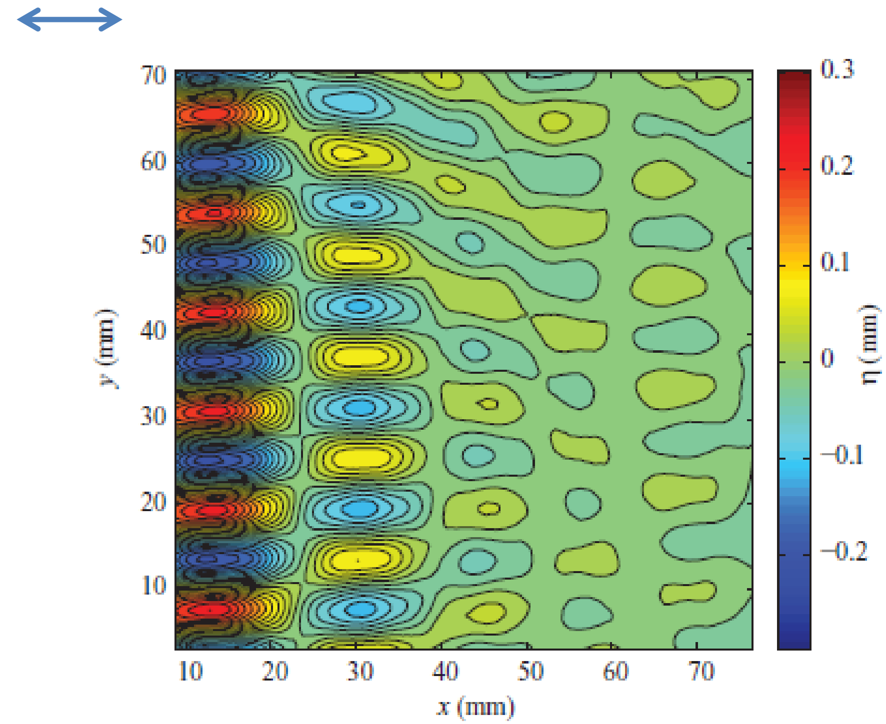
Z = perpendicular to wall

Y = Parallel to wall

X = Vertical

Cross waves generated by the vibrating wall

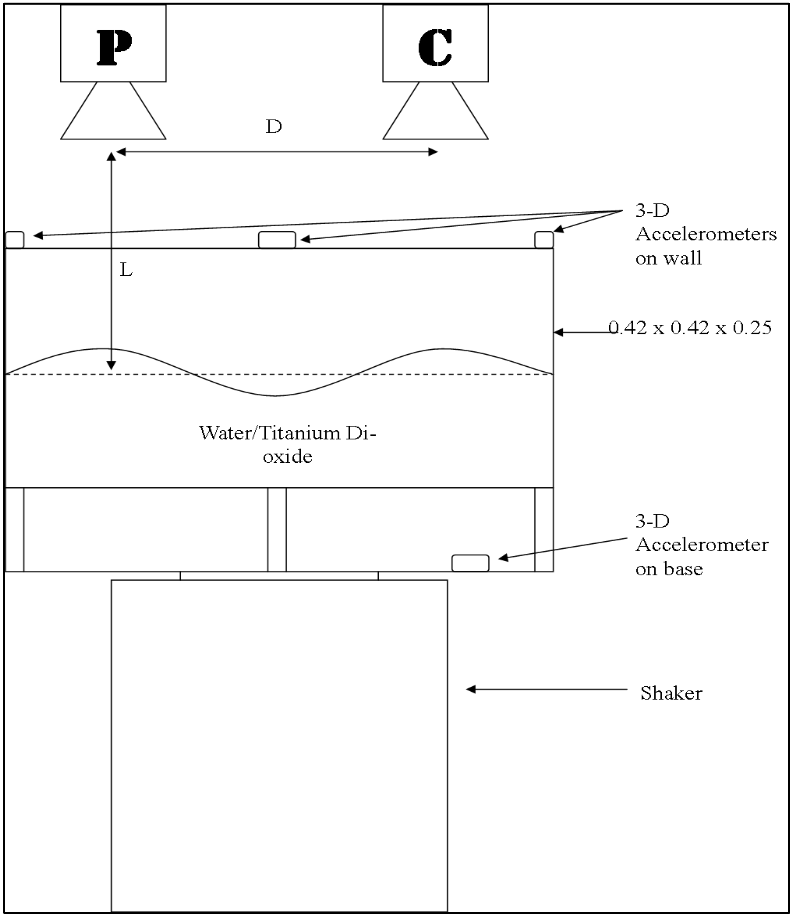




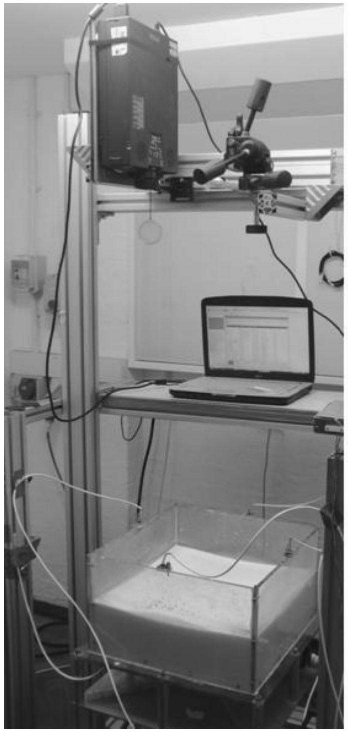
F. Moisy et al. Phys . of Fluids (2012)

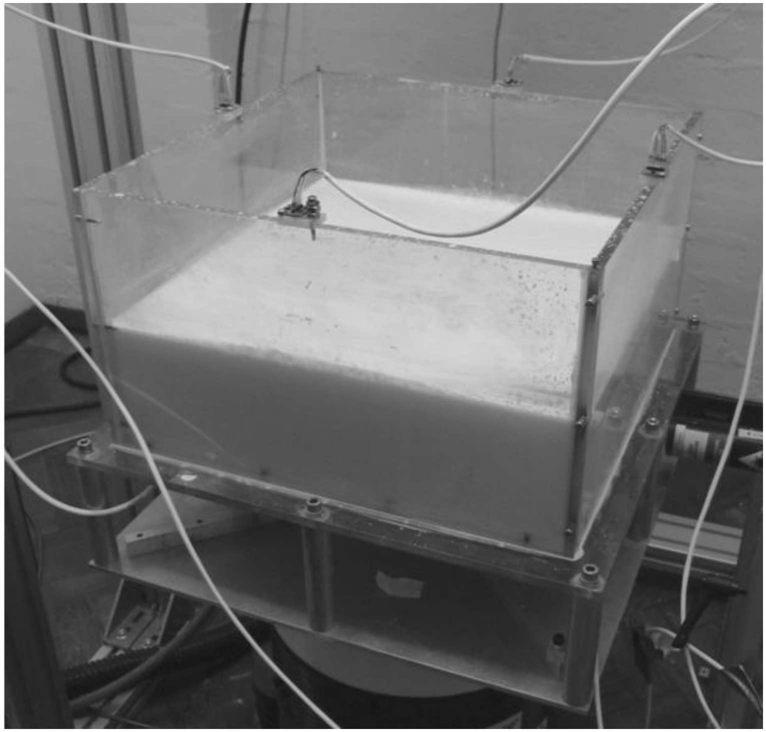
- Low frequency mode appears when lateral walls are oscillated in resonance (80-85 Hz).
- Strong transversal wall oscillations generate the cross waves localized at the near. the central area of the walls.
- The cross waves with the standard parametric waves form inhomogeneous wave field
- The low frequency mode appears.

Profilometry method for 3D characterisation of wave fields



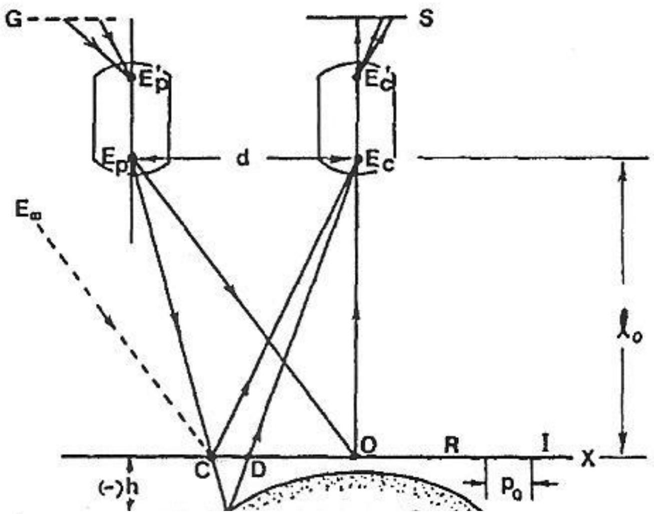
- Working liquid – water with 2% TiO_2 600 nm powder
- Surface Illumination by a structured light
- Image processing to get surface elevation, $\eta(x,y)$
- Video recording up to 1000 fps -> $\eta(x,y,t) \rightarrow \eta(\vec{k},\omega)$



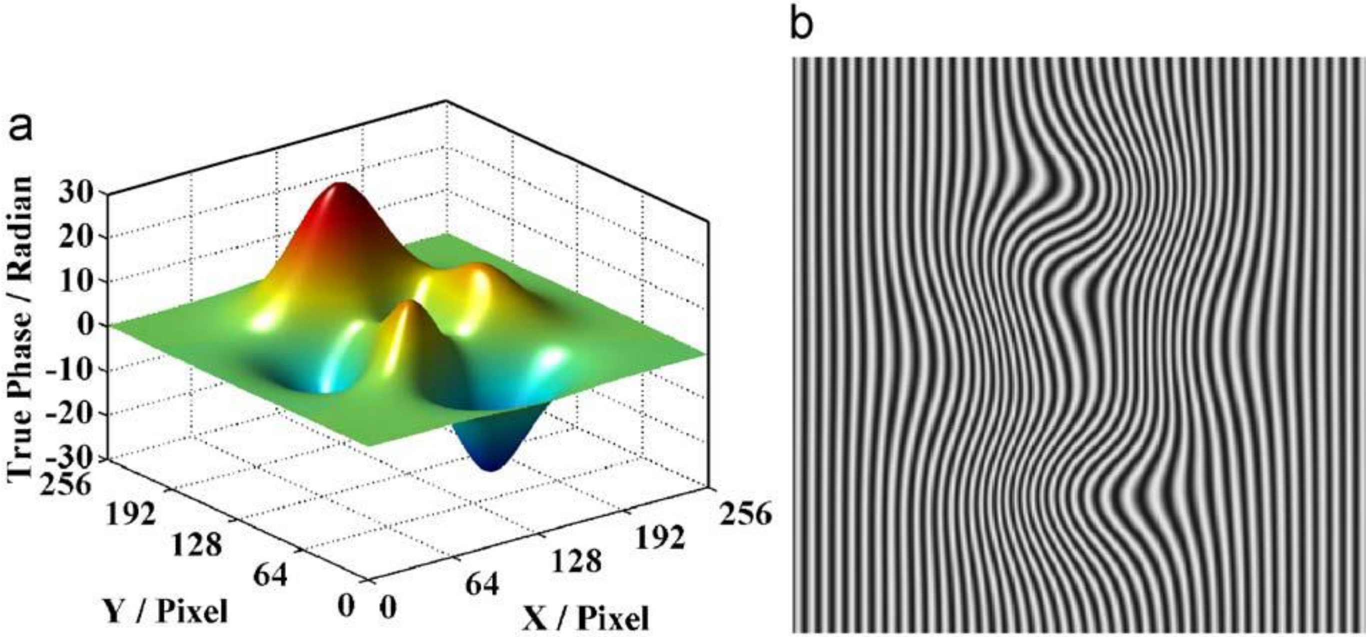


Profilometry: simulation example

Tasks: Wave interactions in k- and ω-domains
Direct and inverse cascades
Measurement of the energy fluxes
Wave – flow interactions



Method: FFT profilometry
works on diffusively reflected
surfaces, e.g. milk, water or
TiO_2 nanoparticles etc.



The phase of the pattern contain information about the surface height

Fourier Transform Profilometry

A fringe pattern:

$$g(x,y) = a(x,y) + b(x,y)\ \cos(2\pi f_0 x + \phi(x,y) = \ a(x,y) + c(x,y) \exp 2\pi i f_0 x + c.c.$$

where $$c(x,y) = \frac{1}{2} b(x,y) \exp[i\phi(x,y)]$$

FFT: $$g(x,y) \rightarrow \text{FFT} \rightarrow G(f,y), \ldots$$

$$G(f,y) = A(f,y) + C(f - f_0, y) + C^*(f + f_0, y)$$

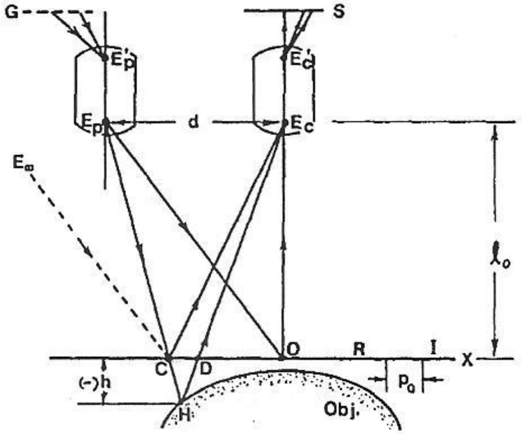
Frequency shift and filtering:

$$C(f - f_0, y) \rightarrow \text{shift by } f_0 + \text{bandpass filtering at } f_0 \rightarrow C(f,y)$$

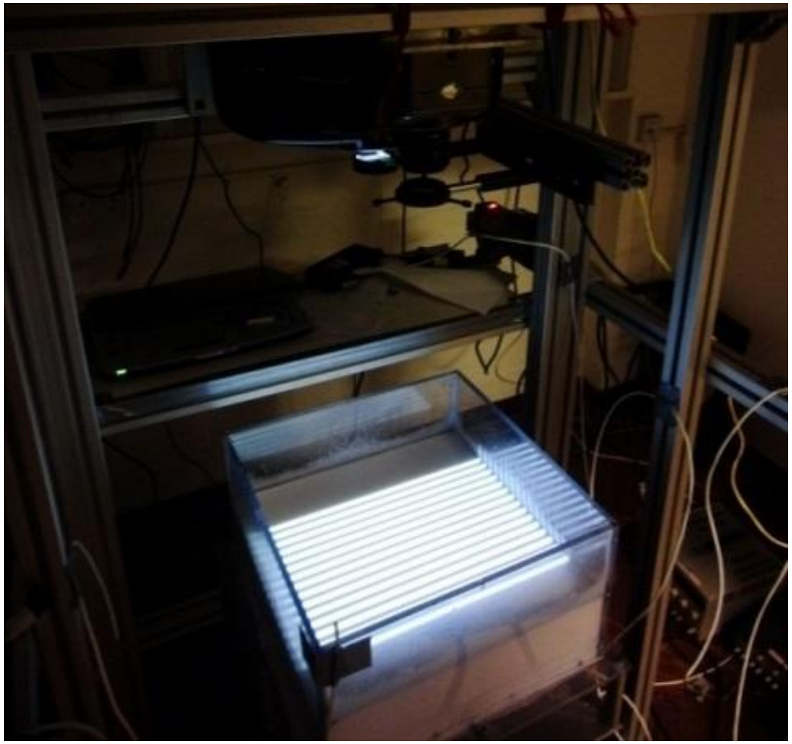
Inverse FFT: $$C(f,y) \rightarrow \text{IFFT} \rightarrow c(x,y)$$

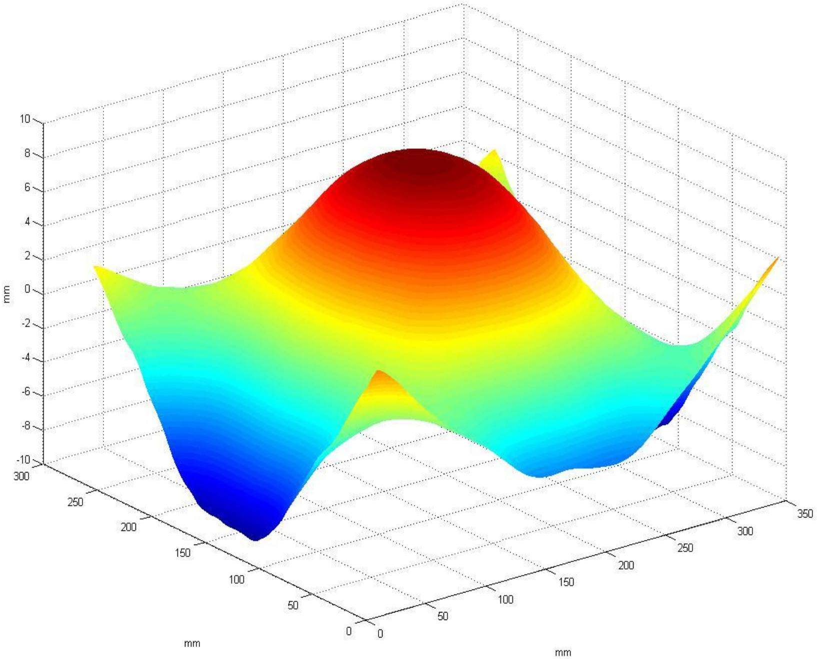
Phase calculation: $$\phi(x,y) = Im\{\log[c(x,y)]\}$$

Wave elevation: $$\eta(x,y) = \frac{l_0 \phi(x,y)}{\phi(x,y) - 2\pi f_0 d}$$



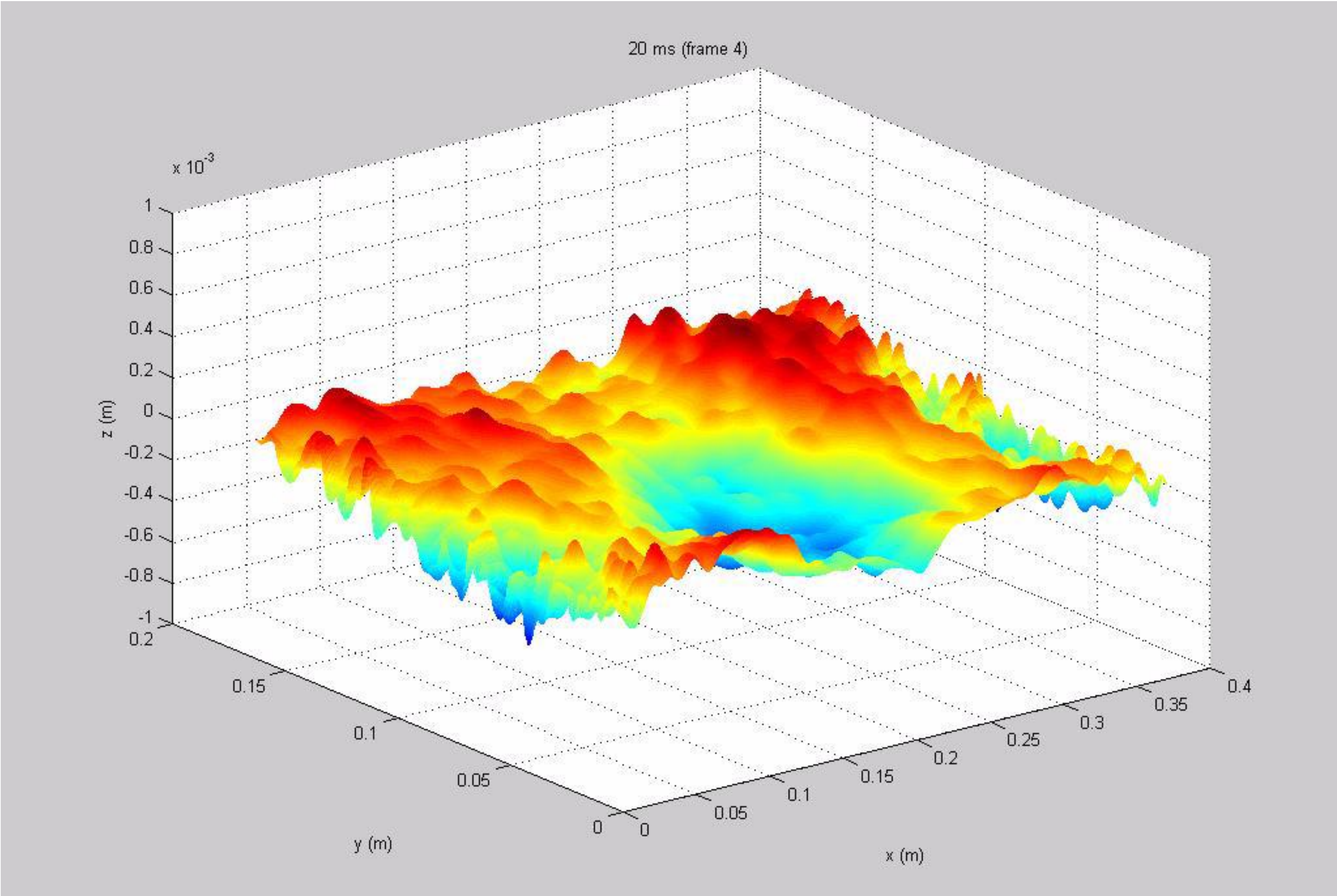
Low frequency mode





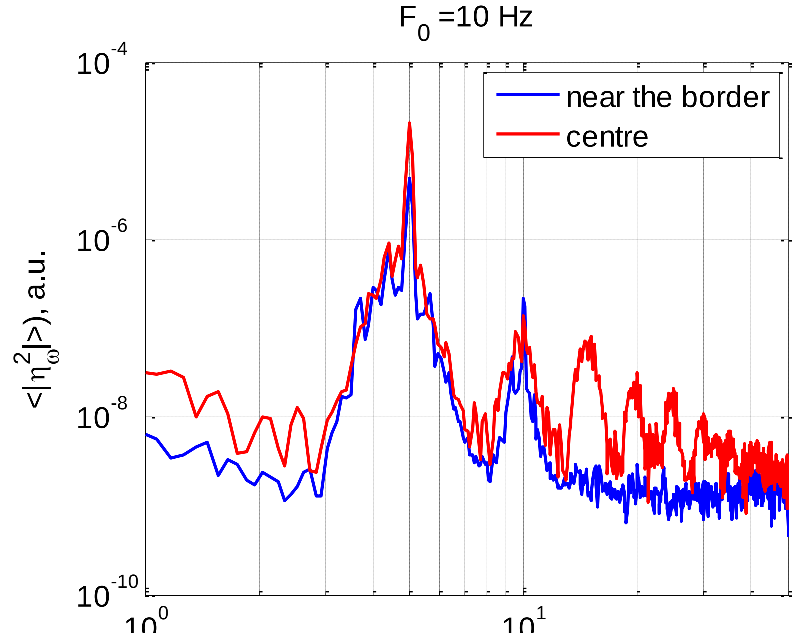
Depth 12.5 cm; Excitation 82 Hz;

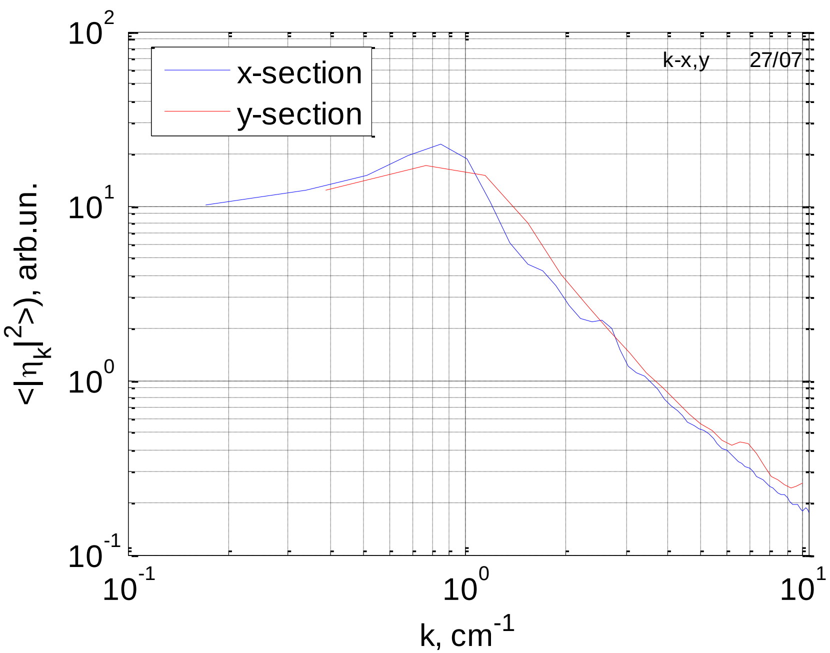
Casio camera; Projector 800x600; 3/05/12



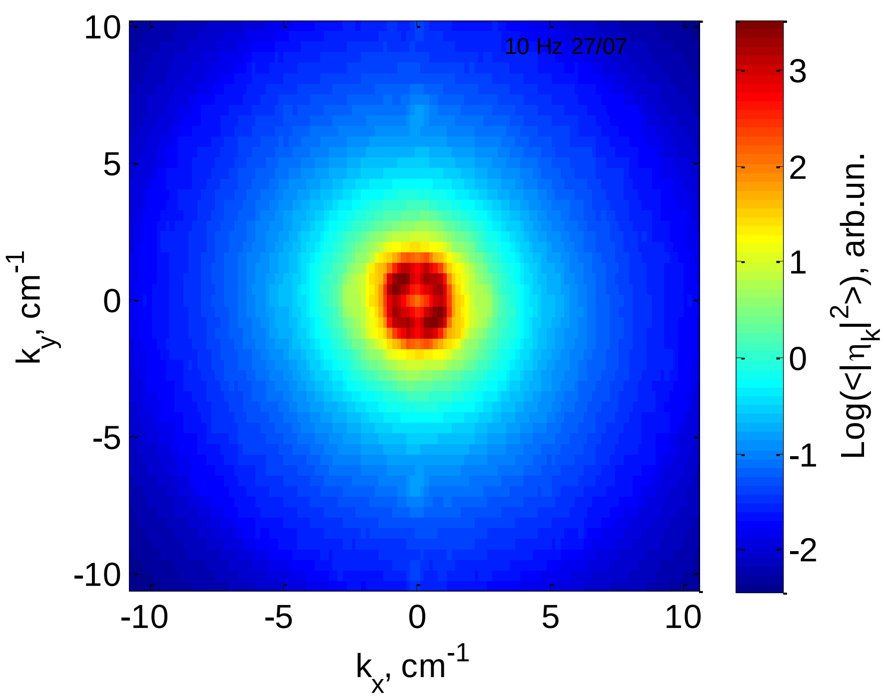
20 ms (frame 4)
x 10^-3
z (m)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0.2
0.15
0.1
0.05
0
y (m)
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
x (m)

Primary parametric mode is not homogeneous in space

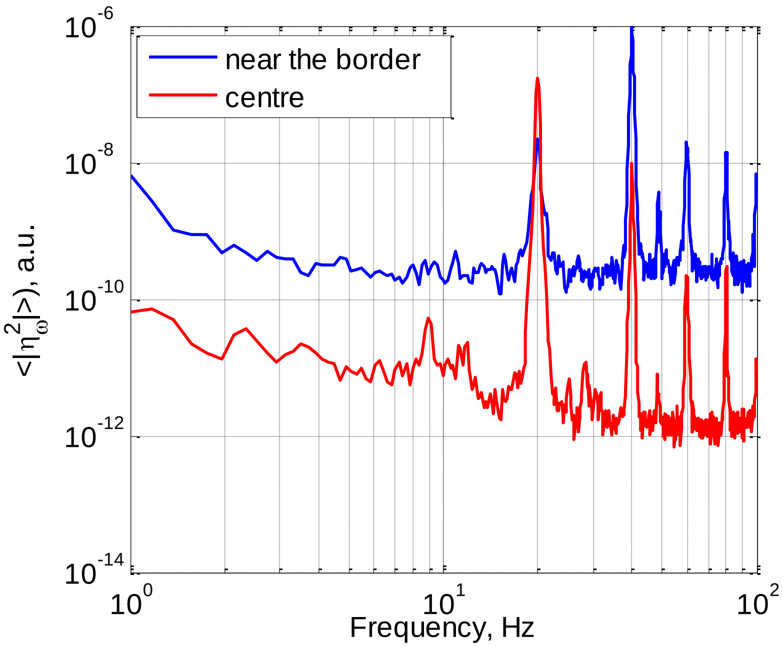


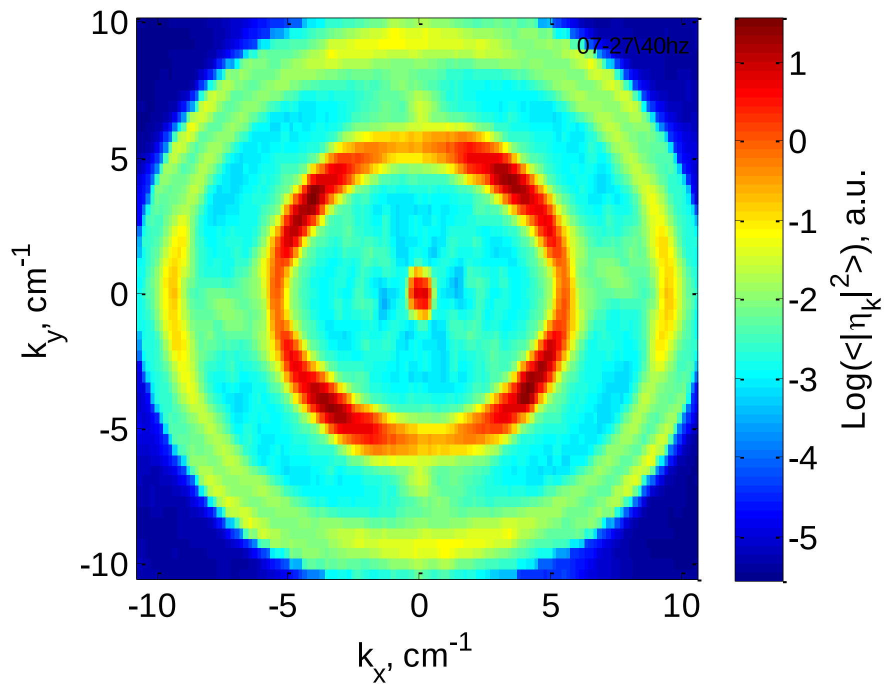


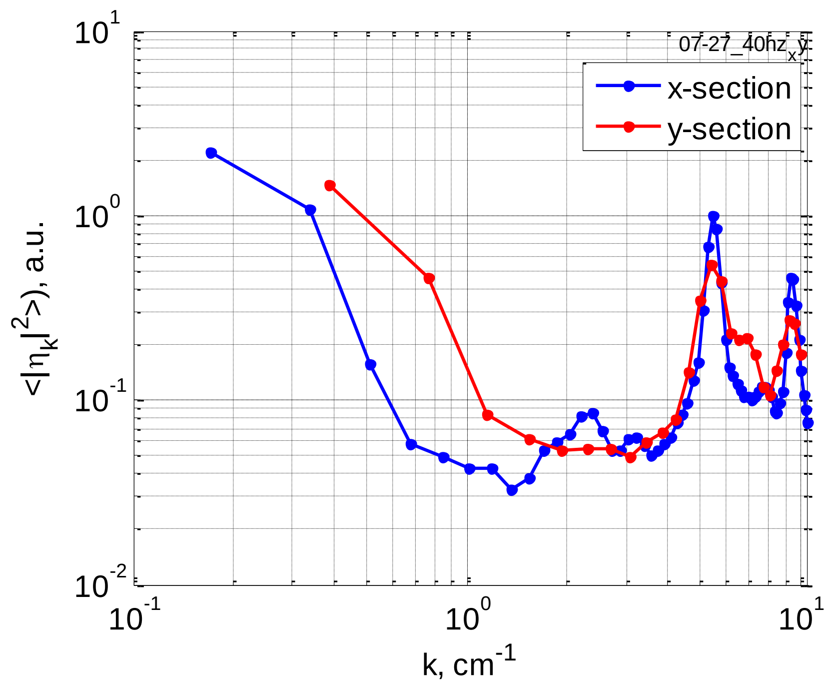
f_0 = 10 Hz, $\lambda = 10$cm



Primary parametric mode is not isotropic

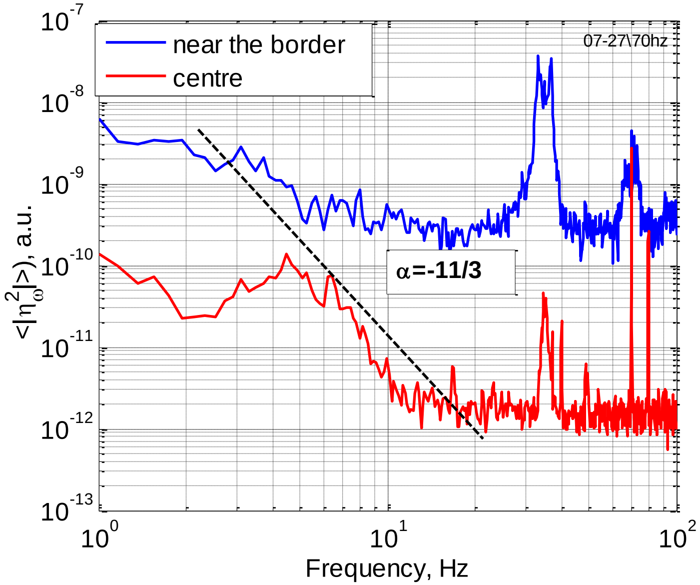


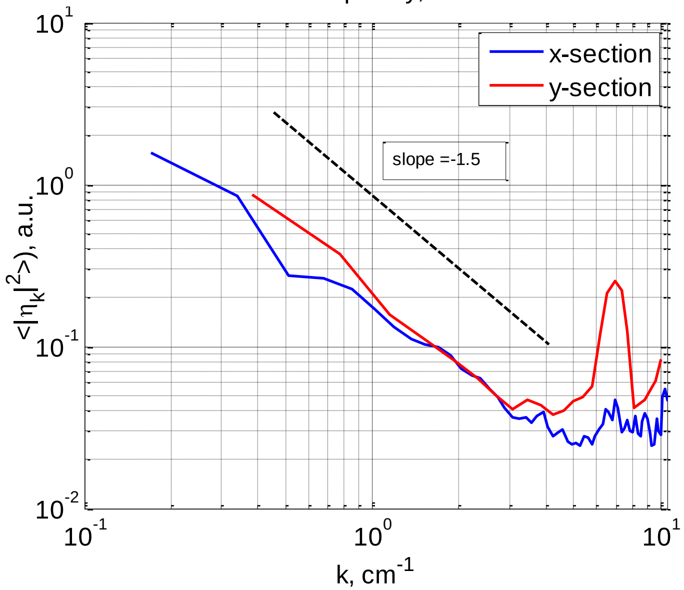


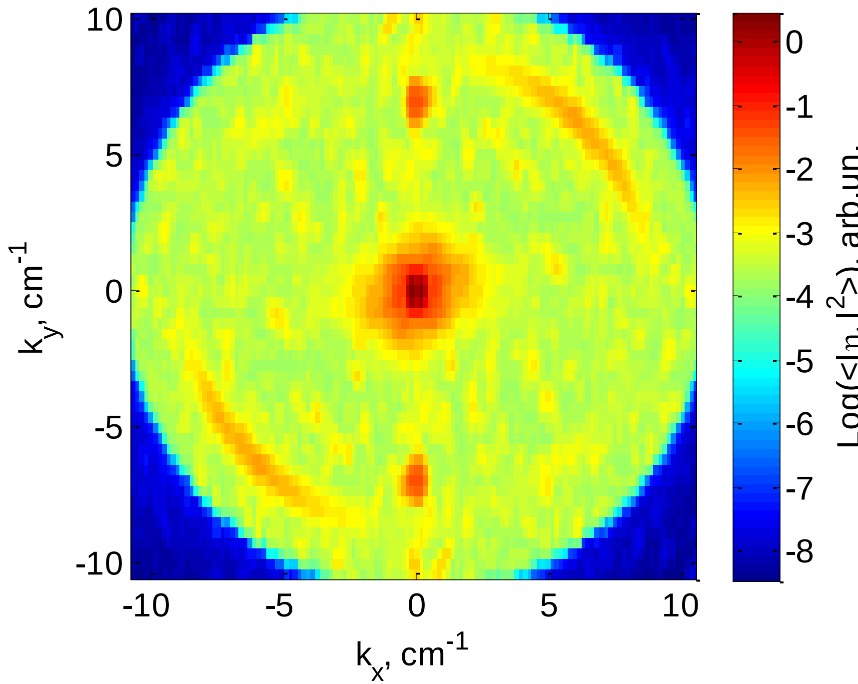


$f_0 = 40$ Hz, $\lambda = 0.9$ cm

Primary parametric mode is supress in the centre of the cell, while the low frequency mode demonstrates a wide peak



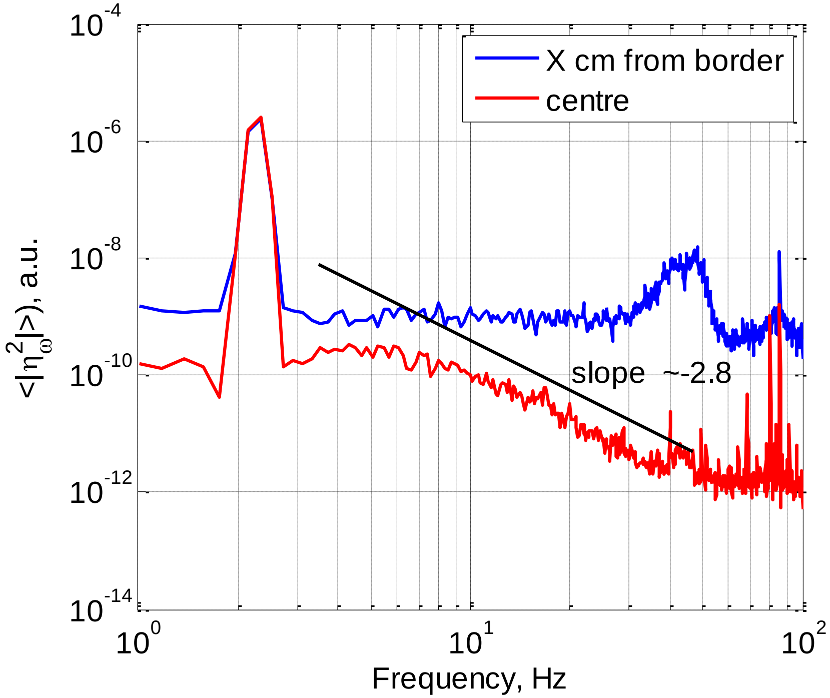




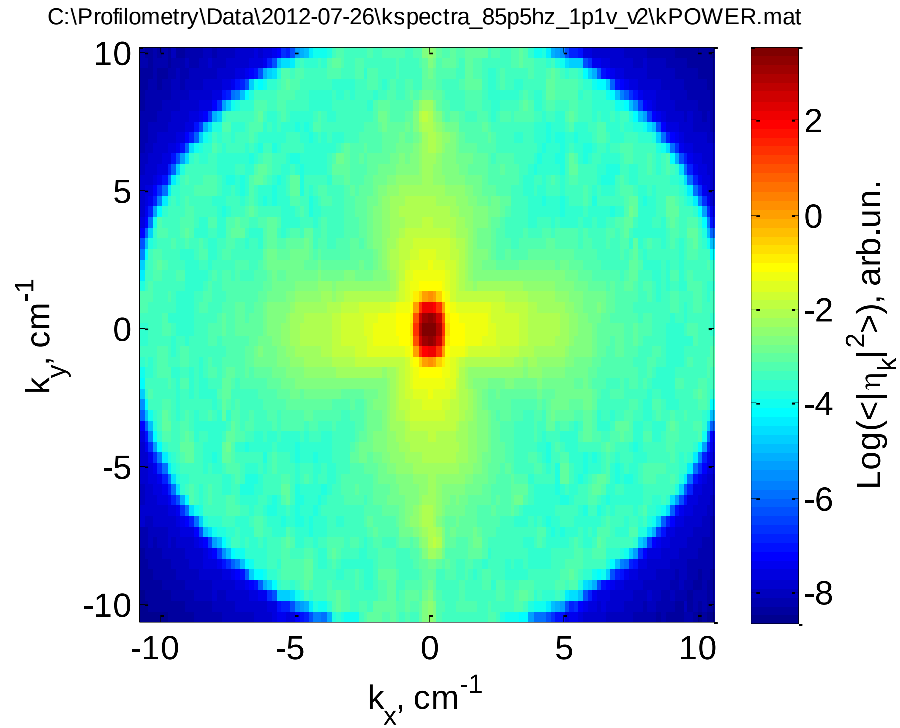
f_0 = 70 Hz

(frequency is far from the resonant)

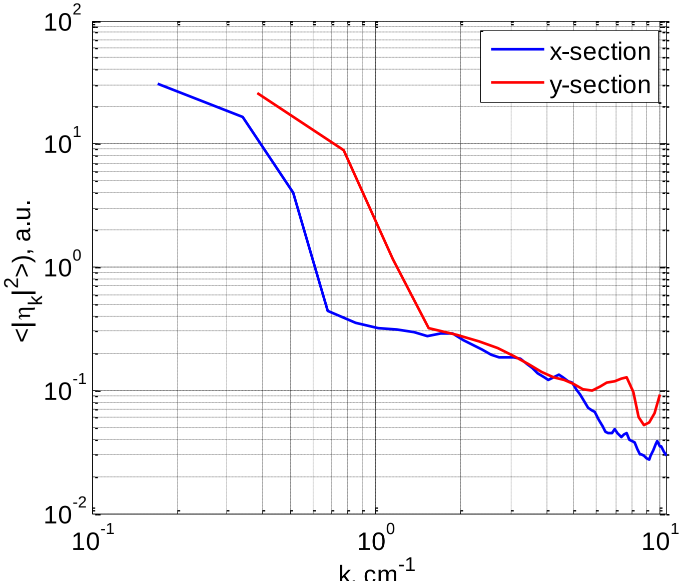
f_{ex} = 85.5 Hz



X cm from border
centre
slope ~-2.8
Frequency, Hz
$<|\eta_\omega^2|>$, a.u.

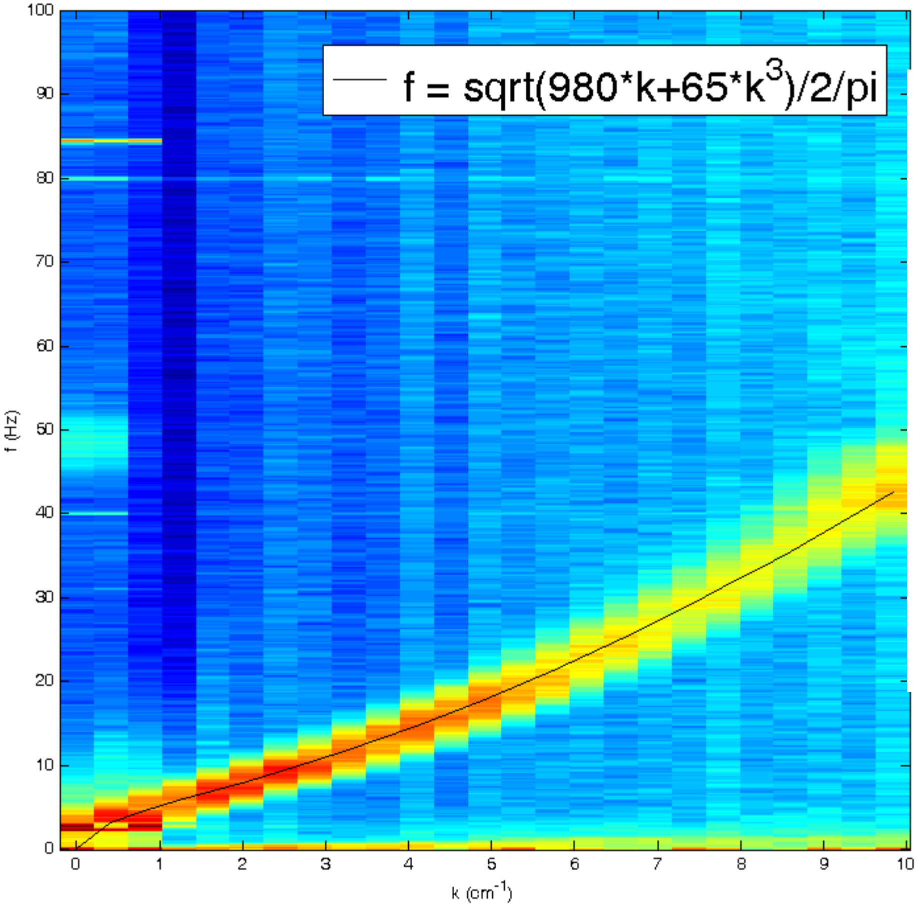


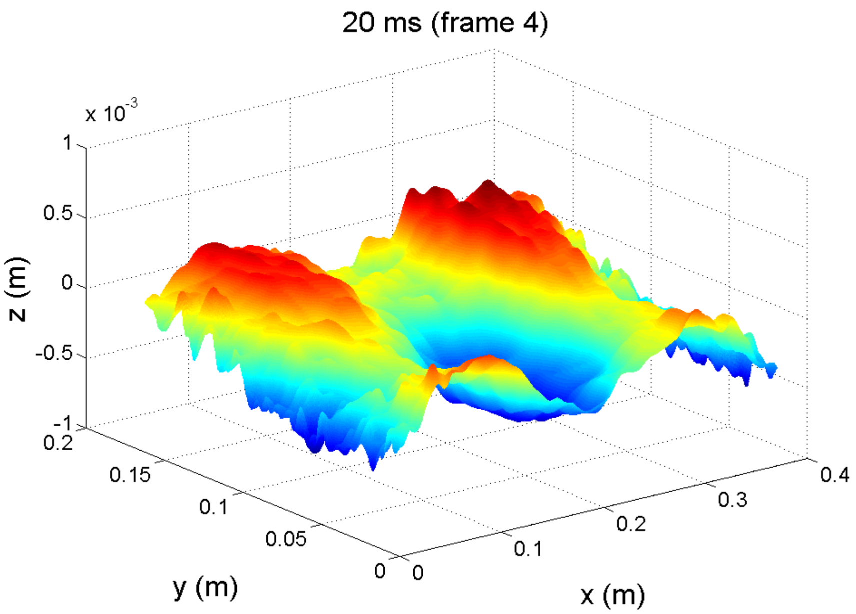
C:\Profilometry\Data\2012-07-26\kspectra_85p5hz_1p1v_v2\kPOWER.mat
k_y, cm^{-1}
k_x, cm^{-1}
Log($<|\eta_k|^2>$), arb.un.



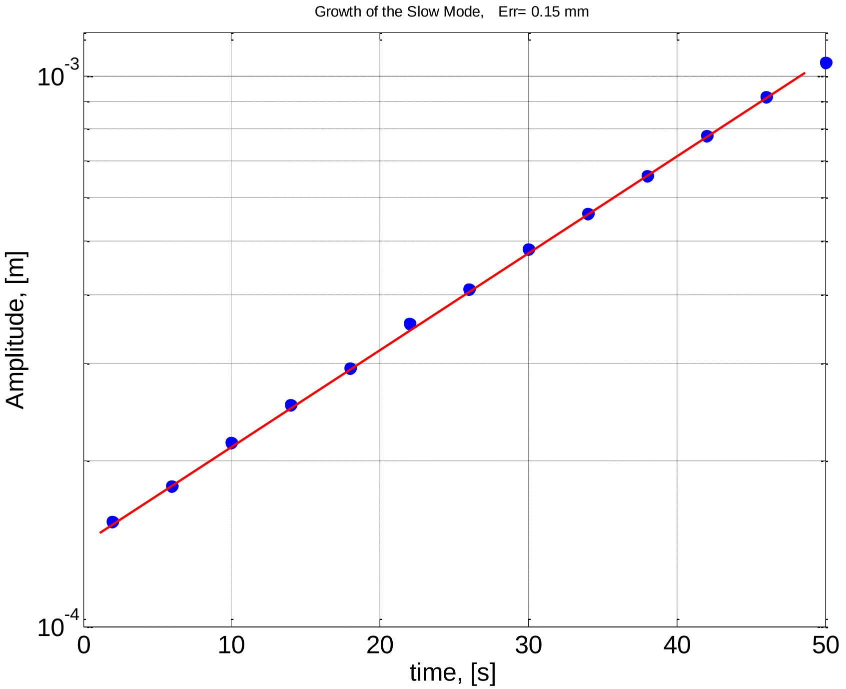
x-section
y-section
$<|\eta_k|^2>$, a.u.
k, cm^{-1}

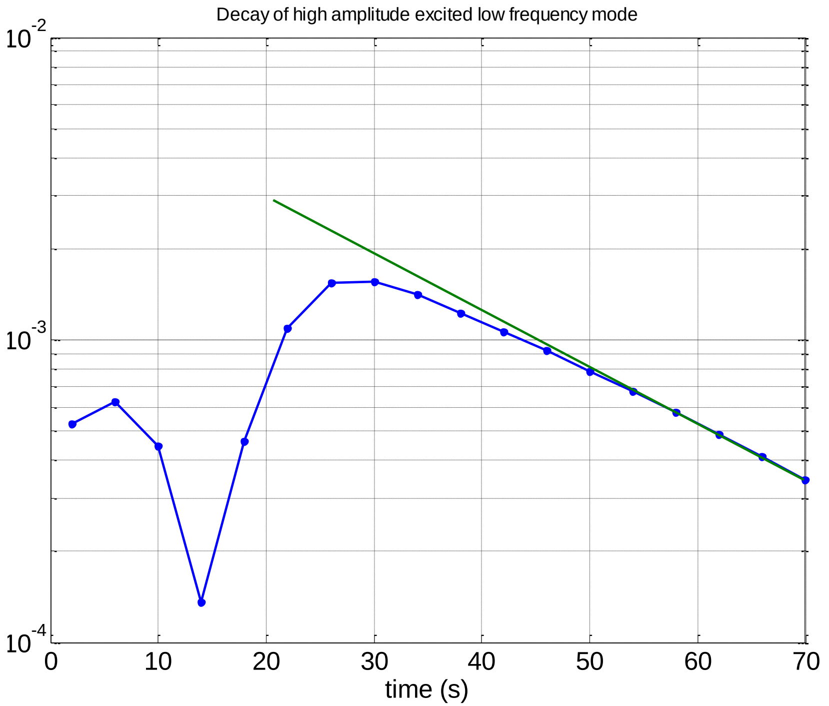
k-ω spectrum





Growth and Decay of the Slow Mode





What is the reason for a drop in low frequency Mode amplitude?

Discussion: other wave systems demonstrating the low frequency instability

- Dragon Wash phenomenon

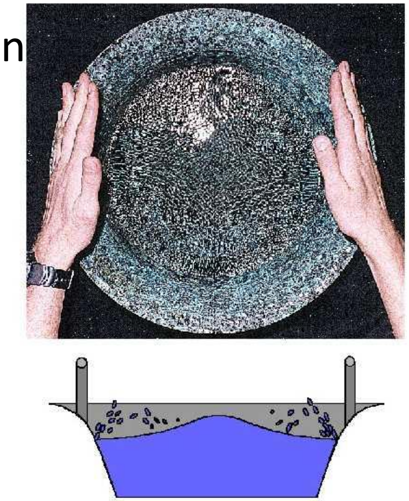
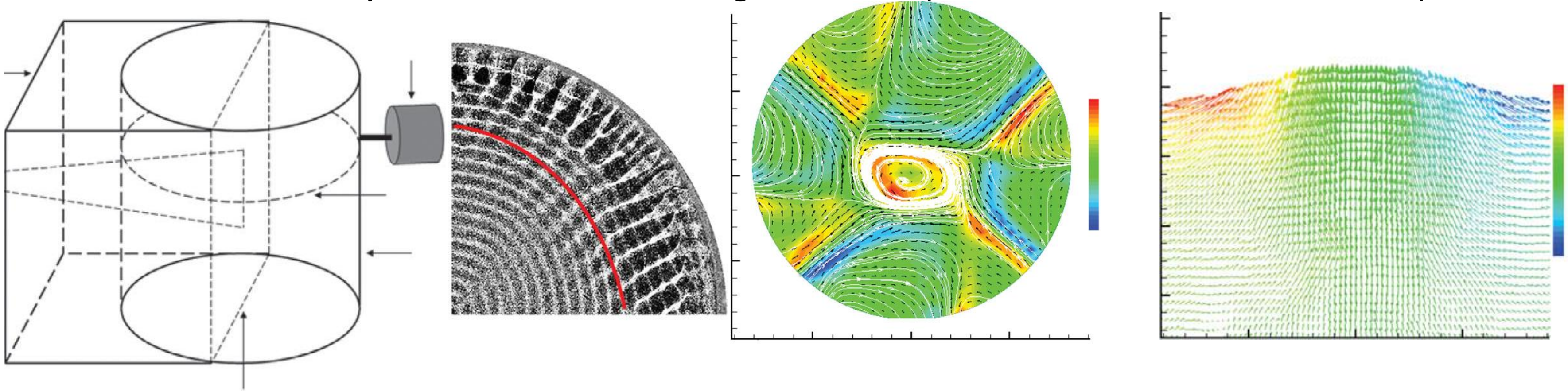
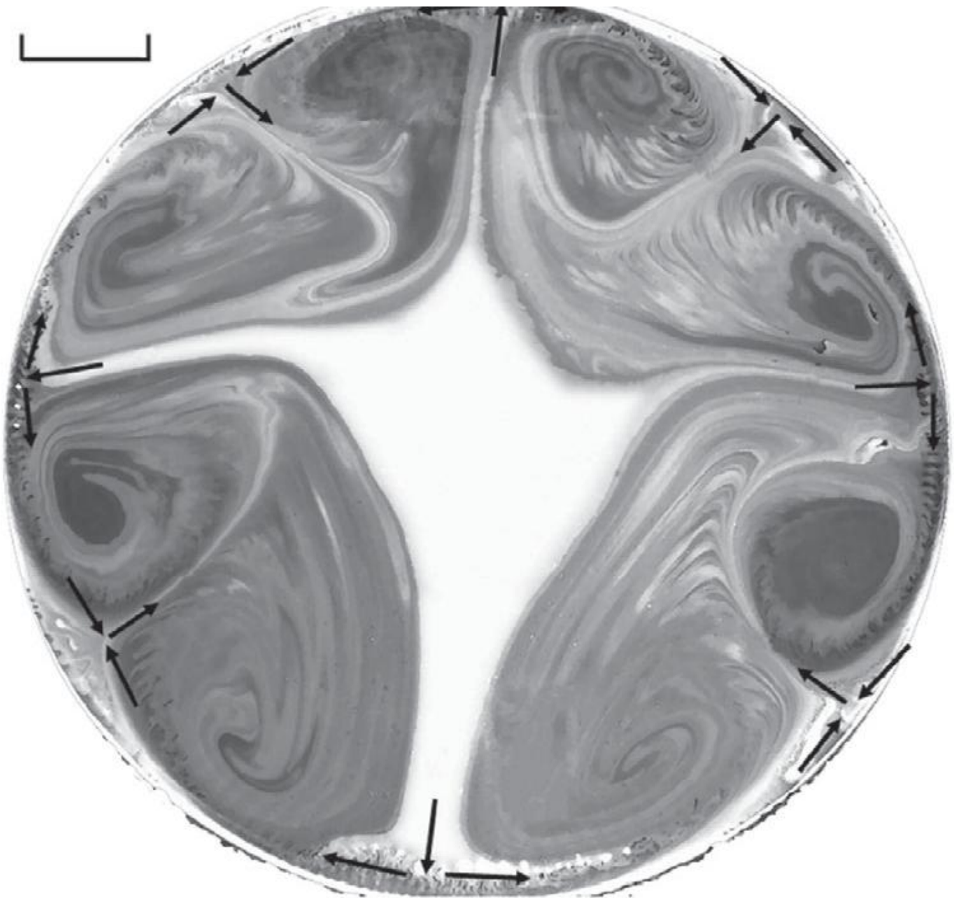


Fig.1. Dragon Wash.
Brass vessel with hollow handles.
Jumping droplets and
the low frequency gravity wave.

- Thin wall cylinder with oscillating side wall (Cunbiao Lee et al, JFM 2011)

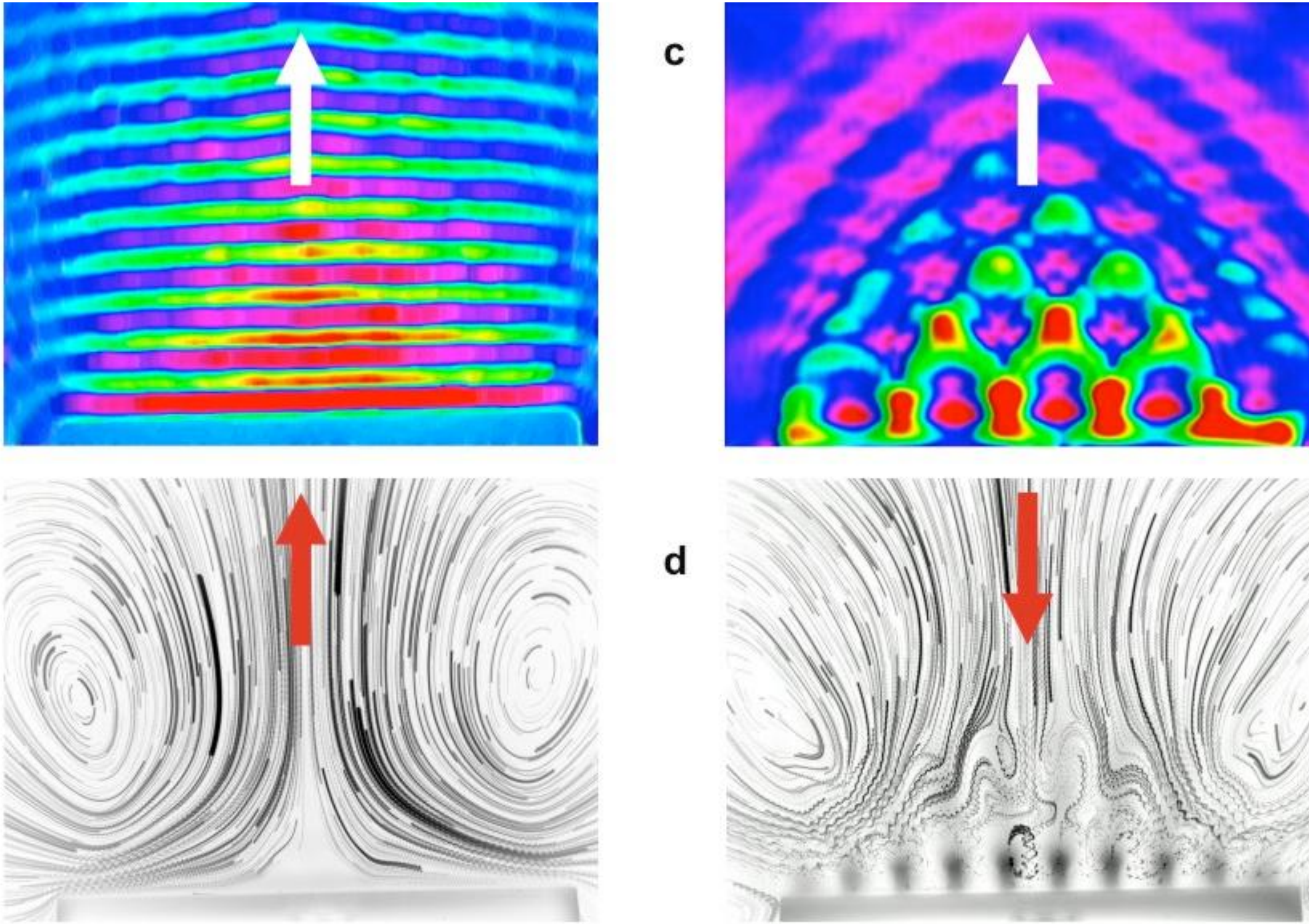


Discussion: Effect of flow generated by external wall

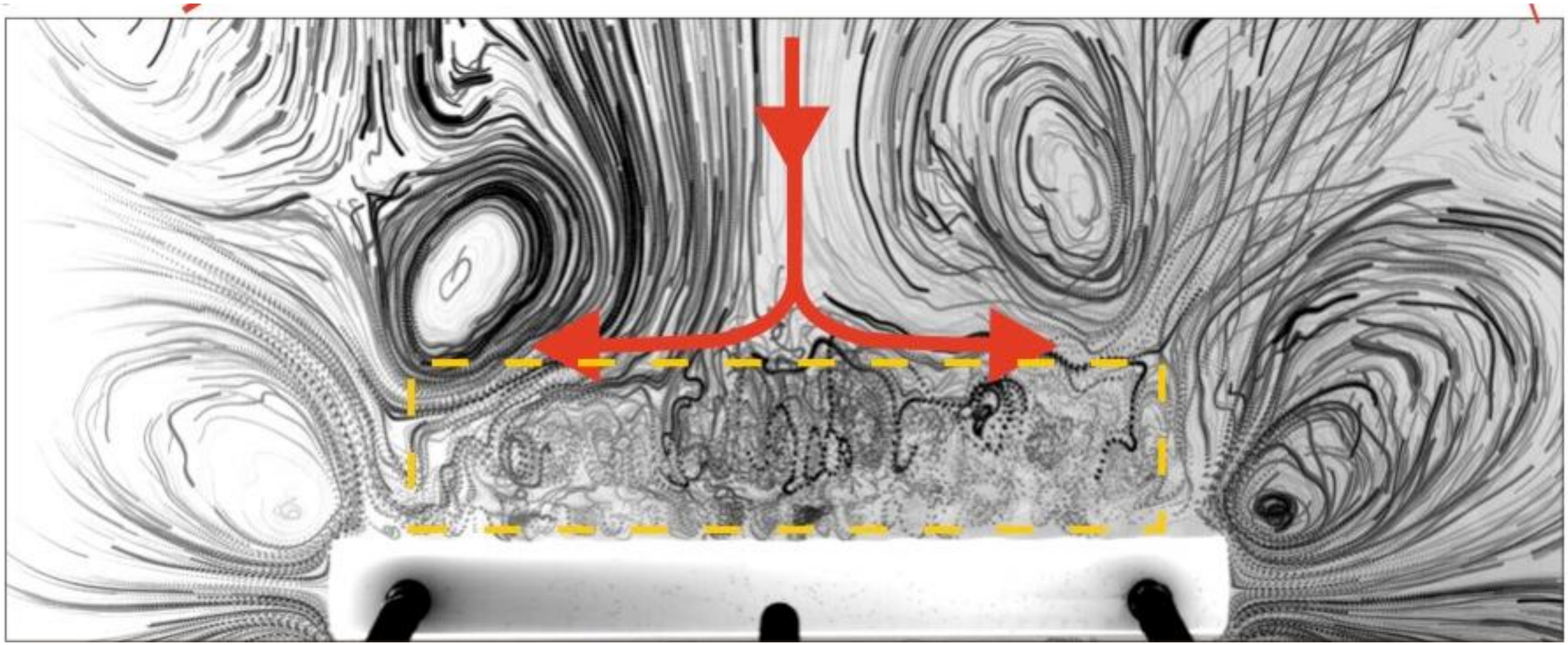


Cunbiao Lee et al, JFM 2011

From “Generation and reversal of surface flows by propagating waves”, Nature Physics, August 12, 2014, courtesy of G Falkovich



Creation of chaotic region as a mechanism for jet formation (from"Generation and reversal of surface flows by propagating waves", Nature Physics, August 12, 2014, courtesy of G Falkovich)

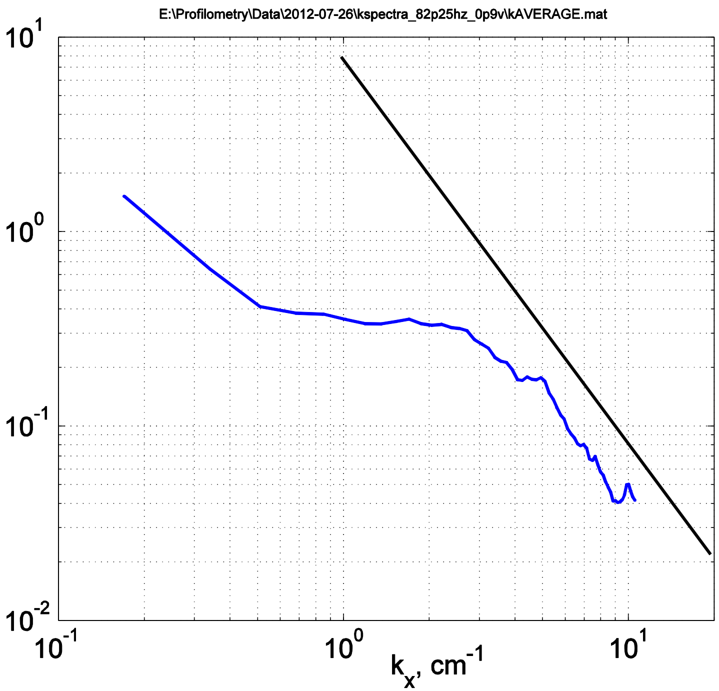


Conclusion

- No inverse cascade were found for parametrically generated low frequency waves. A possible reason for that might be the spectral narrowness of the parametrically excited waves.

- Instead we observe the strong low frequency mode generated by the intense cross waves excited by transversal vibrations of the cell walls.

- A missing link between the cross wave instability and the low frequency mode might be the jets. The jets are generated by cross wave instability by the recently discovered mechanism of chaotic pumping.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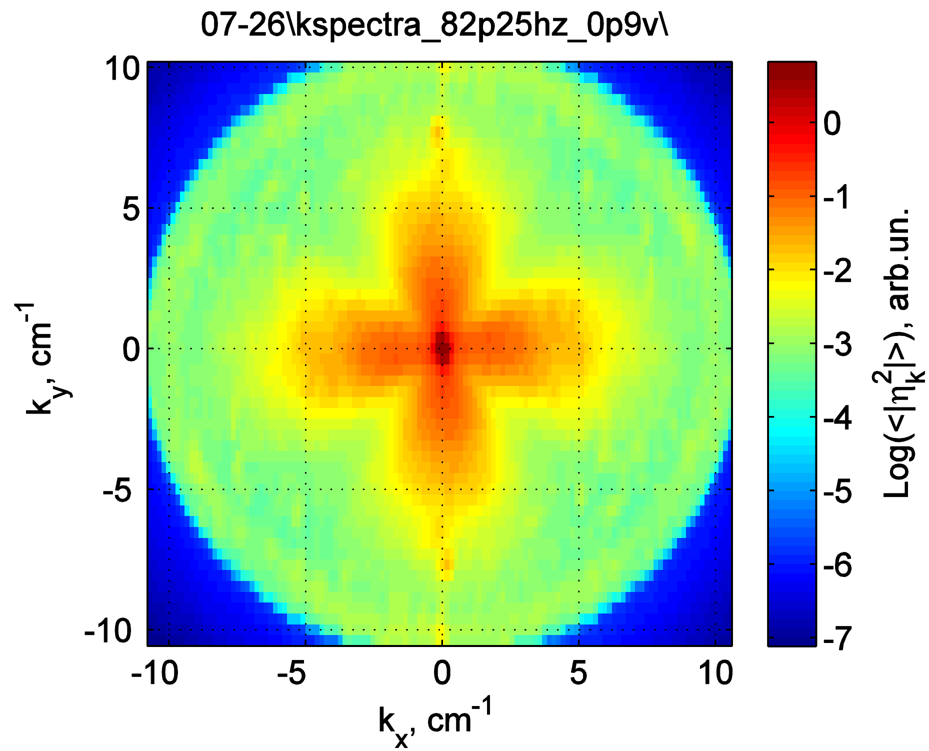
10^1
10^0
10^{-1}
10^{-2}
10^{-1}
10^0
10^1
k_x, cm^{-1}


07-26\kspectra_82p25hz_0p9v\
10
5
0
-5
-10
-10
-5
0
5
10
k_y, cm^{-1}
k_x, cm^{-1}
0
-1
-2
-3
-4
-5
-6
-7
$Log(<|\eta_k^2|>)$, arb.un.

Gravity Wave Turbulence

Dispersion relation for surface waves:

$$\omega^2 = \left(gk + \frac{\sigma}{\rho} k^3 \right) \tanh(kh)$$

Sources of energy

- natural: winds, tides or ships
- laboratory: air flow, wavemakers, parametric resonance.

Energy dissipation mechanisms

- energy transport to smaller scales and viscous dissipation
- wave breaking

Goals:

Short-term: to characterize statistical properties of waves in a finite system

Long-term: to study transport and mixing generated by wave turbulence