Self-similarity and universality of wind wave growth

V. E. Zakharov[1−5], S. I. Badulin[1−3], P. A. Hwang[6] and G. Caulliez[7]

[1]P.P.Shirshov Institute of Oceanology, Moscow, Russia
[2]Lab. of Nonlinear Wave Processes, Novosibirsk State University, Russia
[3]P.N. Lebedev Physical Institute, Moscow, Russia
[4]University of Arizona, Tuscon, USA
[5]Waves and Solitons LLC, Phoenix, Arizona, USA
[6]Remote Sensing Division, Naval Research Laboratory, Washington, D.C.
[7]Institut Méditerranéen d'Océanologie, Institut Pythéas, Marseille, France

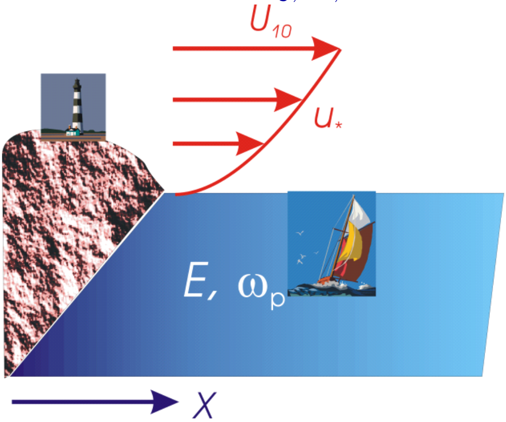
6 August 2014

Why “Wind rules waves”?

Even in the idealized setup the wind wave evolution is determined by a long set of physical quantities:

“external” U_{wind}, g, x and

“intrinsic” H_s, T, ...



The law of growth of self-similar wind-driven seas

$$\mu^4 \nu = \alpha_0^3$$

$\mu = ak_p = \frac{\omega_p^2 \sqrt{<\eta^2>}}{g}$ – wave steepness

ν – number of waves ($\nu = \omega_p t$ or $\nu = 2\mathbf{k}_p x$)

α_0 – a constant ($\alpha_{0(d)} = 0.7$ or $\alpha_{0(f)} = 0.62$)

Wave growth is wind-free!?

We change the concept ?

Waves evolves on their own

instead of conventional

Wind rules waves

Try to show consistency of our paradoxical results with previous studies

In this talk

1. Self-similarity of wind-driven seas
2. Simulations of duration- and fetch-limited setups
3. Sea wave growth in field measurements
4. Wind wave tank experiments
5. Summary

You are welcome to copy this presentation

badulin@ioran.ru

Theory of self-similar wind-driven seas

V. Zakharov, S. I. Badulin, P. A. Hwang and G. Caulliez

1. The kinetic equation (Hasselmann, 1962)

$$dE/dt = S_{nl} + S_{in} + S_{diss} \Rightarrow$$

   $S_{in} + S_{diss}$ – wave input and dissipation, mostly empirical;
   S_{nl} – 4-wave resonances – explicit expression from the first principles

2. Dominating nonlinearity – key assumption (or key fact for sea waves)

$$dE/dt = S_{nl} \qquad (1)$$

$$\langle dE/dt \rangle = \langle S_{in} + S_{diss} \rangle \qquad (2)$$

NB from VZ: Our Lord is graduated in theoretical physics. Hence, the Nature is described by a set of asymptotic models

Families of self-similar solutions

1. Homogeneity of the collision integral S_{nl} (deep water)

$$S_{nl}\left(cE(d\mathbf{k})\right) = c^3 d^{17/2} S_{nl}\left(E(\mathbf{k})\right)$$

2. Self-similar power-law solution $(\nabla_\xi \equiv 0)$
For Eq. 1 – conservative KE in terms of energy

$$E(\mathbf{k}, t) = a t^{p_\tau + 4q_\tau} \Phi(b\mathbf{k}t^{q_\tau})$$

4 free parameters !!!

3. Closure condition – integral balance is consistent with Eq. 1 when wave input is power-law function of time

$$\partial E/\partial t = \langle S_{in} + S_{diss} \rangle = At^{-q_\tau - 1}$$

2 free parameters only !!!

Scale invariance and magic numbers

1. From homogeneity of the collision term S_{nl}

$$a = b^{17/4}; \qquad q_\tau = \frac{2p_\tau + 1}{9} \qquad (3)$$

2. Equation for the 'shape function' $\Phi(\xi)$

$$(p_\tau + 4q_\tau)\Phi(\xi) + 2q_\tau \nabla_\xi \Phi(\xi) = S_{nl}(\Phi(\xi))$$

Shape function depends on parameter p_τ only !

3. From eq. 3 one can get the stationary combination

$$E^2 \omega_p^9 t = (ak_p)^4 (\omega_p t) = \mu^4 \nu = \mathrm{const}$$

The invariant depends on integrals of the shape function only (i.e. on p_τ) !!!
And this dependence is weak – spectral shape invariance

An alternative form – weakly turbulent law of wind-wave growth, Badulin et al., J.Fl.Mech. 2007

An alternative use of homogeneity properties

$$\mu^2 = \frac{E\omega_p^4}{g^2} = \alpha_{ss}\left(\frac{\omega_p^3 dE/dt}{g^2}\right)^{1/3}$$

with rate-dependent parameter of self-similarity

$$\alpha_{ss} \approx \frac{\alpha_0}{p^{1/3}}$$

Build an adiabatic approach to switch between different exponents or to consider the new formulation

$$\mu^4\nu = \alpha_0^3$$

as an adiabatic invariant that does not contain a parameter of adiabaticity?

Wave invariant

$$\mu^4 \nu = \alpha_0^3$$

Life is smoother – life is longer

$\mu = \frac{\omega_p^2 \sqrt{<\eta^2>}}{g}$ – steepness; $\nu = \omega_p t$ – lifetime

Lifetime is counted in instant relaxation times

$\nu \sim \tau_{nl}$ – lifetime is proportional to the instant nonlinear relaxation scale $\tau_{nl} \sim \mu^{-4}$

The wind-free scaling of wind wave growth

Conventional wind speed scaling

$$\tilde{H} = gH_s/U_{10}; \qquad \tilde{T} = gT/U_{10}$$

Duration-limited

$$q_\tau = \frac{2p_\tau + 1}{9}$$

$$\tilde{H} = \frac{H_s}{gt^2};$$

$$\tilde{T} = \frac{T_p}{2\pi t}; \qquad \tilde{T} = \nu$$

$$\tilde{H} = 4\alpha_{0(d)}^{3/4}\,\tilde{T}^{9/4} \approx 3.06\,\tilde{T}^{9/4}$$

Fetch-limited

$$q_\chi = \frac{2p_\chi + 1}{10}$$

$$\tilde{H} = \frac{H_s}{x};$$

$$\tilde{T} = T_p\sqrt{\frac{g}{8\pi^2 x}}; \qquad \tilde{T} = \nu^{-1/2}$$

$$\tilde{H} = 8\alpha_{0(f)}^{3/4}\,\tilde{T}^{5/2} \approx 5.59\,\tilde{T}^{5/2}$$

Self-similar solutions in simulations. Pushkarev et al. 2004, Badulin et al, 2002, 2005, 2008

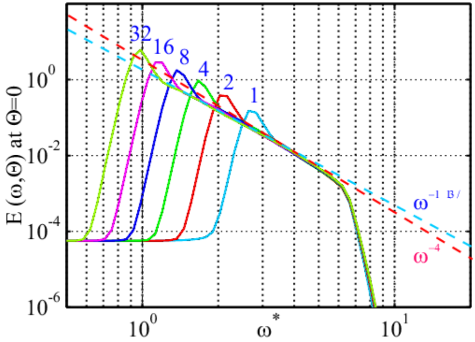
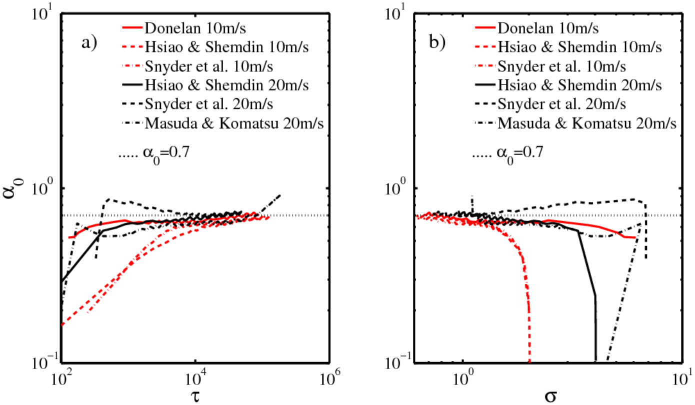


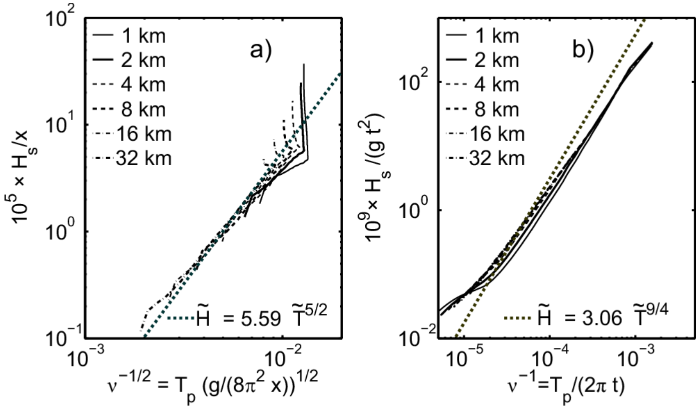
Figure : Wave spectra for duration-limited growth. Wave input by Hsiao & Shemdin, 1981, $U_{10} = 10$m/sec, time in hours

Simulations. Duration-limited growth in terms of the invariant $\mu^4\nu = \alpha_0^3$



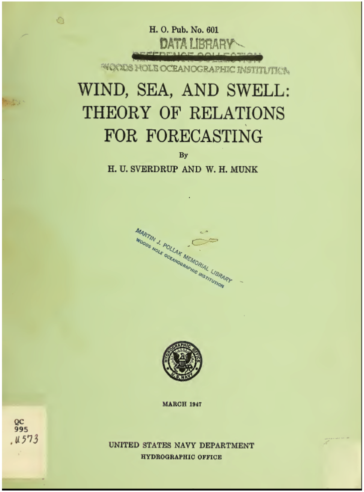
All curves are collapsing to the invariant value.
Simulations from our previous papers:
Pushkarev et al. 2004, Badulin et al. 2002, 2005, 2007.

Simulations. Fetch-limited growth in terms of $\tilde{H}_s = 5.59\,\tilde{T}^{5/2}$ and $\tilde{H}_s = 3.06\,\tilde{T}^{9/4}$



Recent results by Zakharov & Pushkarev, 2012.
There is no “pure fetch-limited” growth.
Wind-free scalings work as intermediate asymptotics

Sea wave growth in field measurements since the Overlord operation



Sverdrup & Munk (1947) and their concept of significant wave height

Data by Sverdrup & Munk (1947) within wind speed and wind-free scalings

Self-similarity and universality of wind wave growth

V. Zakharov, S. I. Badulin, P. A. Hwang and G. Caulliez

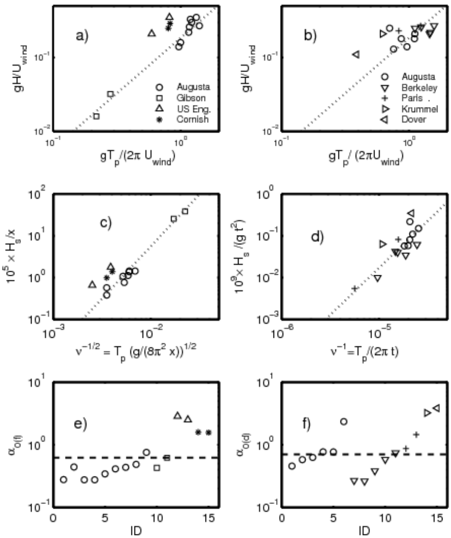
Theory

Simulations

Sea measurements

Wind wave tank

Summary

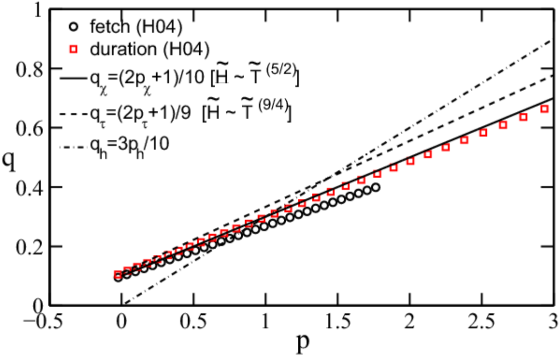


Sea wave growth in field measurements. Hwang & Wang (2004) power-law fits

Exponents of HW(2004) power-law fits satisfy (cf. our theory)

$$q_\chi = \frac{2P_\chi p_\chi + Q_\chi}{10}; \qquad q_\tau = \frac{2P_\tau p_\tau + Q_\tau}{9}$$

where $P_\chi = 0.85$; $Q_\chi = 0.99$; $P_\tau = 0.85$; $Q_\tau = 0.99$

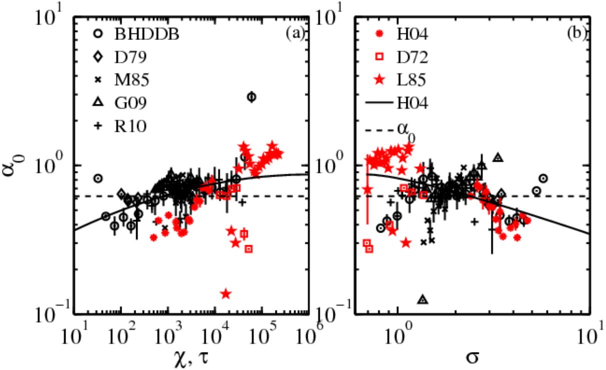


Sea wave growth in field measurements. Hwang & Wang power-law fits and our invariant

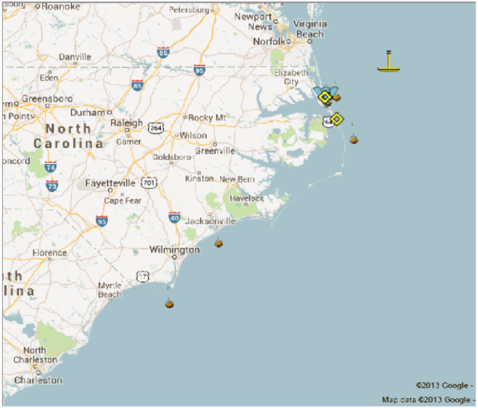
"The Hwang & Wang (2004) invariant"

$$(\mu^4 \nu)\chi^{-0.54+0.039\ln\chi} = I_{fetch}$$

factor 2.7 for the range of dimensionless fetch $10 - 10^5$!!!



Waverider data of Field Research Facility of the US Army Corps of Engineers



5 FRF wave riders

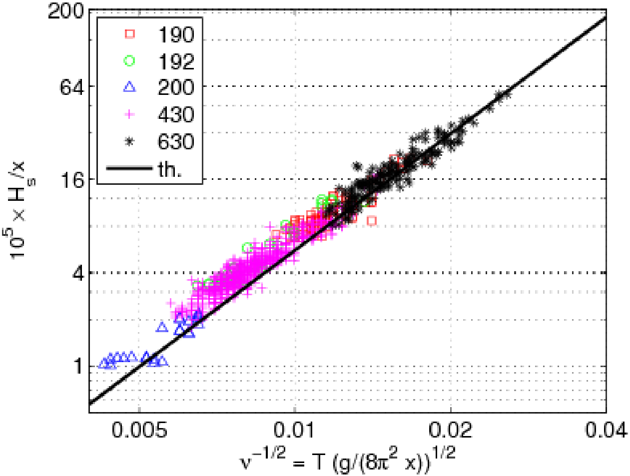
ID	Fetch	N
190	6.1 km	102
192	18.5 km	35
200	18.5 km	24
430	18.5 km	606
630	3.0 km	171

Nothing but data selection in wave direction $\pm 30^\circ$ to the off-shore (exc. B200) !!!

Wave rider data of Field Research Facility of the US Army Corps of Engineers

Wind-free law of wind wave growth

$$\tilde{H} = 5.59\,\tilde{T}^{5/2}$$



Wind wave tank experiments
Beyond the theory or beyond the reality ?

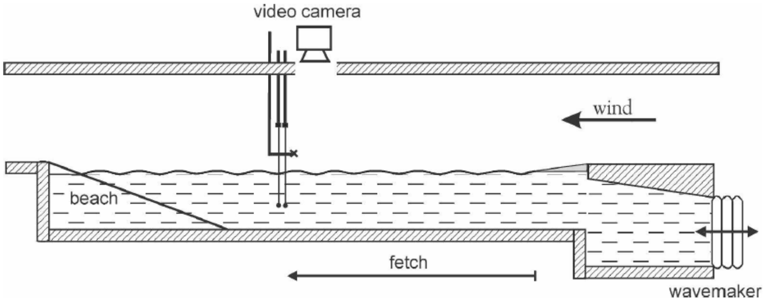


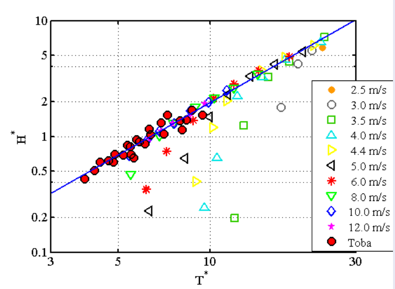
FIG. 1. Schematic view of the large IRPHE-Luminy wind wave tank.

Depth 0.9 m, width 2.6 m, air tunnel height 1.5 m
Fetches 2 – 26.2 meters; Wind speeds 2.5 – 12 m/s;

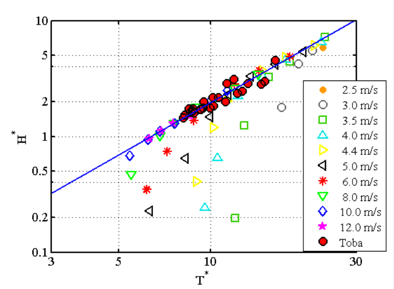
Wave age $C/U > 5$;
Number of wave periods $x/\lambda = 15 - 500$

Wind wave tank experiments
Wind speed scaling for Toba (1972) and G. Caulliez

No corrections for Toba's data



Corrections for Toba's u^*

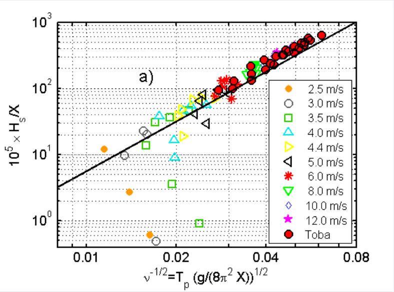


- Data by Caulliez "see" a transition to the Toba law;
- Parameter ranges for Y. Toba and G. Caulliez are close;
- Corrections for capillarity and drift current are necessary

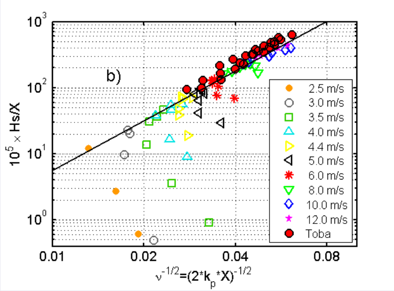
Wind wave tank experiments

Wind free scaling for Toba (1972) and G. Caulliez

No data corrections for drift current and capillarity



Corrections moves the Caulliez data to the wind-free law



- Corrections make data closer to the theoretical line;
- Toba's data have shorter lifetime than ones by Caulliez: within the new scaling data ranges are not the same

Summary

- Self-similar solutions for duration- and fetch-limited wind wave growth are generalized in the form of the wind-free invariant;
- Wind-free scaling of wind-wave growth is proposed;
- Consistency of the theory with previous results of theoretical, experimental (sea and wind wave tank) and numerical studies is demonstrated

Analytical wind wave theory is running

THANK YOU

Our special thanks for data provided by the Field Research Facility, Field Data Collections and Analysis Branch, US Army Corps of Engineers, Duck, North Carolina

to Andrey Pushkarev

P. N. Lebedev Physical Institute

for data of simulation of fetch-limited growth