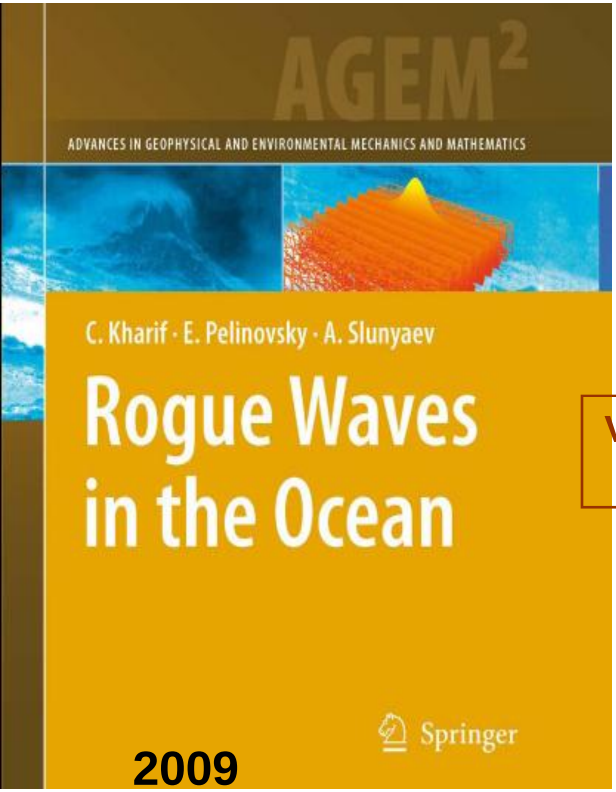
Soliton interaction and turbulence in KdV-like models



2009

Grant No. FP7 GA-234175

Volkswagen Foundation, Germany

РОССИЙСКИЙ ФОНД ФУНДАМЕНТАЛЬНЫХ ИССЛЕДОВАНИЙ
РФФИ

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Int Conf in honor of **Vladimir Zakharov**, Russia, 5 August 2014

Contemporary Physics
Vol. 52, No. 6, November–December 2011, 571–590
Taylor & Francis
Taylor & Francis Group

Rogue waters

Alexey Slunyaev[a,b]*, Ira Didenkulova[a,c] and Efim Pelinovsky[a,d]

Two review papers

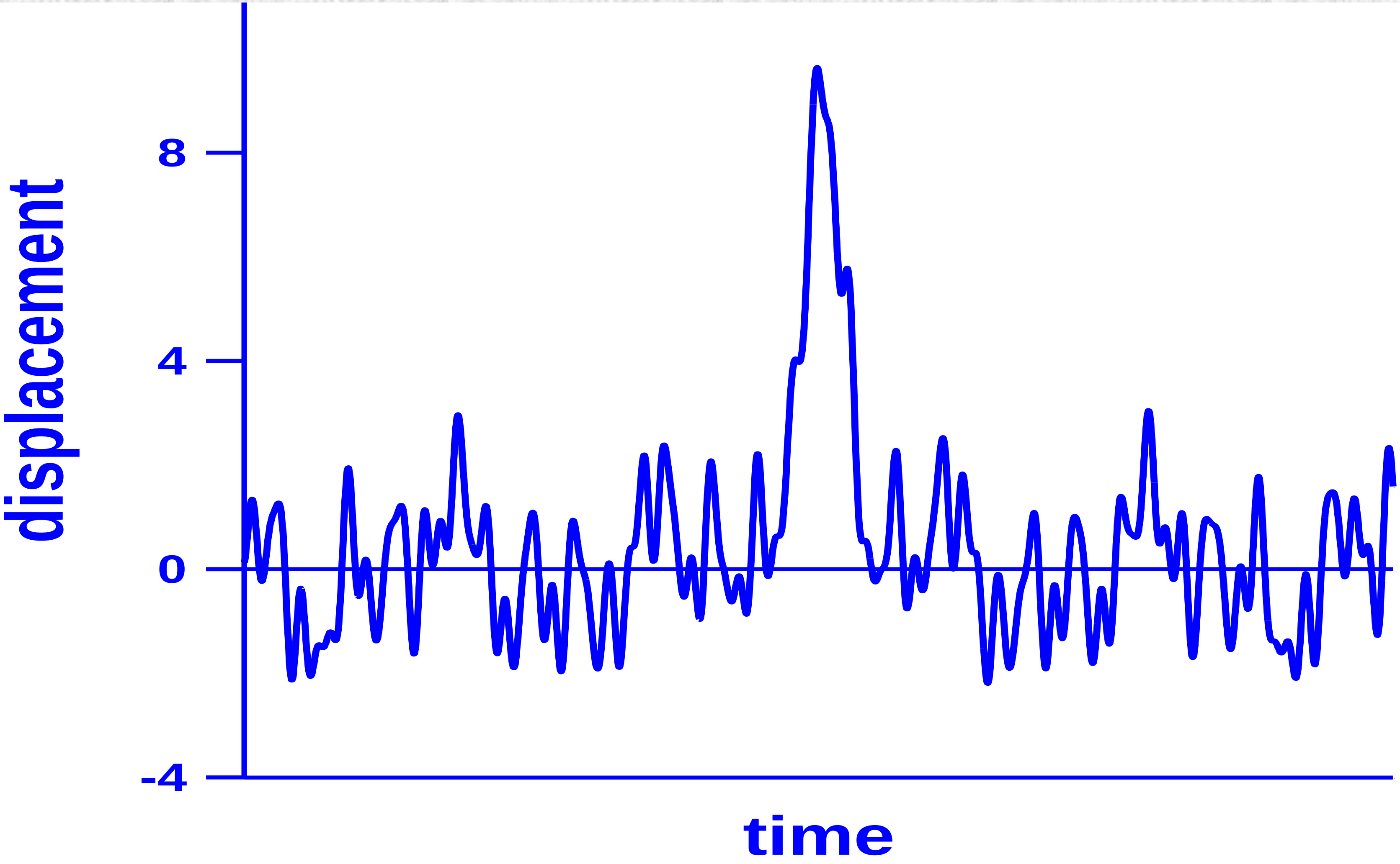
IOP PUBLISHING NONLINEARITY
Nonlinearity **24** (2011) R1–R18 doi:10.1088/0951-7715/24/3/R01

INVITED ARTICLE

Rogue waves in nonlinear hyperbolic systems (shallow-water framework)*

Ira Didenkulova[1,2] and Efim Pelinovsky[2]

Freak Wave Definition *(mathematical)*

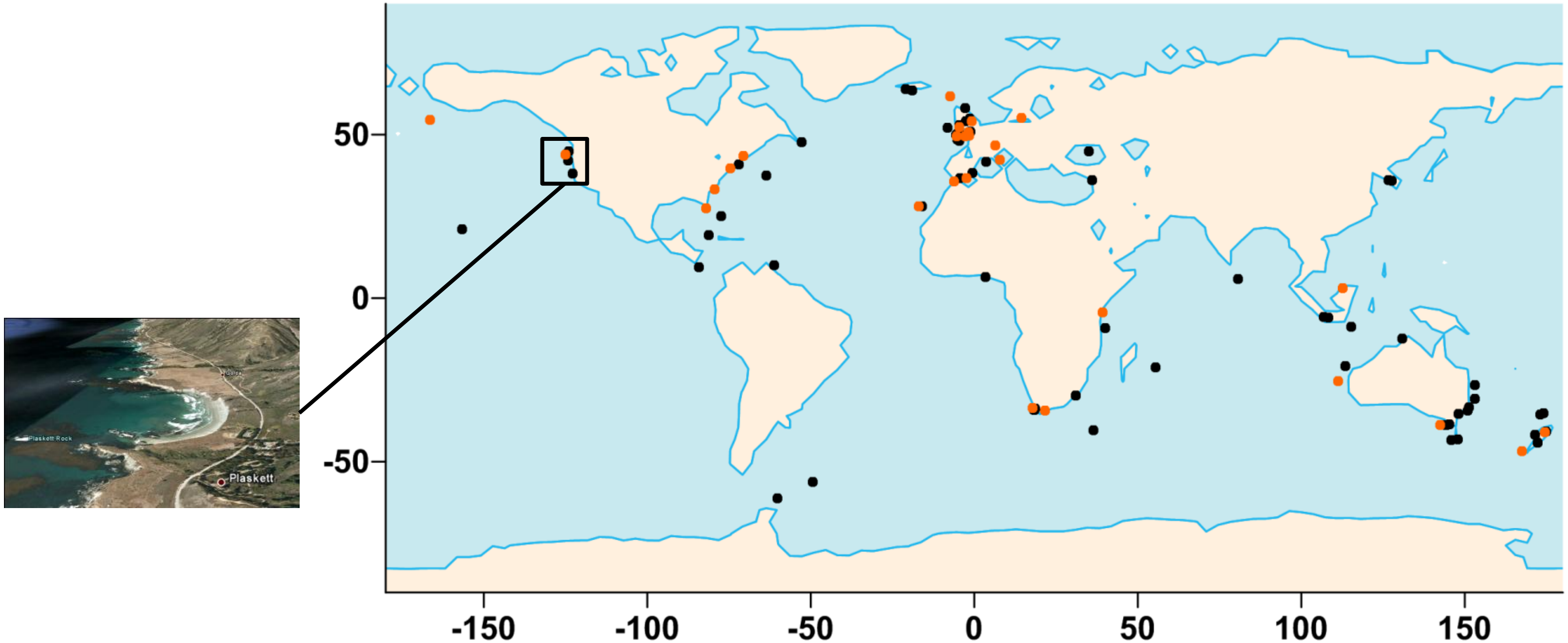


$$H_{freak} > 2\ H_{significant}$$

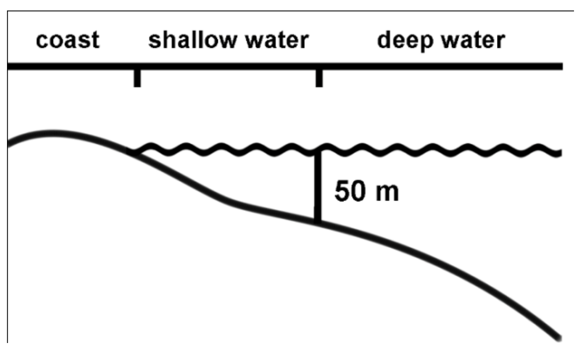
Average on 1/3 highest waves in 20 min record

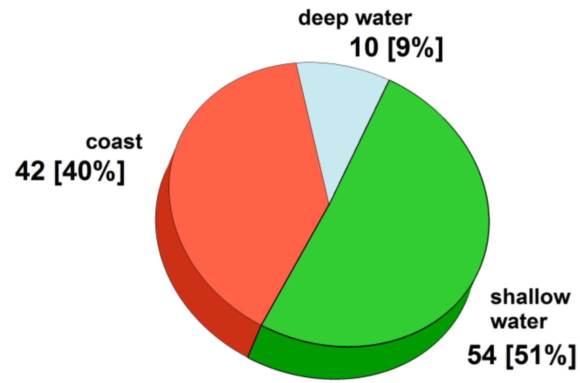
Rogue waves from mass media in 2006-2010

(Nikolkina & Didenkulova, 2011)

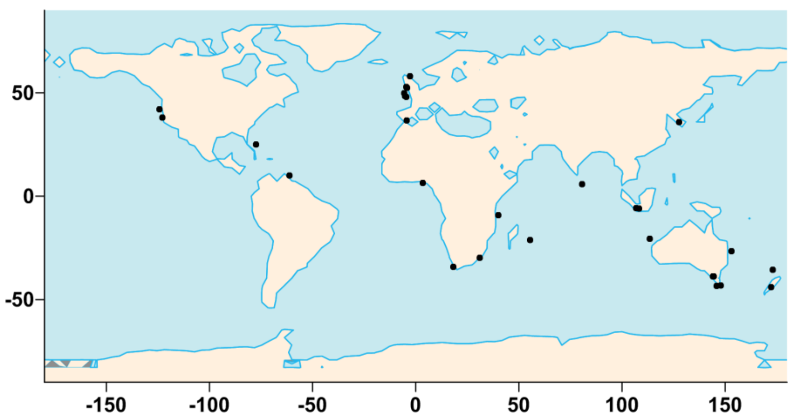


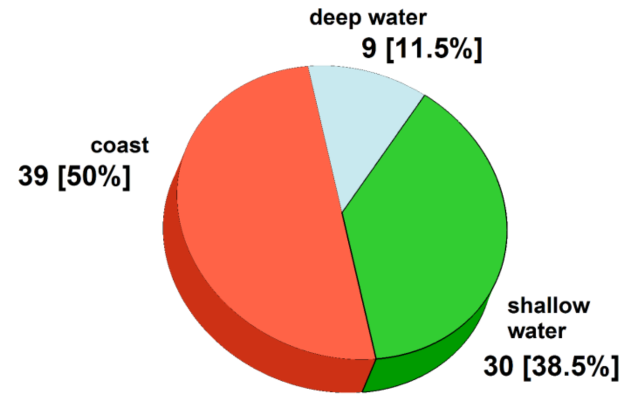
- 78 true events
- 106 possible





Rogue waves in 2006-2010: shallow water



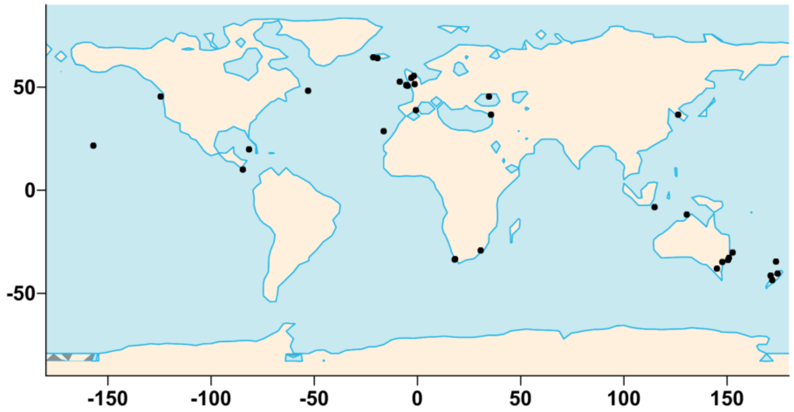


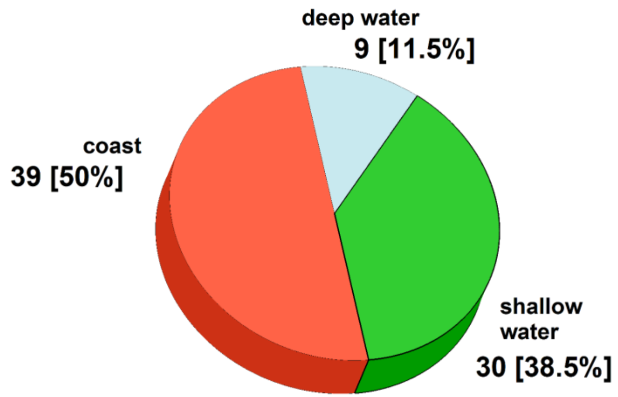
14 of 30 events led to the damage of the vessel, 7 events – to its loss (in deep waters only 5 ships were damaged). These events are also associated with extremely high number of human fatalities (79 persons) and injuries (90 persons). For comparison, the number of human losses in the deep water area is significantly less: 6 fatalities and 27 injuries.

In August 2010 the ship carrying 60 people (only 21 rescued) capsized and sank minutes before arriving in harbour.

Another large loss of lives (11 fatalities) occurred in this area when a fishing boat "Jaya Baru" was engulfed by 6 m waves in May, 2007.

Rogue waves in 2006-2010: coast





Totally, during 2006–2010, 39 such events were reported, which caused 46 fatalities and 79 injuries. Usually such waves appear unexpectedly in calm weather conditions and result in the washing the person off to the sea.

Rogue wave in Kalk Bay on 26 August 2005

The wave over 9 m washed two people off the breakwater in Kalk Bay (South Africa).
The wave overflows the breakwater.

16 October, 2005   Trinidad   3 m

16 October, 2005 Trinidad 3 m

South Korea, May 4, 2008

죽도의 '너울파도' 형성 과정

At least eight people are reported to have been killed after they were swept away by high waves.

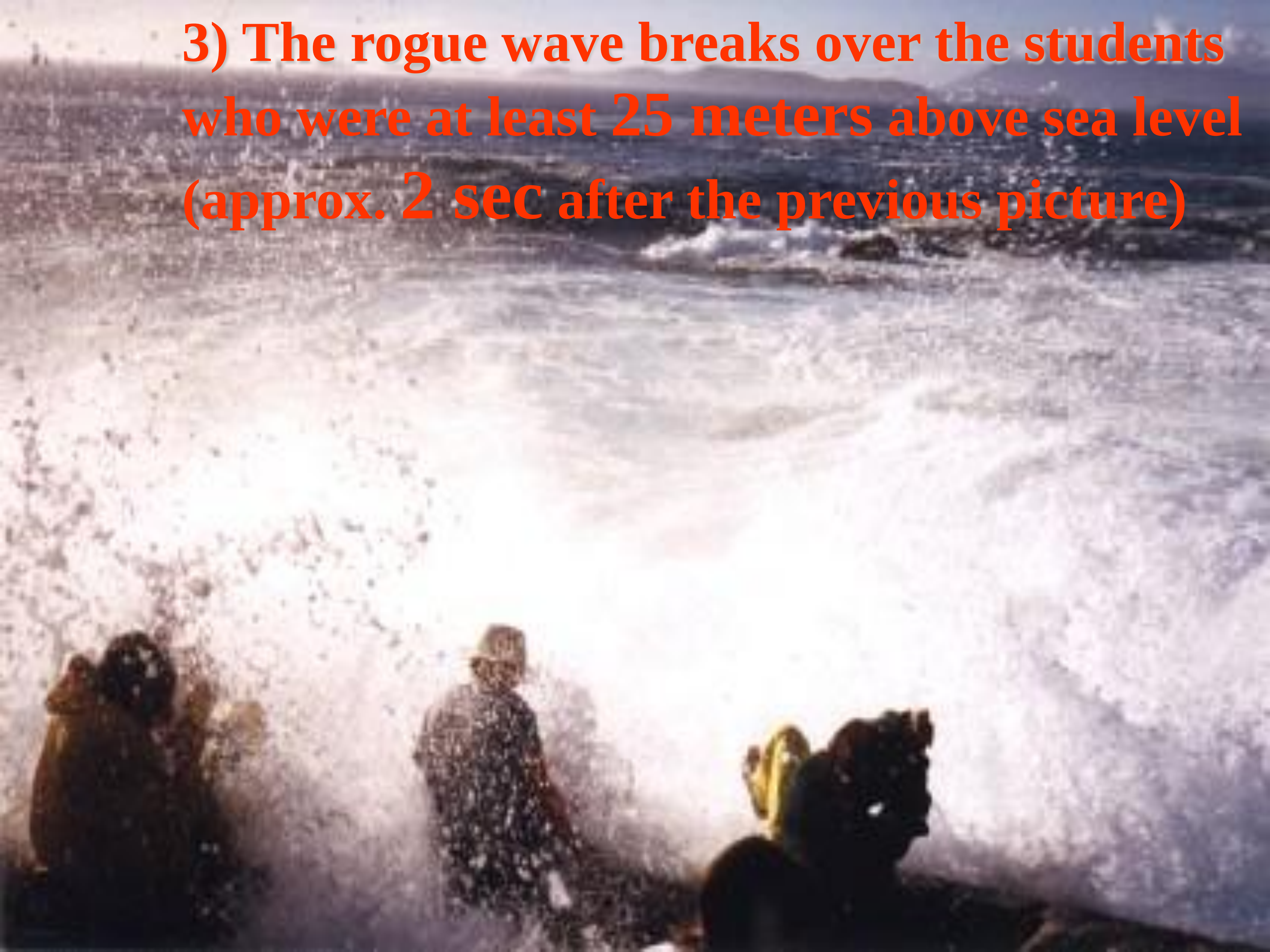
In October, 1998, thirteen students in the Bamfield Marine Station Fall Program were taken on a field trip to Kirby Point, a wave-beaten peninsula on the southwest corner of Dianna Is. (Barkley Sound, Vancouver Island, British Columbia), to view the large open-ocean swell breaking on the shore the day after a very large storm had passed through. The students split into two groups and sat atop two adjacent rock outcrops, at least 25 meters above sea level

After about 45 minutes of wave watching, one student tried to capture the feel of these huge waves thundering onto the shore by taking *three pictures* in quick succession of what looked to be a nice example of a large wave as it started to break


1) A rogue wave starts to break low on the shore
25 m


2) The rogue wave races up the shore
(approximately 2 sec after the first picture)
25 m

3) The rogue wave breaks over the students who were at least 25 meters above sea level (approx. 2 sec after the previous picture)

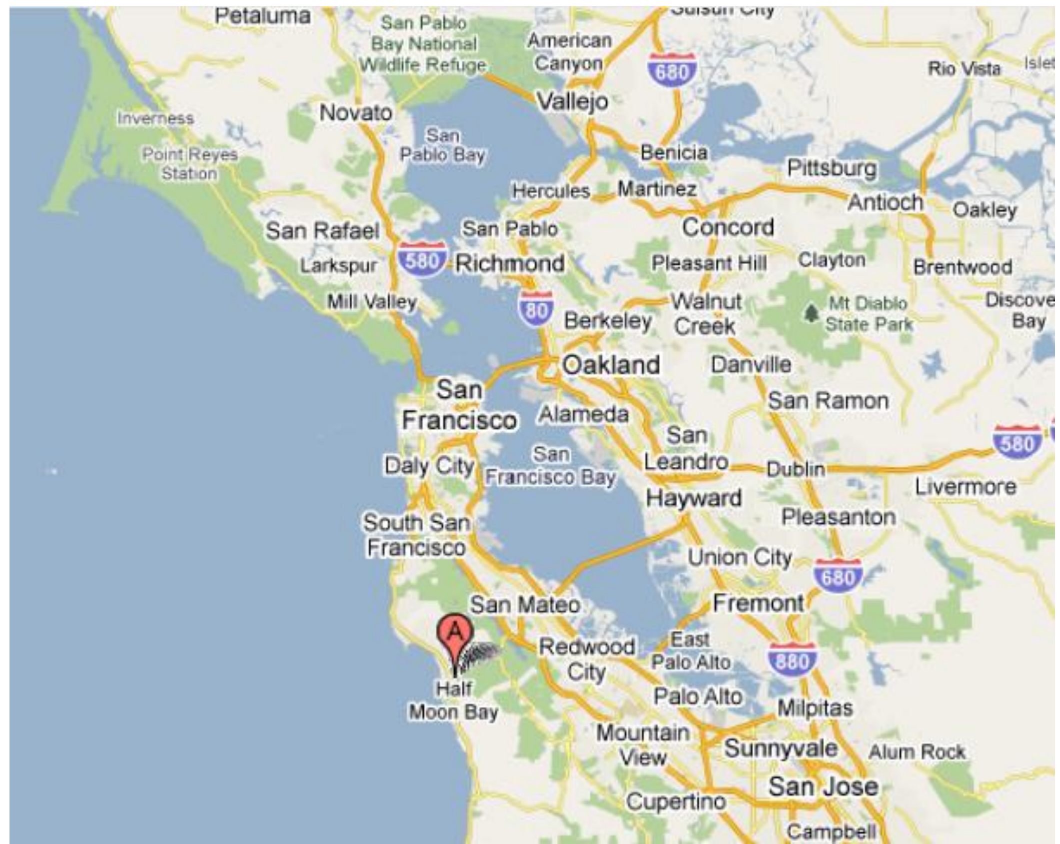


February 14, 2010

CNN producer note

sra3001 biked over to the Mavericks Surf Competiton in Half Moon Bay this morning and noticed some waves splashing over the sea wall. He backed up because he figured other waves would come over. He was right. He captured images of the giant wave surging over the sea wall and onlookers being knocked over.

- **zdan**, CNN iReport producer

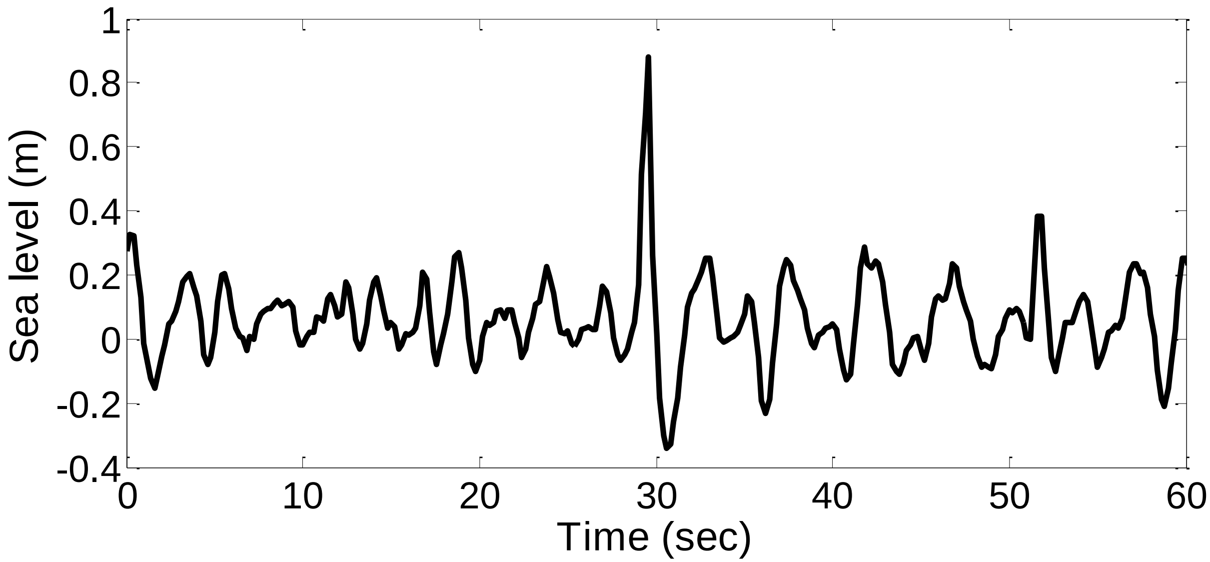




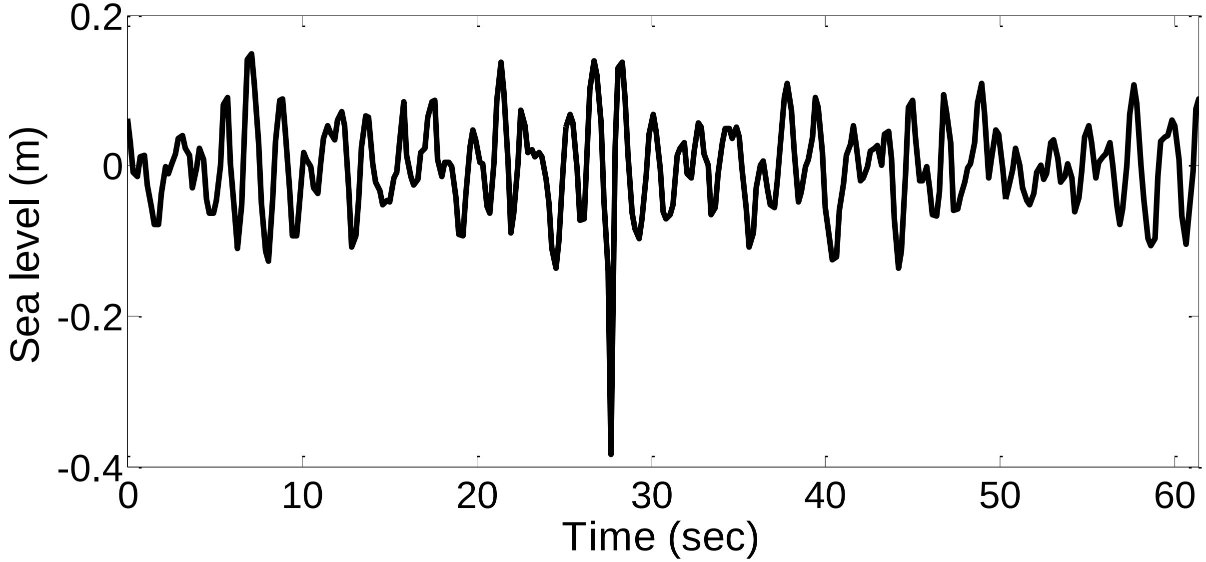








I.Didenkulova, C.Anderson, Freak waves of different types in the coastal zone of the Baltic Sea. Natural Hazards and Earth System Sciences 2010, 10, 2021-2029



I.Didenkulova, Shapes of freak waves In the coastal zone of the Baltic Sea (Tallinn Bay), Boreal Env. Research, 2011, 16, 138-148

Tallinn Bay, Baltic Sea, depth 2 m

Rogue waves in shallow water: possible mechanisms

- Wave-current interaction
  - ✓ Wave blocking
  - ✓ Random caustics
- Wave-bottom (coast) interaction
  - ✓ Focuses
  - ✓ Random caustics
- "Itself" wave dynamics
  - ✓ Nonlinear wave interaction
  - ✓ Dispersive focusing
  - ✓ **Modulational Instability (BF instability)???**
- ~~Wave-atmosphere interaction~~

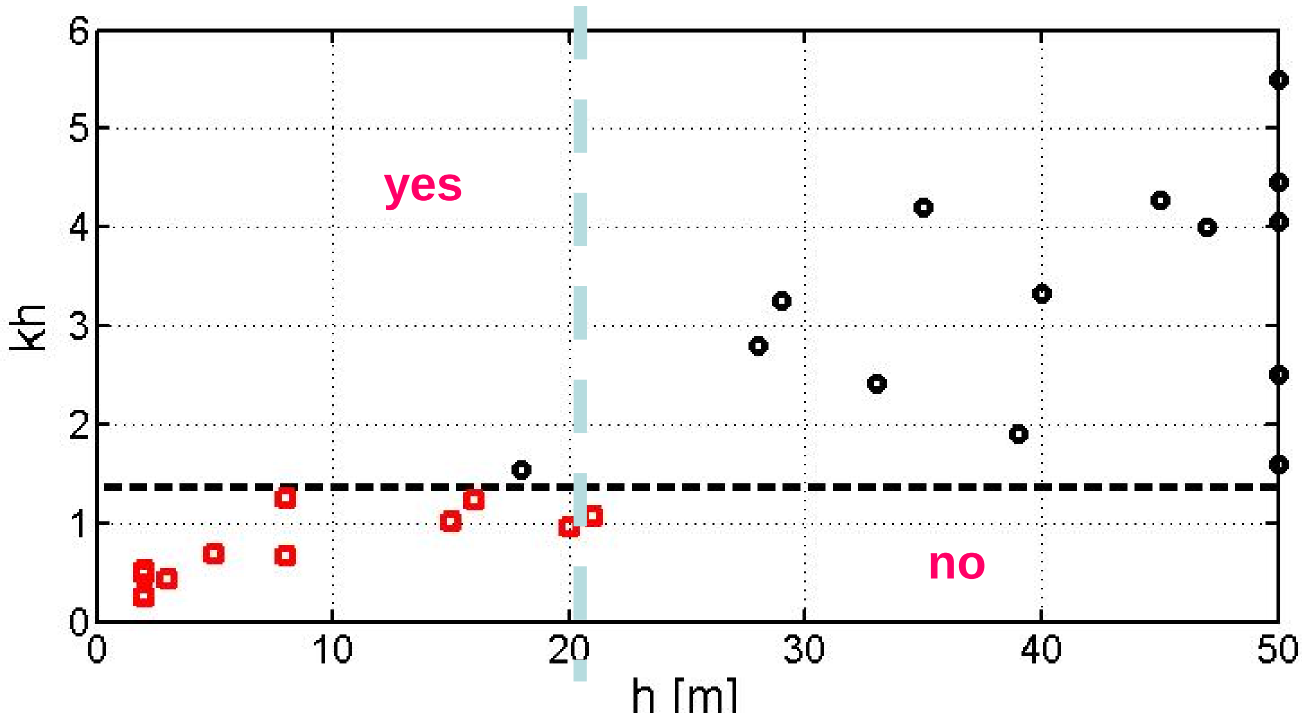
Weak dispersion leads to relatively long life-time of individual waves, which makes them more hazardous!

Rogue waves in intermediate water depth

Where is the border between deep and shallow water and what happens with a decrease in water depth?

Criterion of modulational instability $kh > 1.363$

Border at 20 m depth



ISSN 0021-3640, JETP Letters, 2013, Vol. 97, No. 4, pp. 194–198. © Pleiades Publishing, Ltd., 2013.

Based on the world data 2006-2010

Rogue Waves in the Basin of Intermediate Depth and the Possibility of Their Formation Due to the Modulational Instability¶

I. I. Didenkulova[a–c, *], I. F. Nikolkina[a, c], and E. N. Pelinovsky[c, d]

NLS equation for an arbitrary depth

$$i\frac{\partial A}{\partial t} + \mu\frac{\partial^2 A}{\partial x^2} + \gamma|A|^2 A = 0$$

NLS for complex wave amplitude A

dispersion coeff

nonlinearity coeff

$$\mu = \mu_\infty M(kh),\quad \mu_\infty = \frac{1}{2}\frac{\partial^2\omega}{\partial k^2} = -\frac{1}{8}\frac{g^2}{\omega^3},$$

$$\sigma = \tanh(kh)$$

$$M = [\sigma - kh(1-\sigma^2)]^2 + 4k^2h^2\sigma^2(1-\sigma^2)$$

$$\gamma = \gamma_\infty G(kh),\quad \gamma_\infty = -\frac{\omega k^2}{2} = -\frac{\omega^5}{2g^2},$$

coefficient of NLS in deep waters

$$G = \frac{1}{4\sigma^4}\left\{\frac{1}{c_{gr}^2 - gh}[4c_{ph}^2 + 4c_{ph}c_{gr}(1-\sigma^2) + gh(1-\sigma^2)^2] + \frac{1}{2\sigma^2}(9 - 10\sigma^2 + 9\sigma^4)\right\},$$

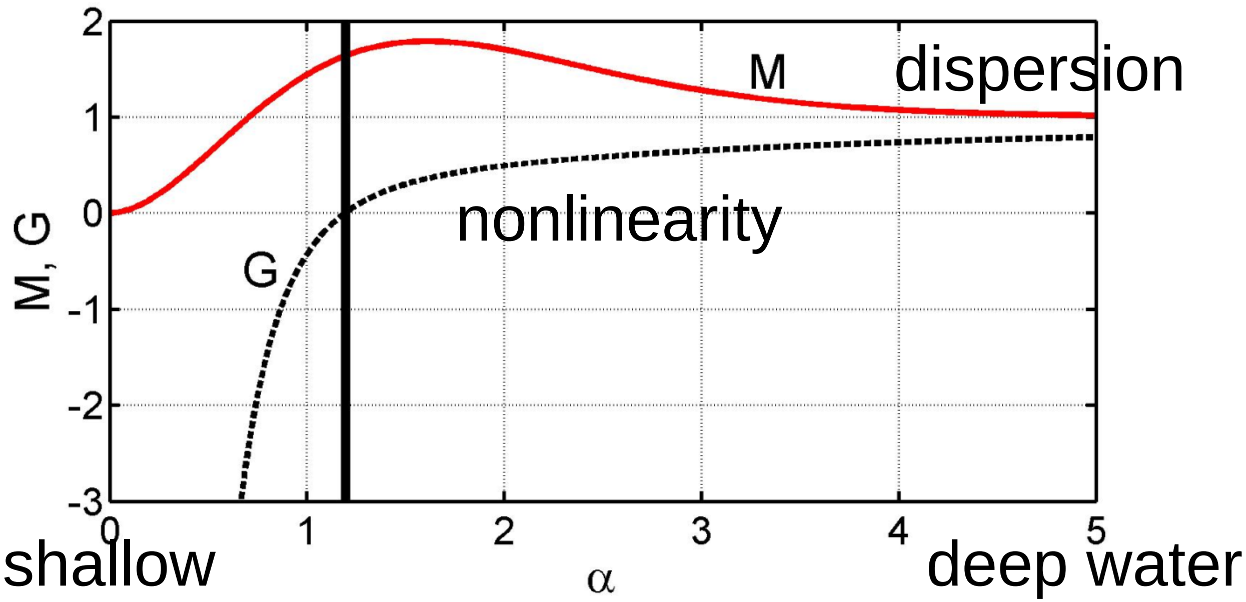
correction, related to the finiteness of the water depth

Coefficients of dispersion and nonlinearity

$$\alpha = \omega^2 h/g,$$

$$a_0 \approx 1.195$$

[Criterion of modulational instability]



• Wave becomes more linear while approaching the modulational instability limit: its nonlinearity decreases and its dispersion is still high and close to its maximum value

• μ is always negative, while γ changes its sign from negative to positive passing through the critical value of *kh*

Modulational instability regime

$$i\frac{\partial u}{\partial \tau} + \frac{\partial^2 u}{\partial y^2} + 2|u|^2 u = 0,$$

$$i\frac{\partial A}{\partial t} + \mu\frac{\partial^2 A}{\partial x^2} + \gamma|A|^2 A = 0$$

Canonical form

$$y = kx,\ u = A\sqrt{\gamma/2\mu k^2},\ \tau = -\mu k^2 t.$$

Family of rational or multi-rational solutions (breathers), which allow different shapes of rogue waves

$$A(x,t) = A_0\left[-1 + \frac{4 - 8i\gamma A_0^2 t}{1 + 2\gamma A_0^2 x^2/\mu + 4\gamma^2 A_0^4 t^2}\right]\exp(-i\gamma A_0^2 t).$$

Peregrine breather

[in original variables]

$$L \sim \frac{1}{A_0}\sqrt{\frac{\mu}{\gamma k_0}}$$ its length

$$T \sim \frac{1}{\gamma A_0^2}$$ duration

Peregrine breather

$$A(x,t) = A_0\Bigg[-1 + \frac{4 - 8i\gamma A_0^2 t}{1 + 2\gamma A_0^2 x^2/\mu + 4\gamma^2 A_0^4 t^2}\Bigg]\exp(-i\gamma A_0^2 t).$$

Peregrine breather

[in original variables]

$$L \sim \frac{1}{A_0}\sqrt{\frac{\mu}{\gamma k_0}}$$

$$T \sim \frac{1}{\gamma A_0^2}$$

Wave steepness

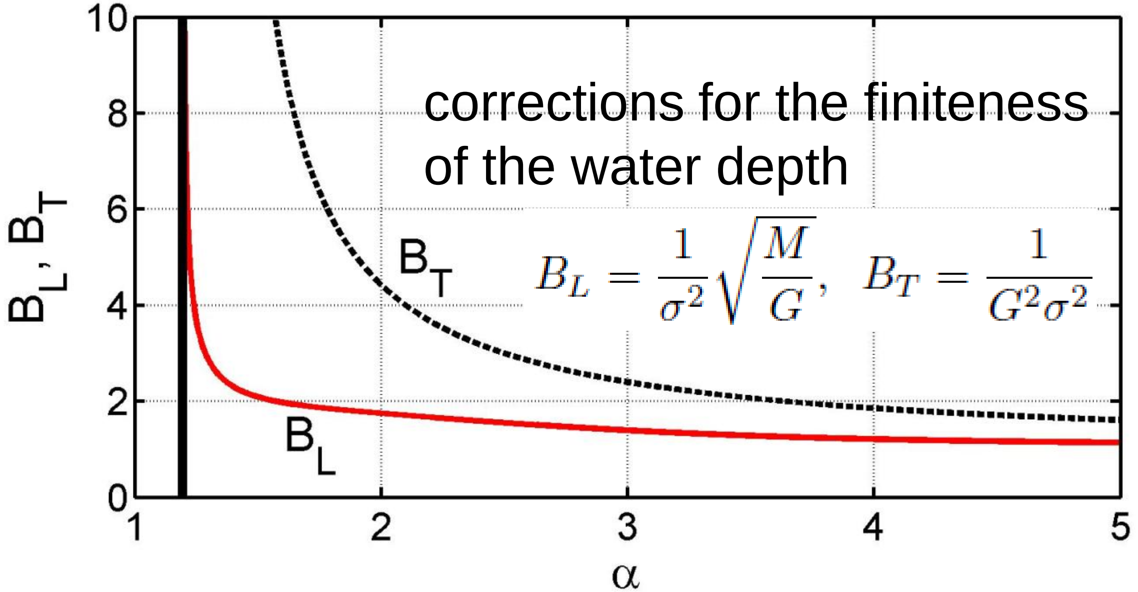
$$n_L \sim kL \sim \frac{B_L}{\varepsilon}, \quad n_T \sim \omega T \sim \frac{B_T}{\varepsilon^2}, \quad \varepsilon = kA,$$

Number of waves is space and in time

$$\alpha = \omega^2 h/g,$$

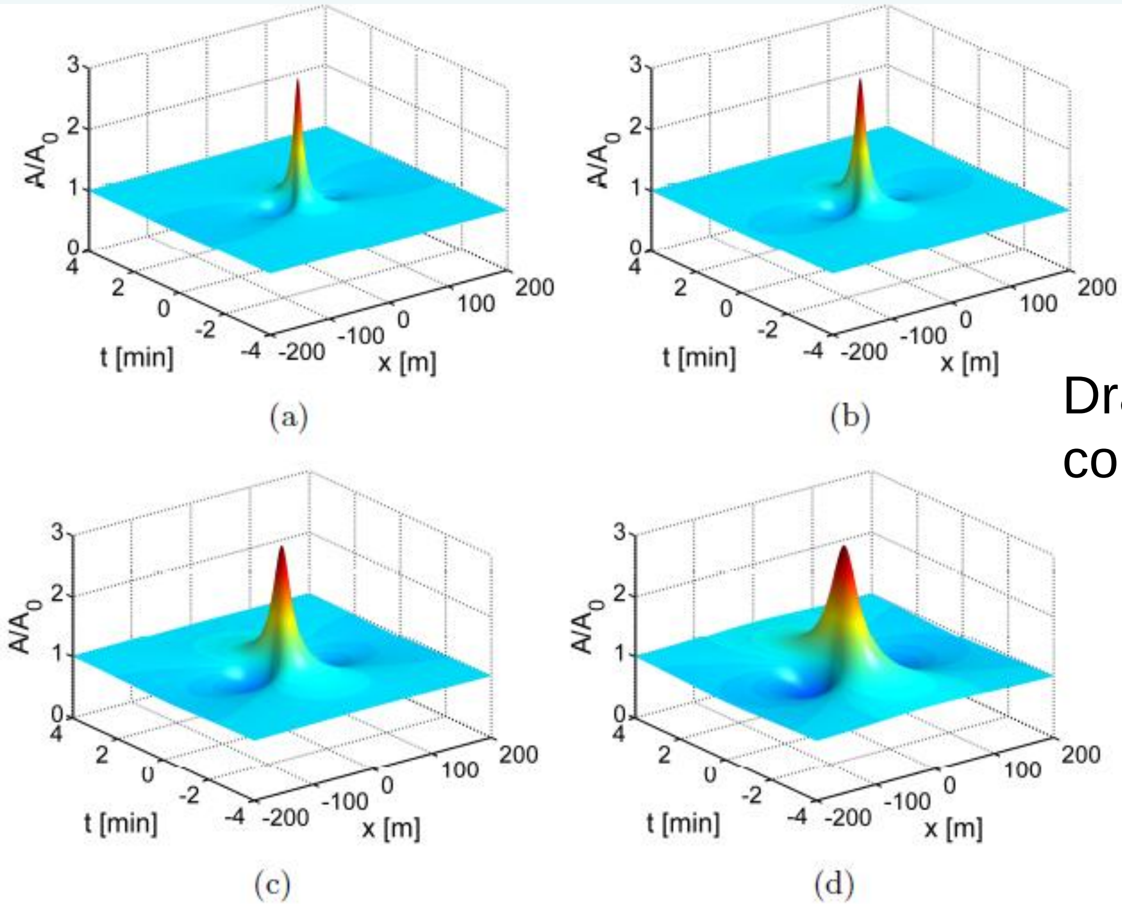
$$a_0 \approx 1.195$$

[Criterion of modulational instability]



Especially intensive increase in breather duration – long life!

Rogue waves in intermediate water depth



Draupner wave conditions

Peregrine breather at the background of the carrier wave with the period 6 s and amplitude 3 m for a) kh = ∞; b) kh = 5; c) kh =2; d) kh =1.6.

Note: for a fixed wave period and amplitude, the number of individual waves within breather increases with a decrease in kh. As a result, the rogue event in shallow water contains more hazardous waves than in deep water

Rogue waves in shallow water: possible mechanisms

- Wave-current interaction
  - ✓ Wave blocking
  - ✓ Random caustics
- <u>Wave-bottom (coast) interaction</u>
  - ✓ Focuses
  - ✓ Random caustics
- "Itself" wave dynamics
  - ✓ <u>Nonlinear wave interaction</u>
  - ✓ Dispersive focusing
  - ✓ ~~Modulational instability~~
- ~~Wave-atmosphere interaction~~

Weak dispersion leads to relatively long life-time of individual waves, which makes them more hazardous!

The most probable mechanism of rogue wave generation in deep water does not work in shallow water!

Shallow Water Equations (Hyperbolic System)

Constant Depth, No boundaries

1D Problem

$$\frac{\partial H}{\partial t} + \frac{\partial}{\partial x}\left[Hu\right] = 0$$

$$\frac{\partial (uH)}{\partial t} + \frac{\partial}{\partial x}\left[Hu^2 + \frac{gH^2}{2}\right] = 0$$

H – total water depth
u – depth averaged velocity
g – gravity acceleration

Unidirectional Riemann Wave (right)

$$\frac{\partial H}{\partial t} + V(H)\frac{\partial H}{\partial x} = 0$$

$$u = 2\left(\sqrt{gH} - \sqrt{gh}\right)$$

$$H(x,t) = H_0\left[x - V(H)t\right]$$

$$V = \sqrt{gh} + \frac{3u}{2} = 3\sqrt{gH} - 2\sqrt{gh}$$

Velocity of points on wave profile

Random Non-breaking Riemann Wave

$$H(x,t) = H_0\left[x - V(H)t\right]$$

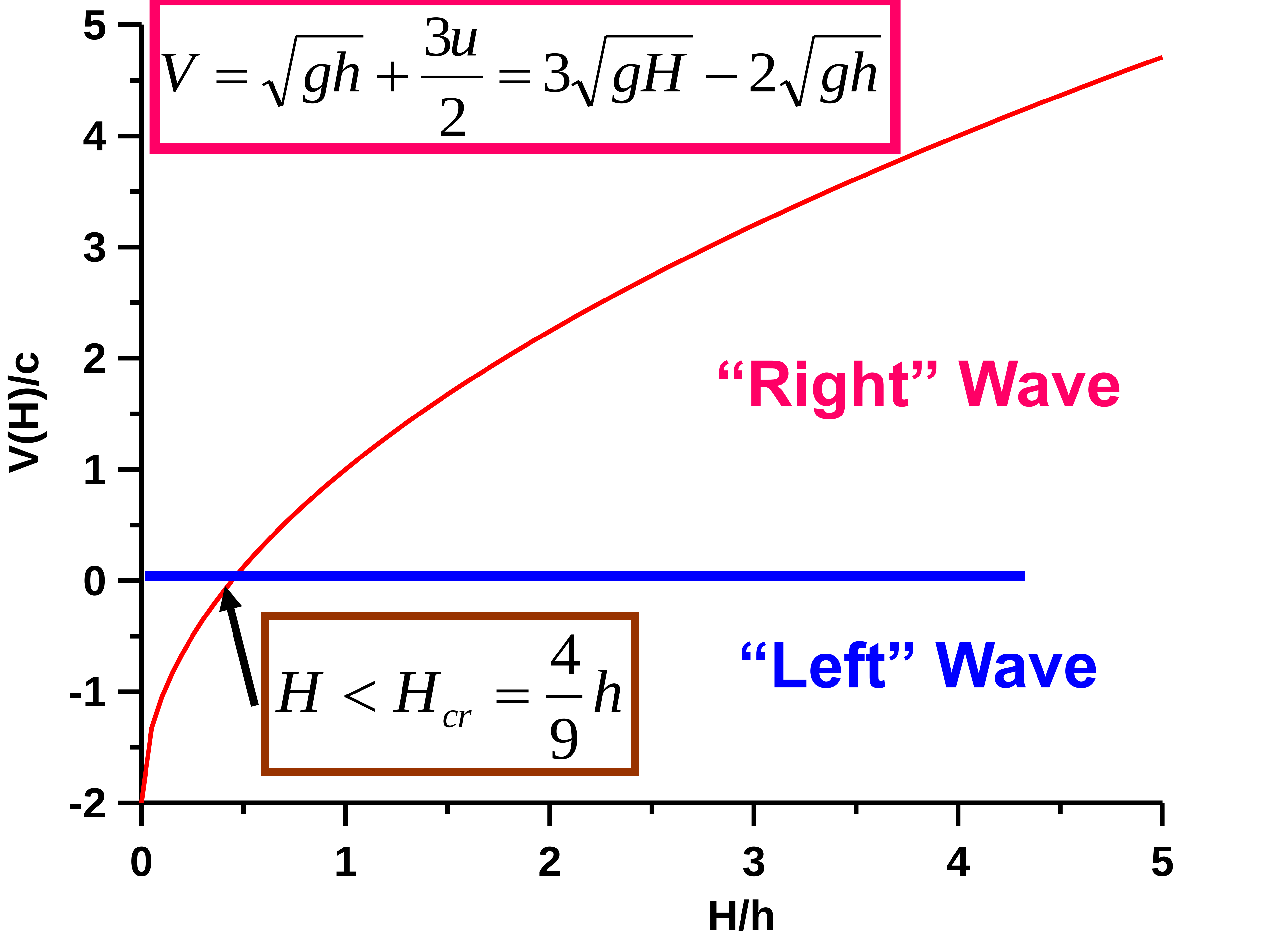
No Variation in Statistics!

$$u = 2\left(\sqrt{gH} - \sqrt{gh}\right)$$

Probability density function W

$$W(u) = W(\eta)\,|\,d\eta/du\,|$$

One of them or both are Non-Gaussian


$V = \sqrt{gh} + \frac{3u}{2} = 3\sqrt{gH} - 2\sqrt{gh}$
"Right" Wave
"Left" Wave
$H < H_{cr} = \frac{4}{9}h$
V(H)/c
H/h
5
4
3
2
1
0
-1
-2
0
1
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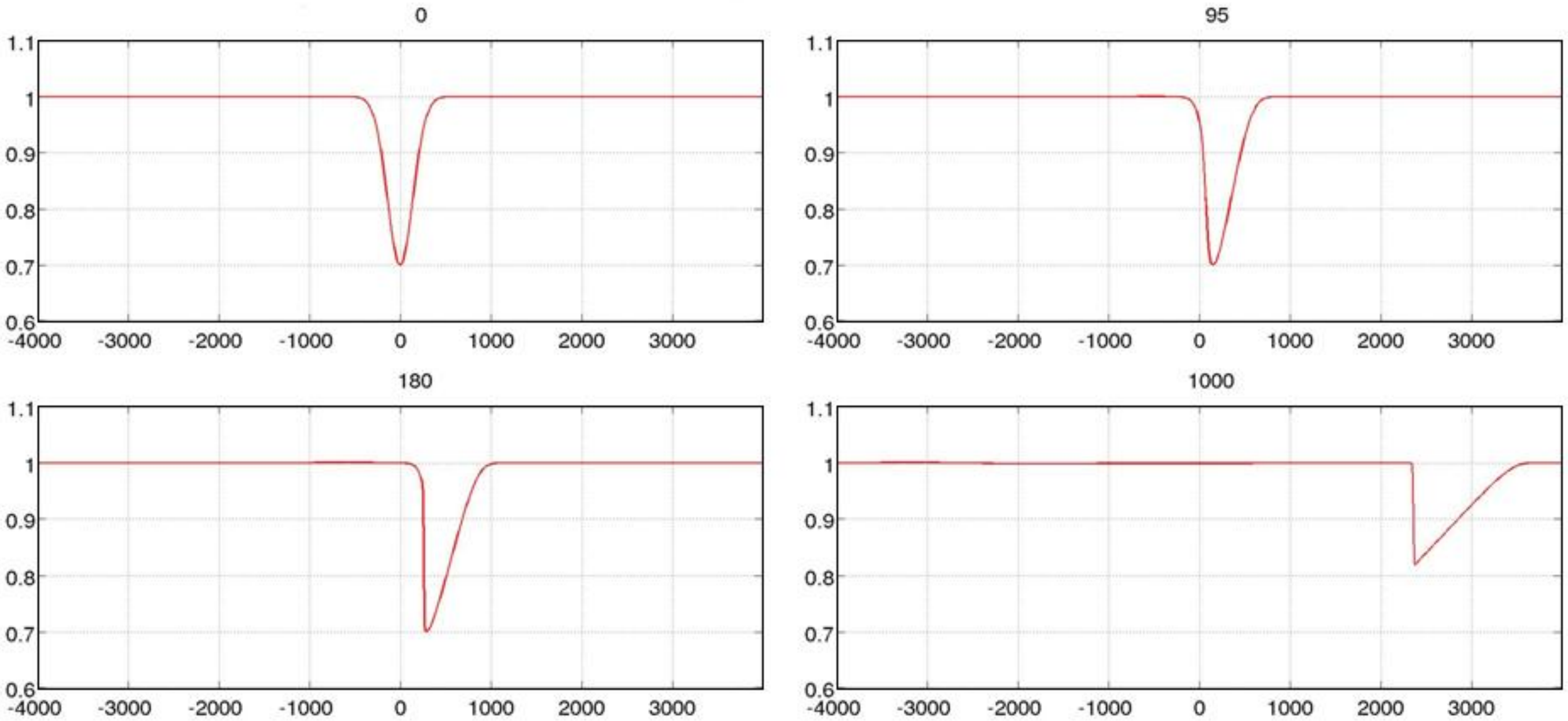
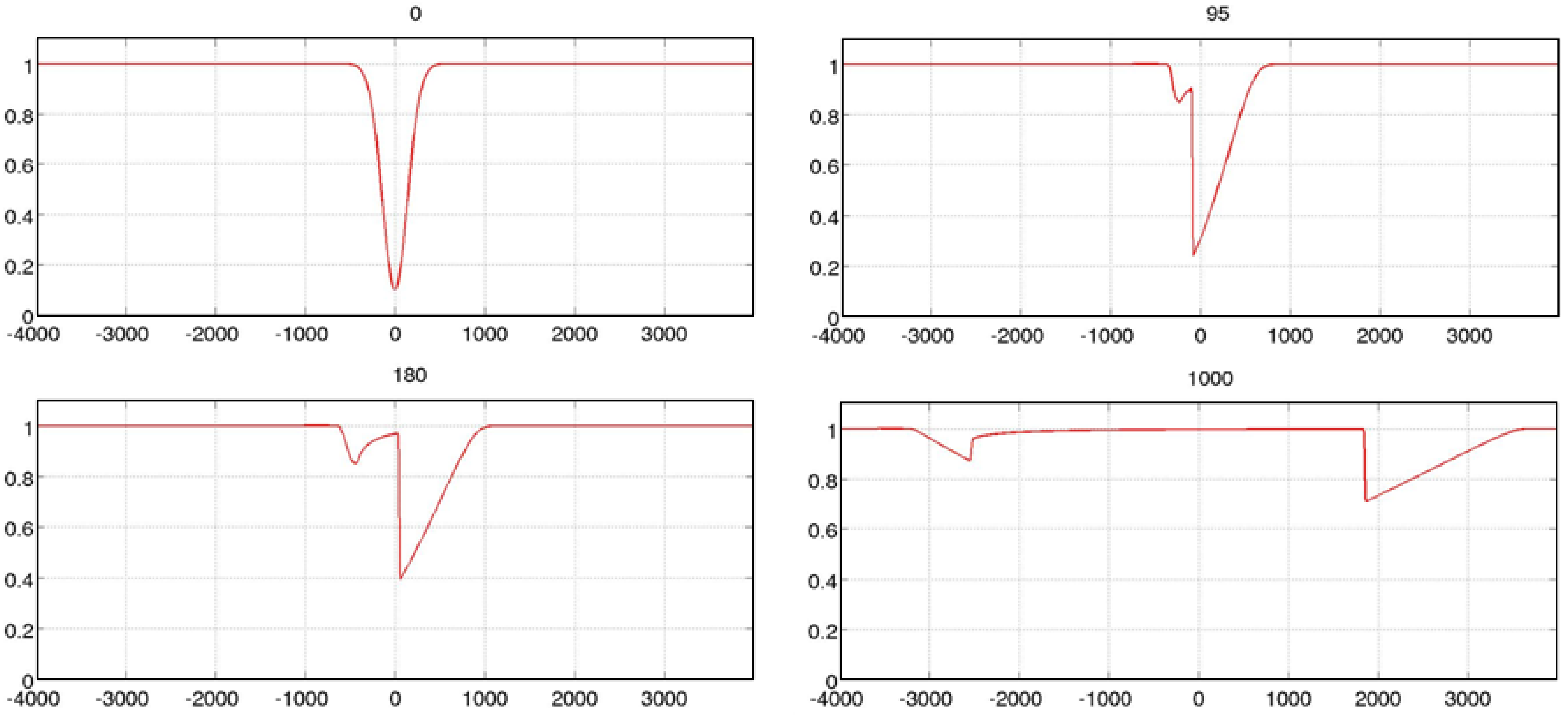
3
4
5



Weak Amplitude – 0.3 m
Depth 1 m

Really, for this condition – soliton generation, no hydraulic jump



Strong Amplitude – 0.9 m
Depth – 1m

Shock formation decreases the rogue wave probability

KdV model for shallow water

$$\frac{\partial \eta}{\partial t} + c\left(1 + \frac{3\eta}{2h}\right)\frac{\partial \eta}{\partial x} + \frac{ch^2}{6}\frac{\partial^3 \eta}{\partial x^3} = 0$$

Inverse scattering method

$$\frac{d^2\Psi}{dx^2} + \left(\lambda - U(x,t)\right)\Psi = 0$$

$$U = -\frac{3\eta}{2h^3}$$

Discrete λ - solitons

Continuous λ – dispersive tail

Korteweg – de Vries equation

Periodic boundary conditions – cnoidal waves

Representation of the solutions through Theta-Function *(Matveev, Osborne et al)*

A.R. Osborne, E. Segre, and G. Boffetta, Soliton basis states in shallow-water ocean surface waves, Phys. Rev. Lett. 67 (1991) 592-595.

A.R. Osborne, Numerical construction of nonlinear wave-train solutions of the periodic Korteweg – de Vries equation, Phys. Rev. E 48 (1993) 296-309.

A.R. Osborne, Behavior of solitons in random-function solutions of the periodic Korteweg – de Vries equation, Phys. Rev. Lett. 71 (1993) 3115-3118.

A.R. Osborne, Solitons in the periodic Korteweg – de Vries equation, the Θ-function representation, and the analysis of nonlinear, stochastic wave trains, Phys. Review E 52 (1995) N. 1 1105-1122.

A.R. Osborne, and M. Petti, Laboratory-generated, shallow-water surface waves: analysis using the periodic, inverse scattering transform. Phys. Fluids 6 (1994) 1727-1744.

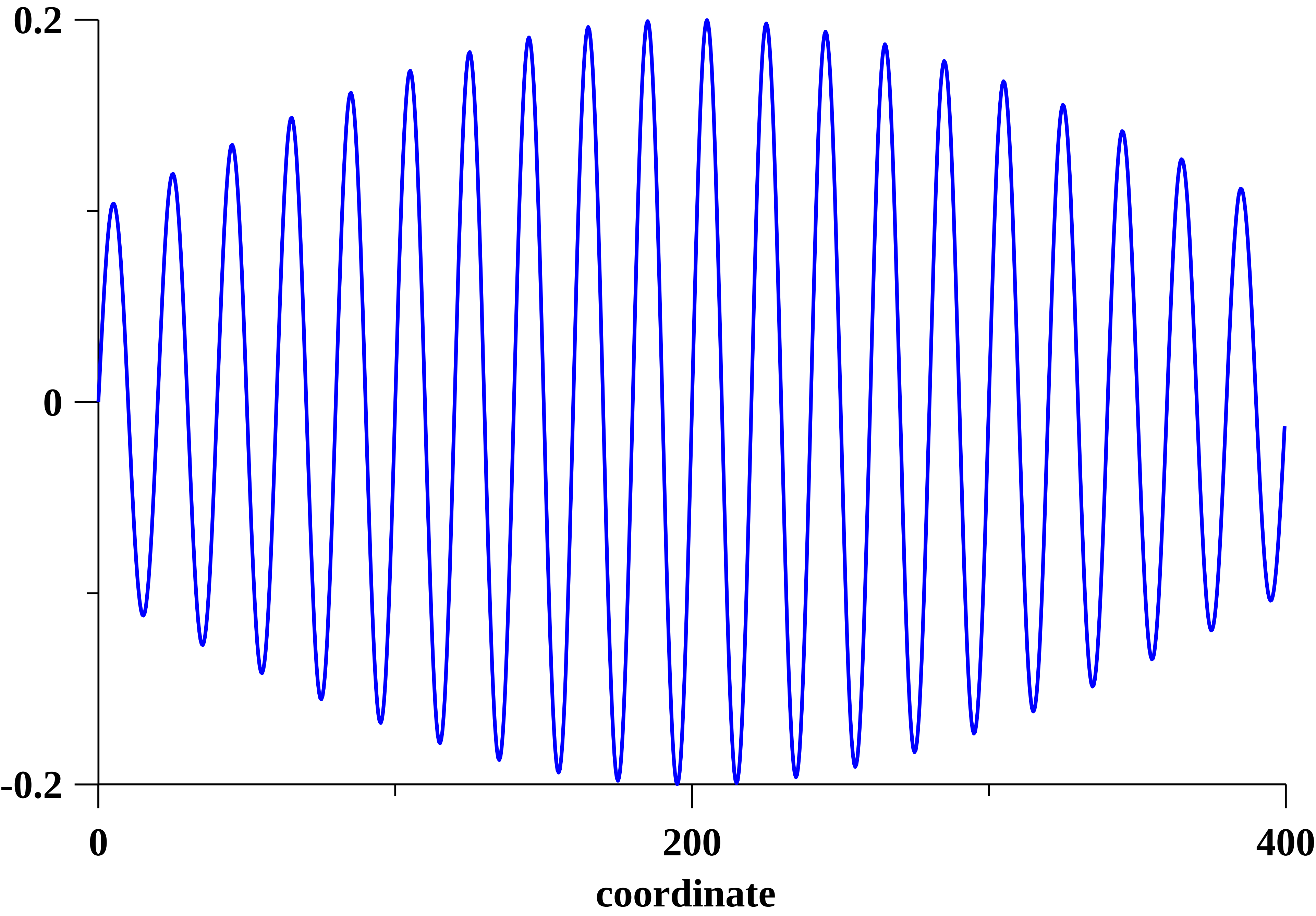
A.R. Osborne, M. Serio, L. Bergamasco, and L. Cavaleri, Solitons, cnoidal waves and nonlinear interactions in shallow-water ocean surface waves, Physica D 123 (1998) 64-81.

Korteweg – de Vries equation

$$\frac{\partial \eta}{\partial \tau}+c\left(1+\frac{3}{2h}\eta\right)\frac{\partial \eta}{\partial y}+\frac{ch^2}{6}\frac{\partial^3 \eta}{\partial y^3}=0$$

Modulated wave field:
no Benjamin – Feir instability

$$A(x,0)=A_0\left(1+m\sin(Kx)\right)$$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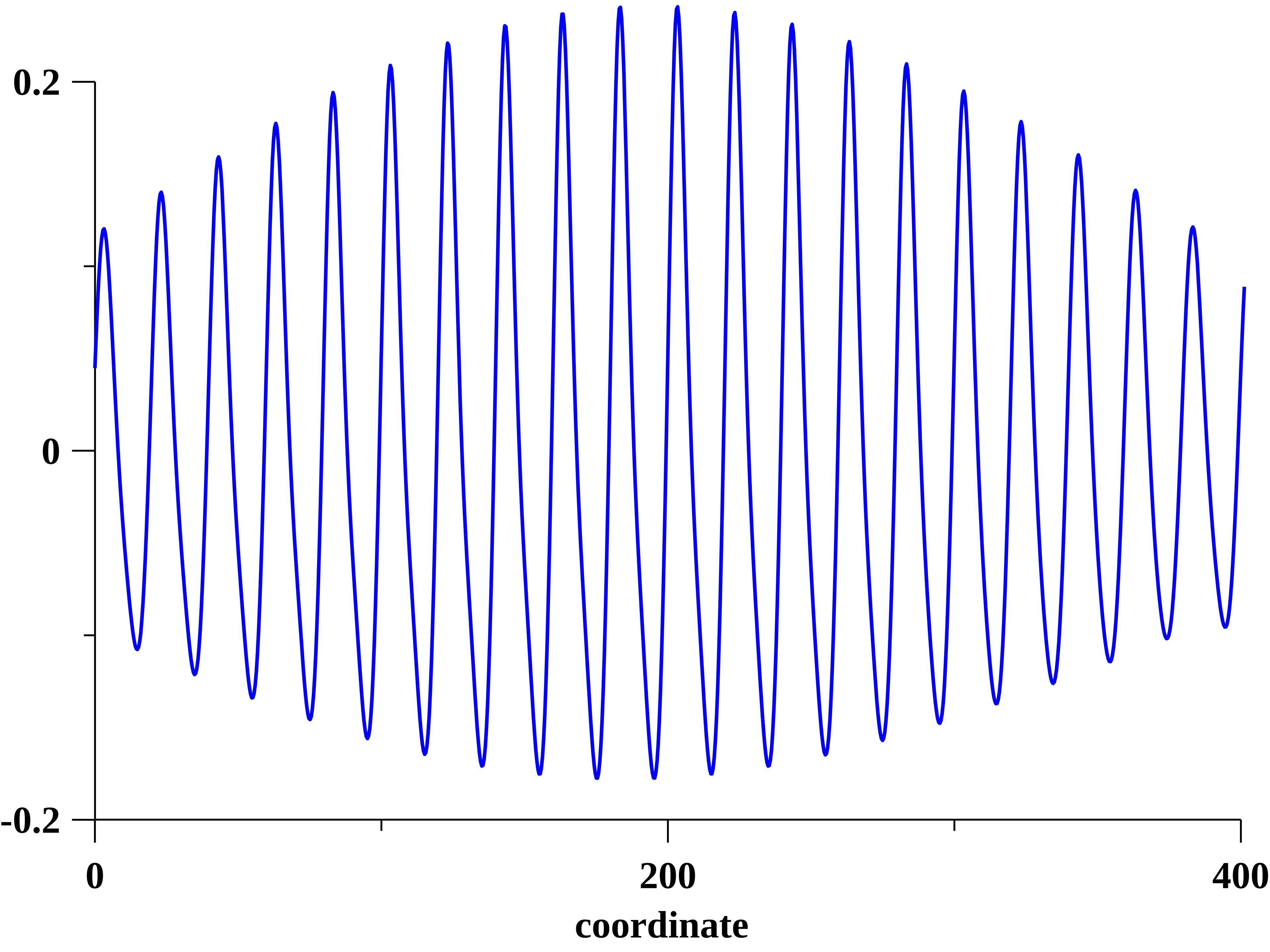

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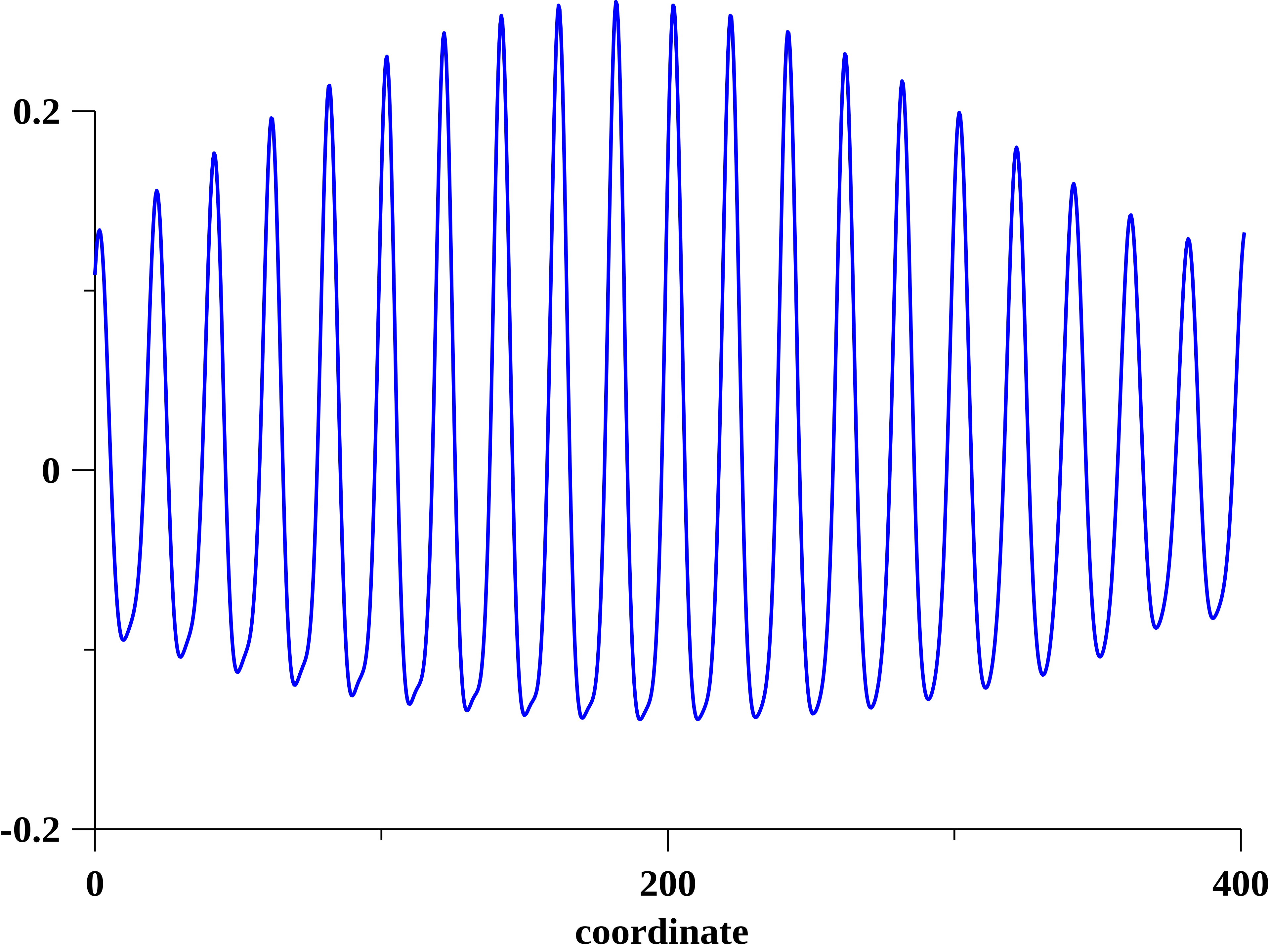

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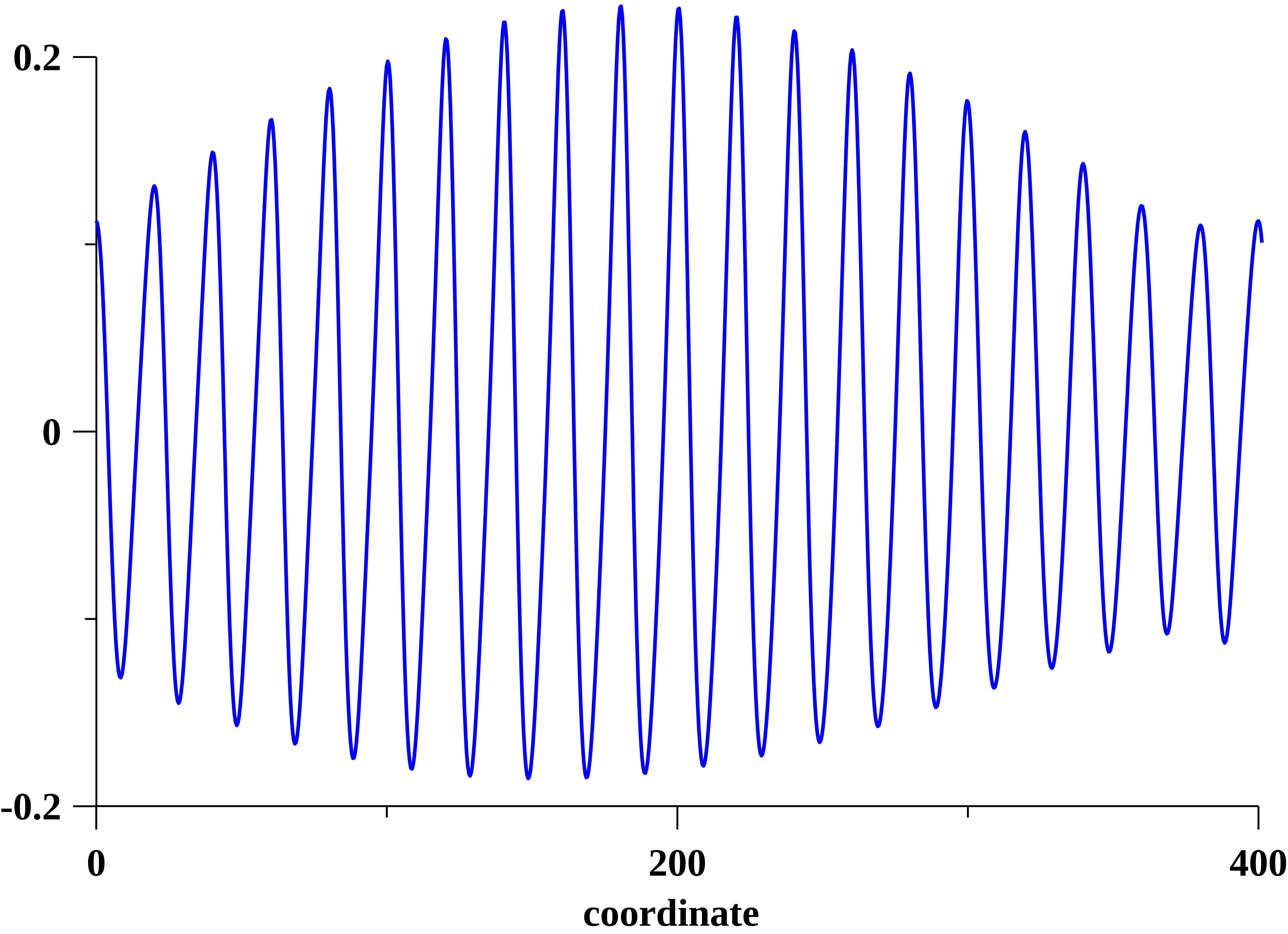

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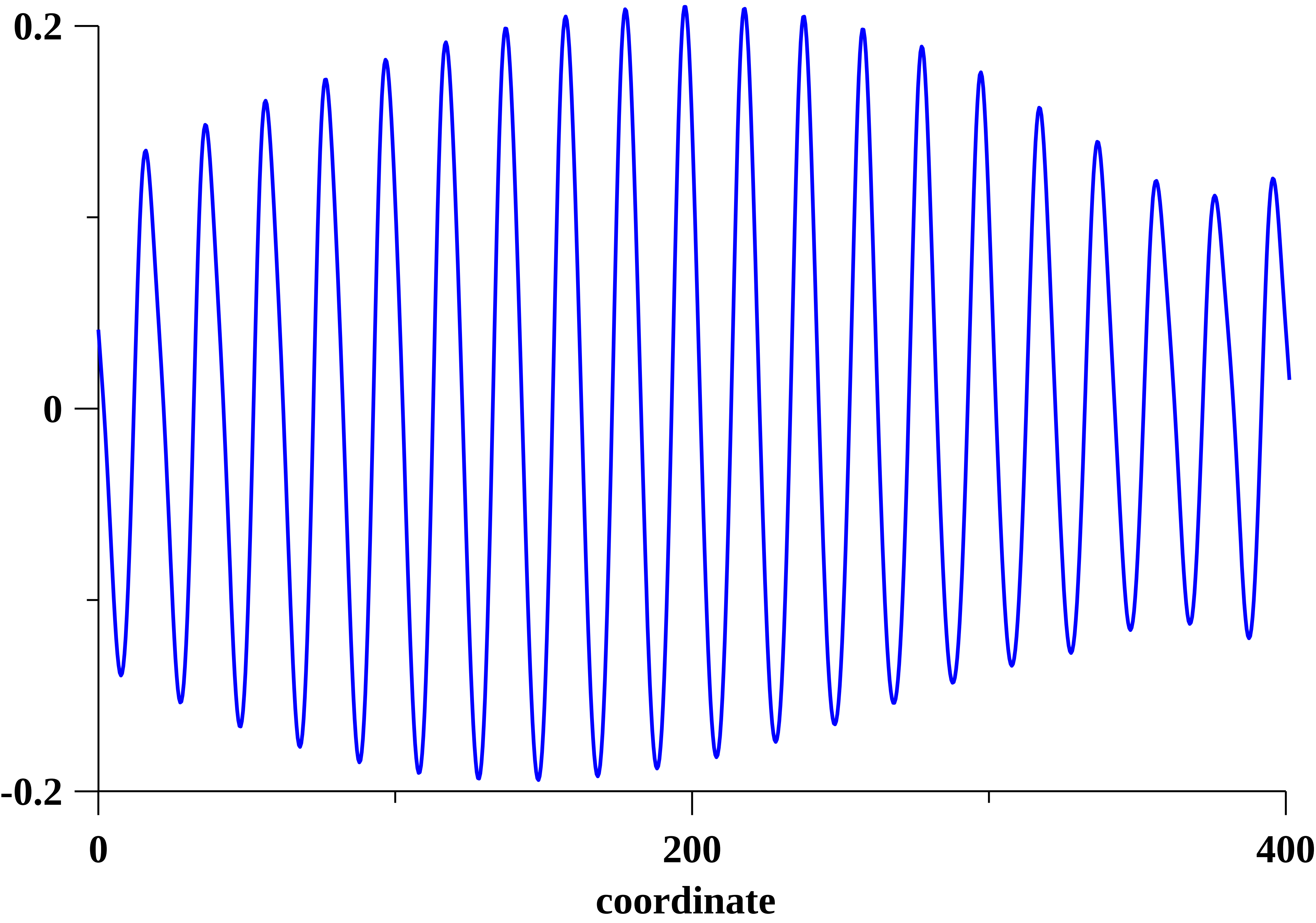
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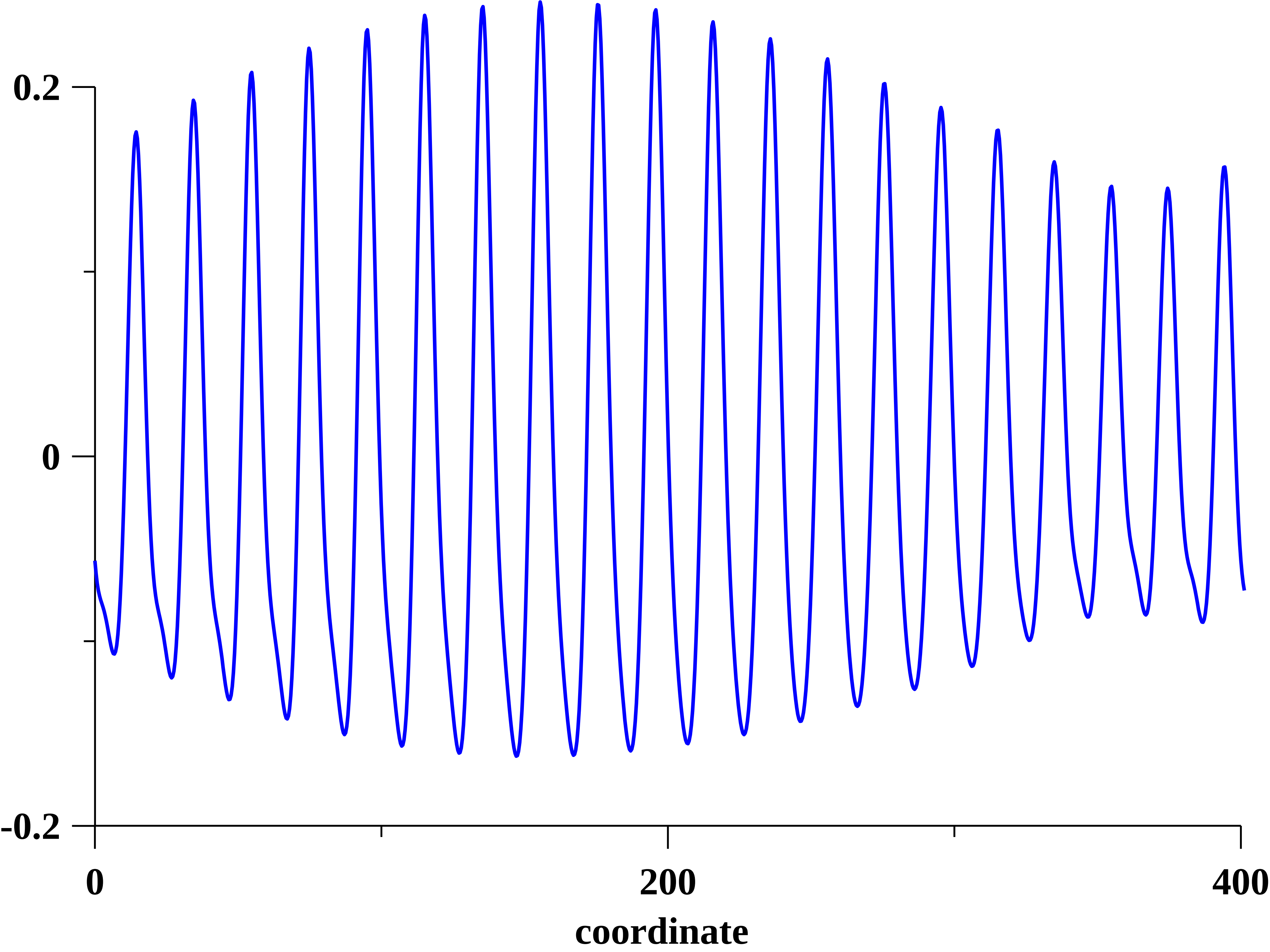
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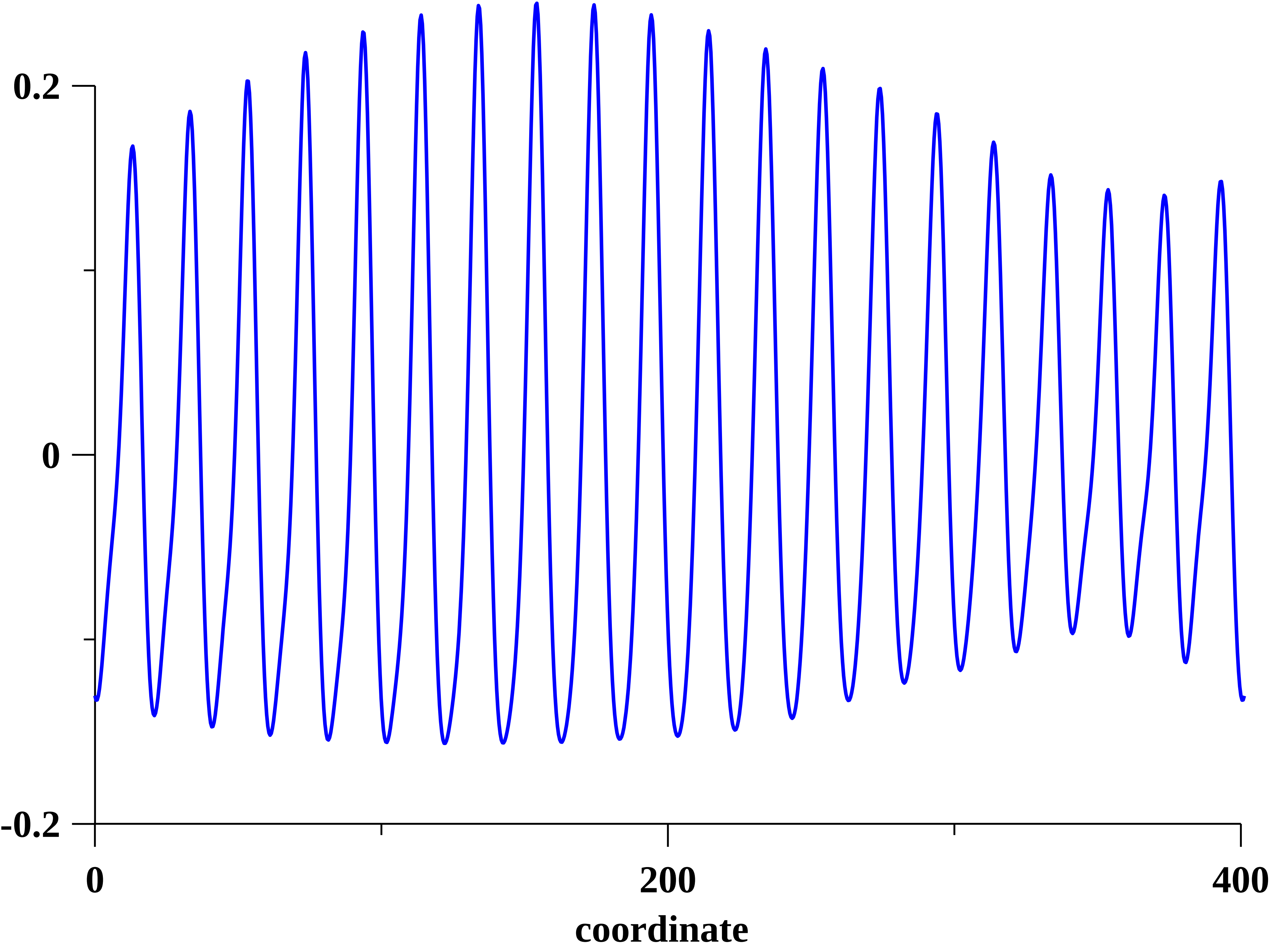

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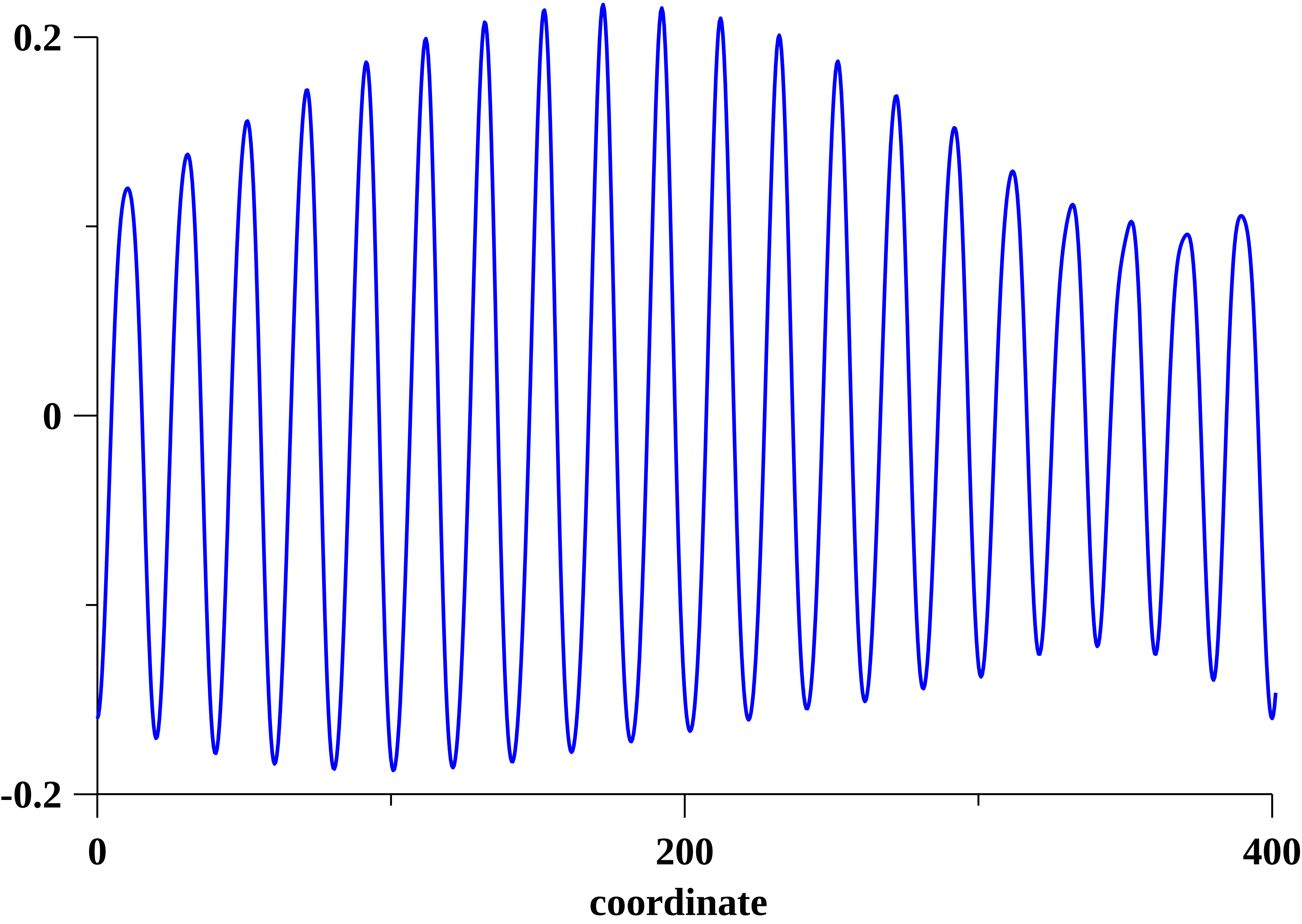

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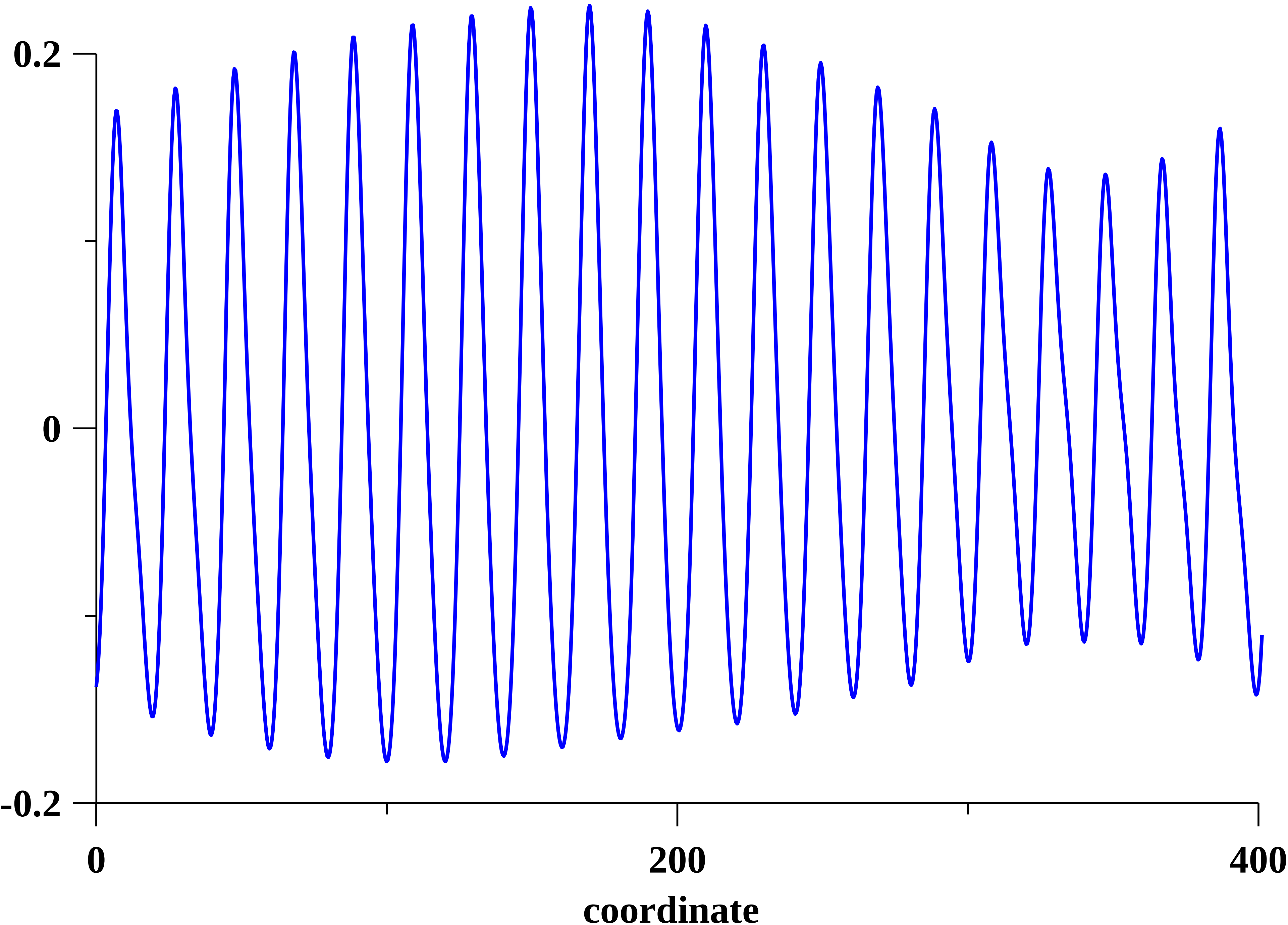
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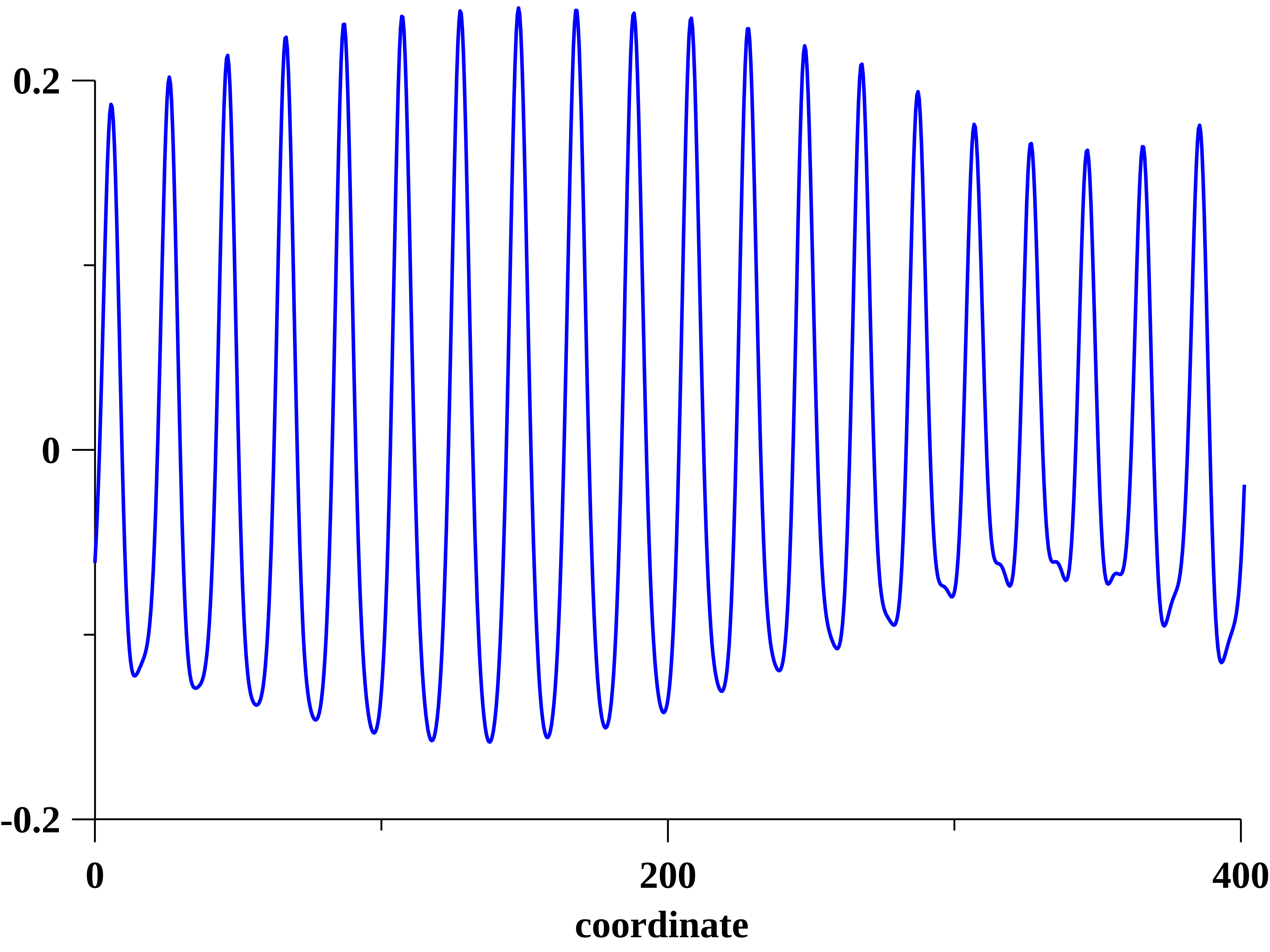
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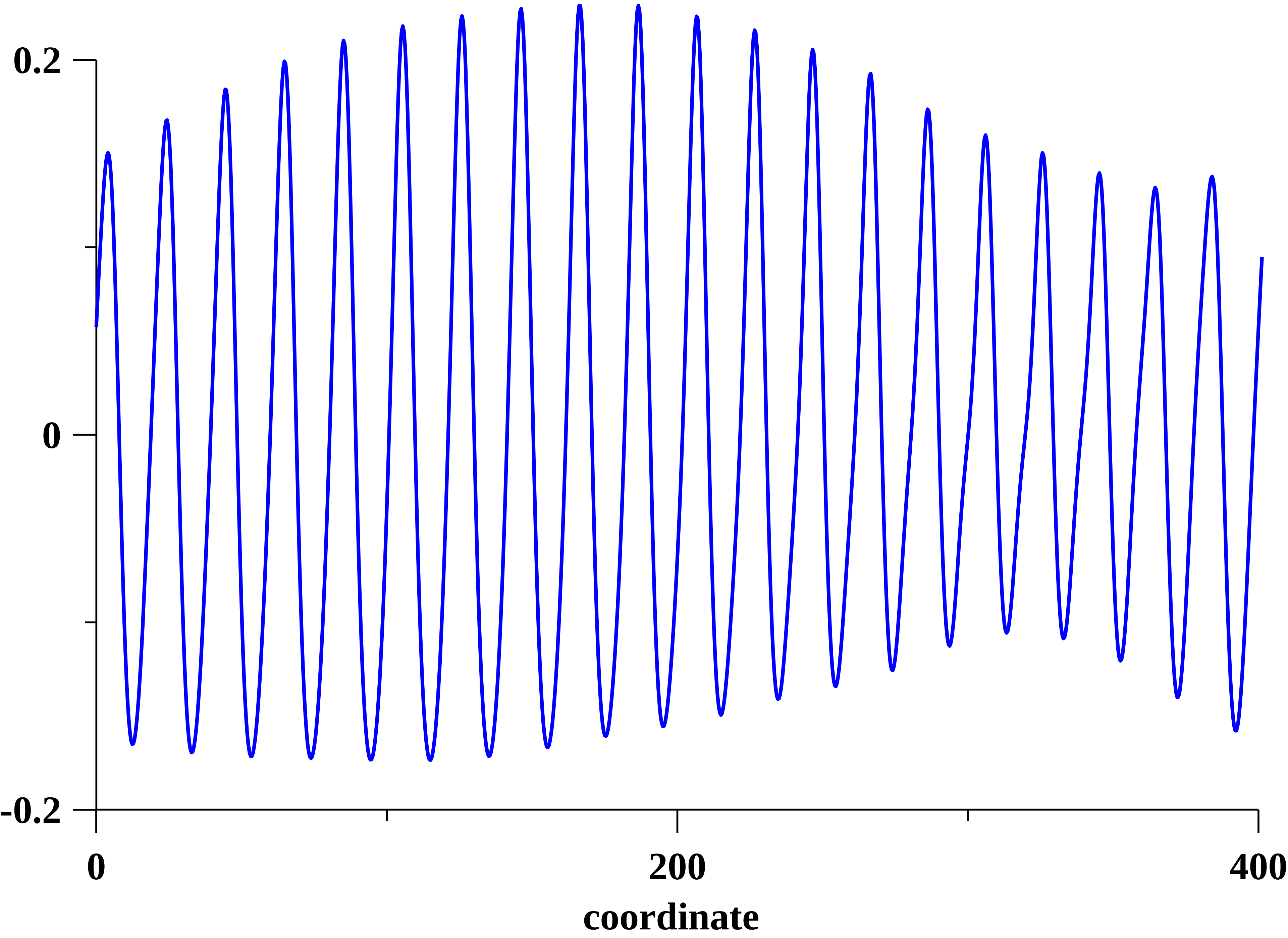

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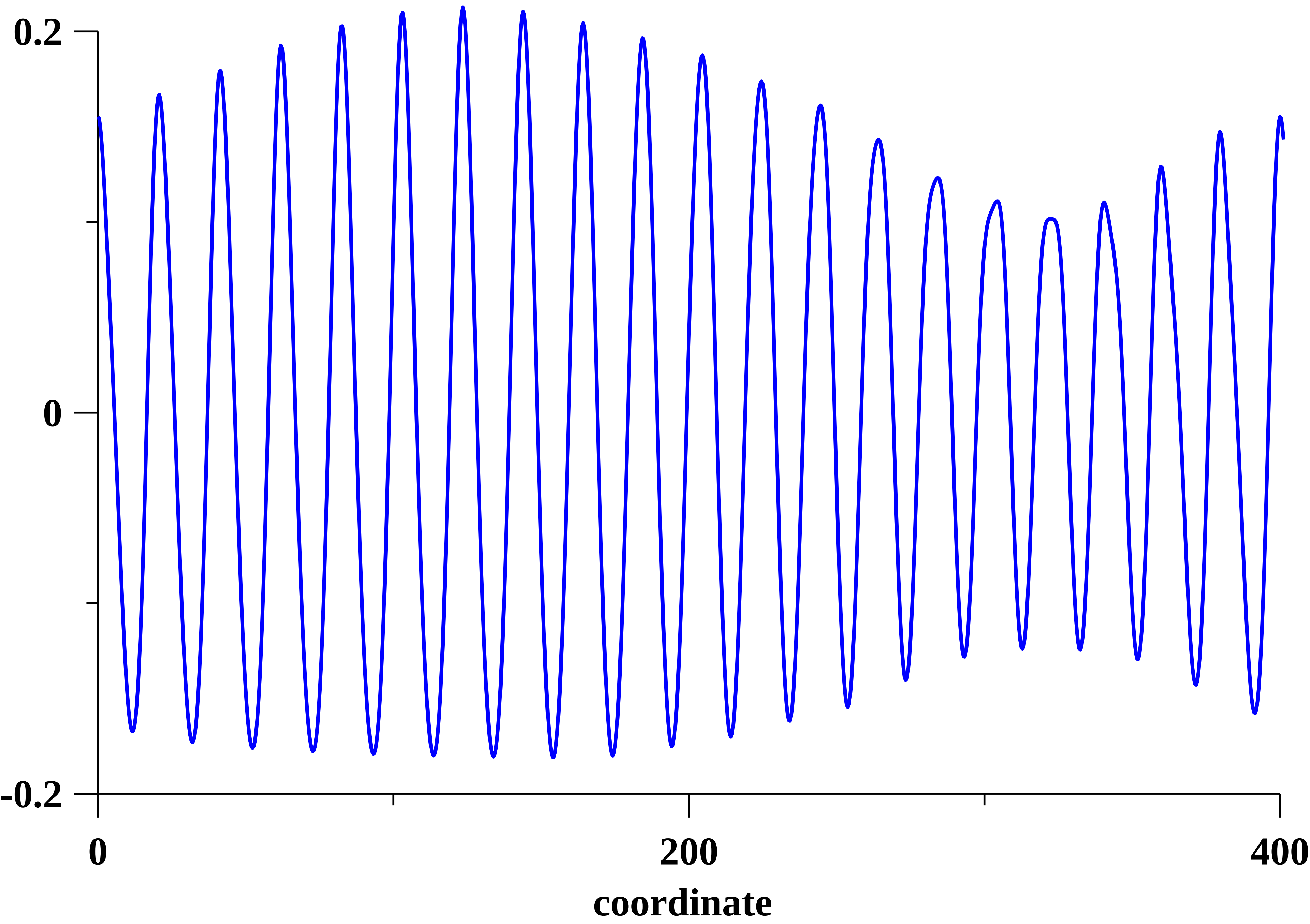
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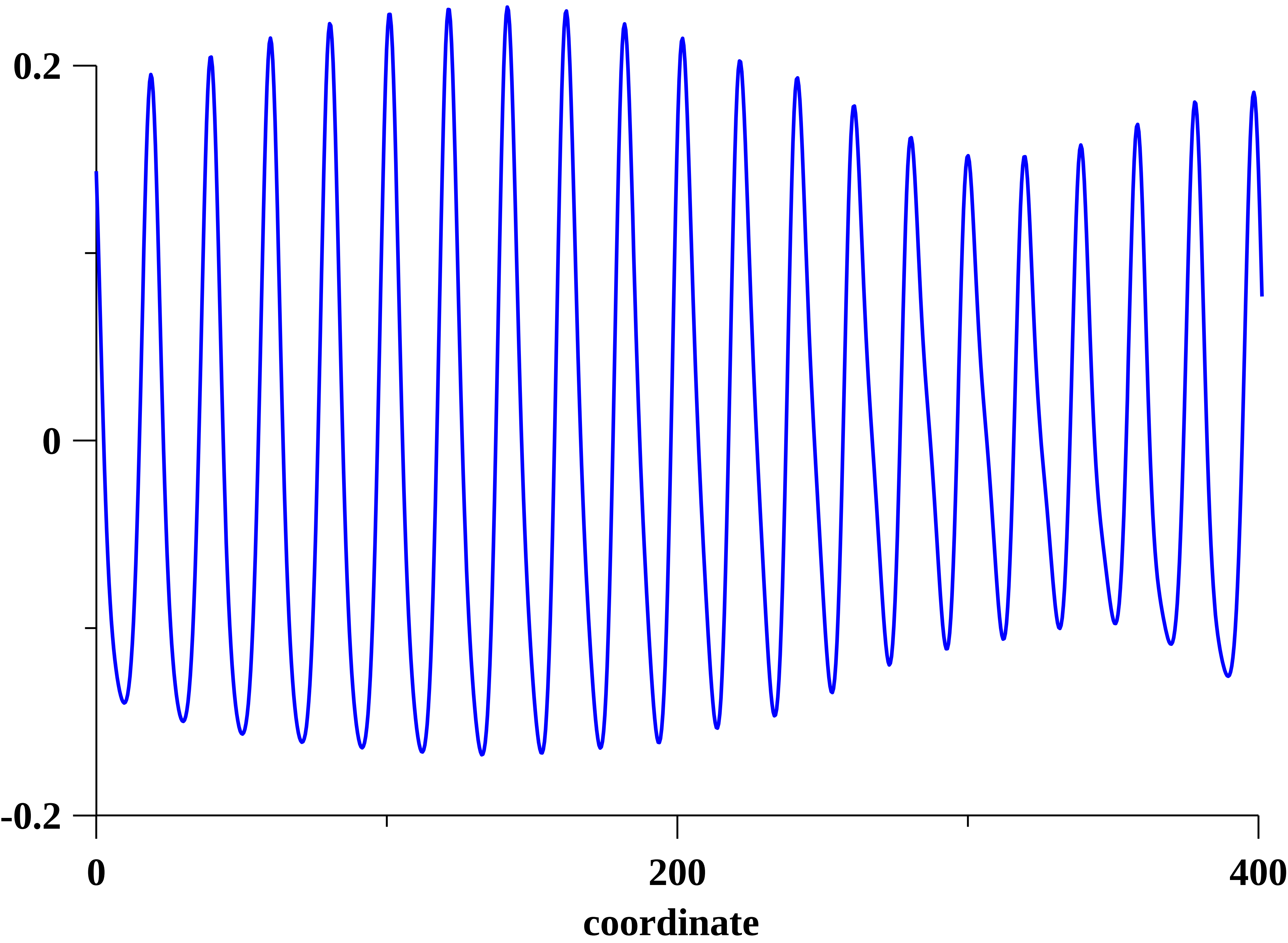
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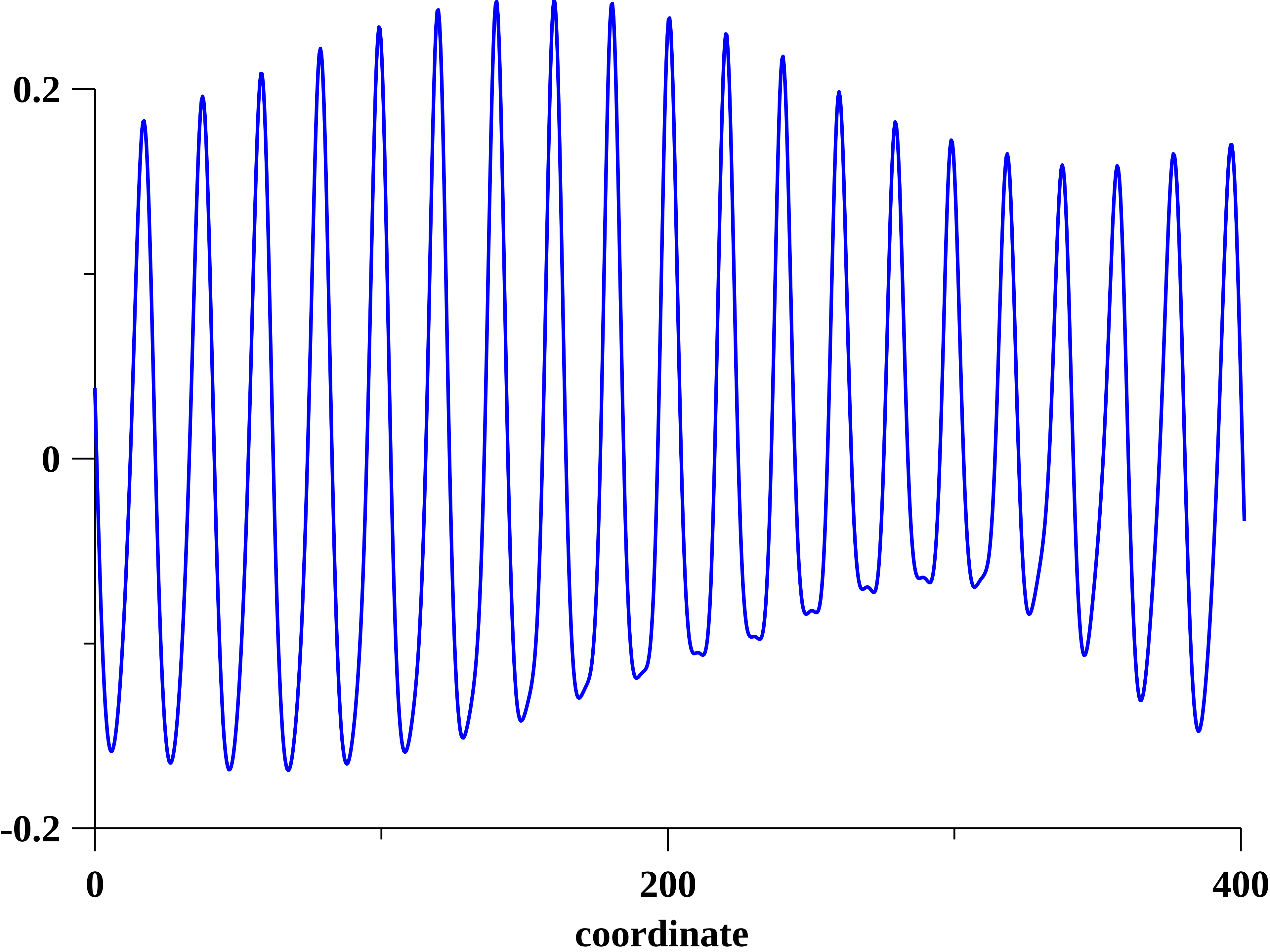
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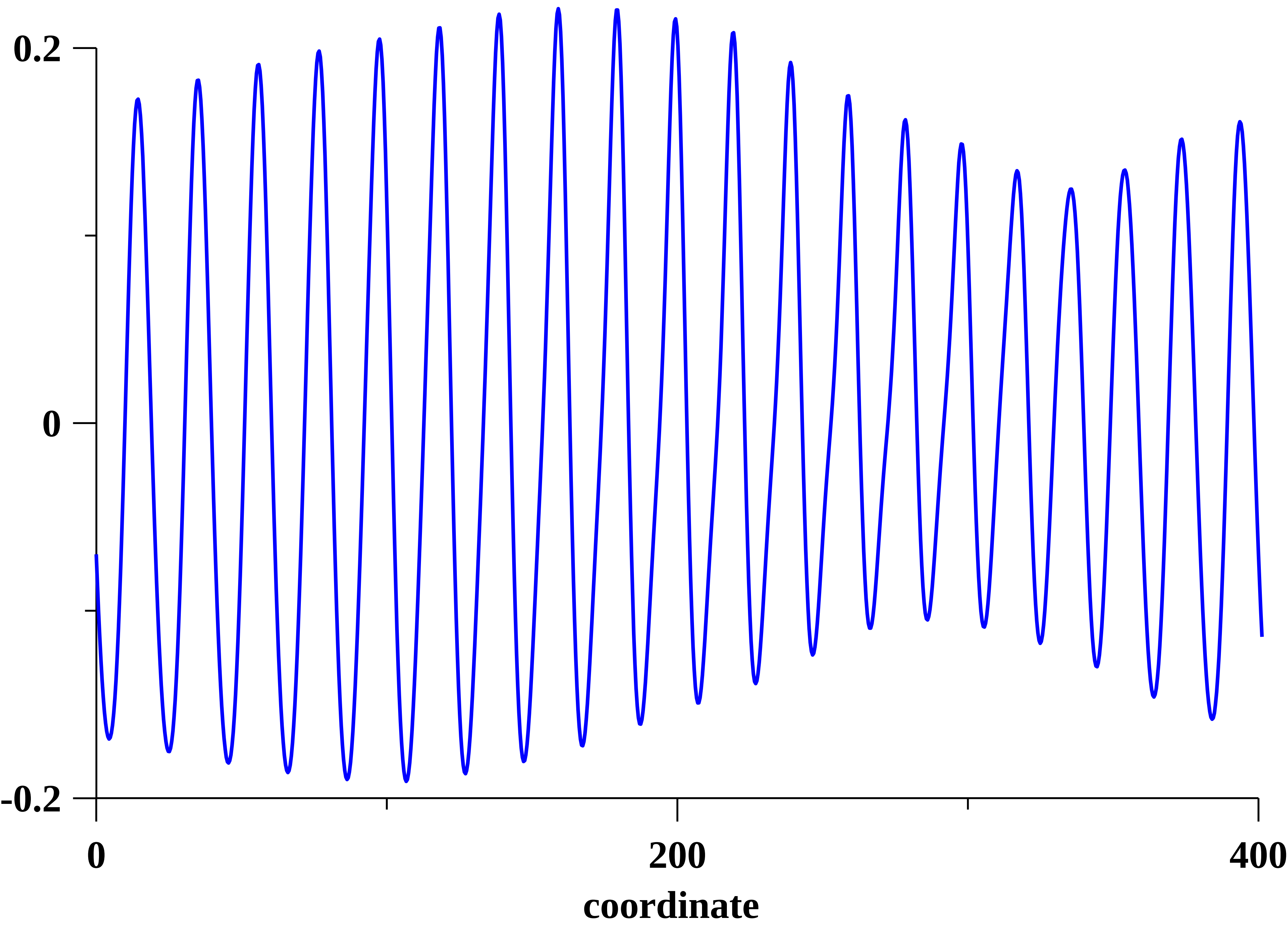
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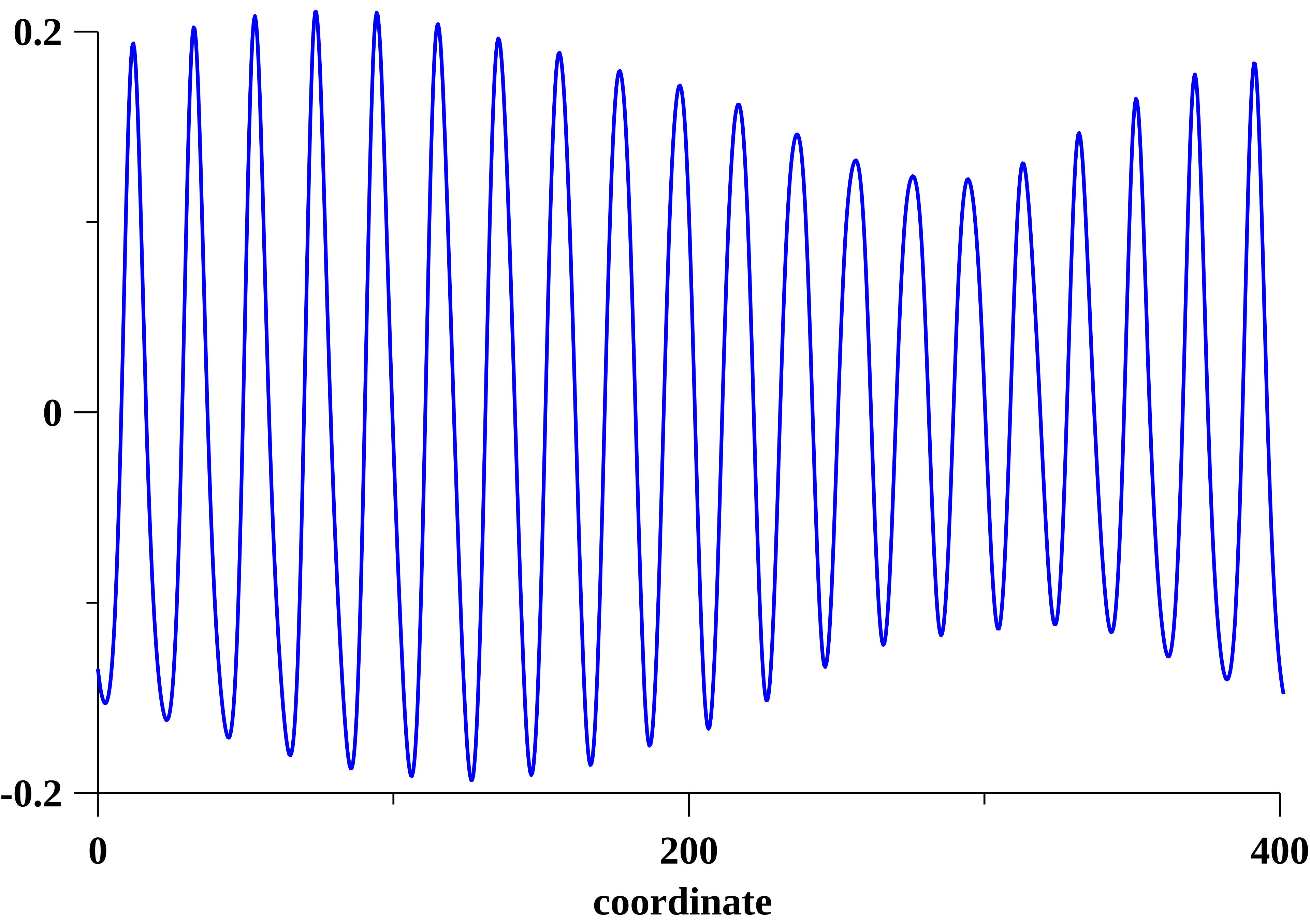
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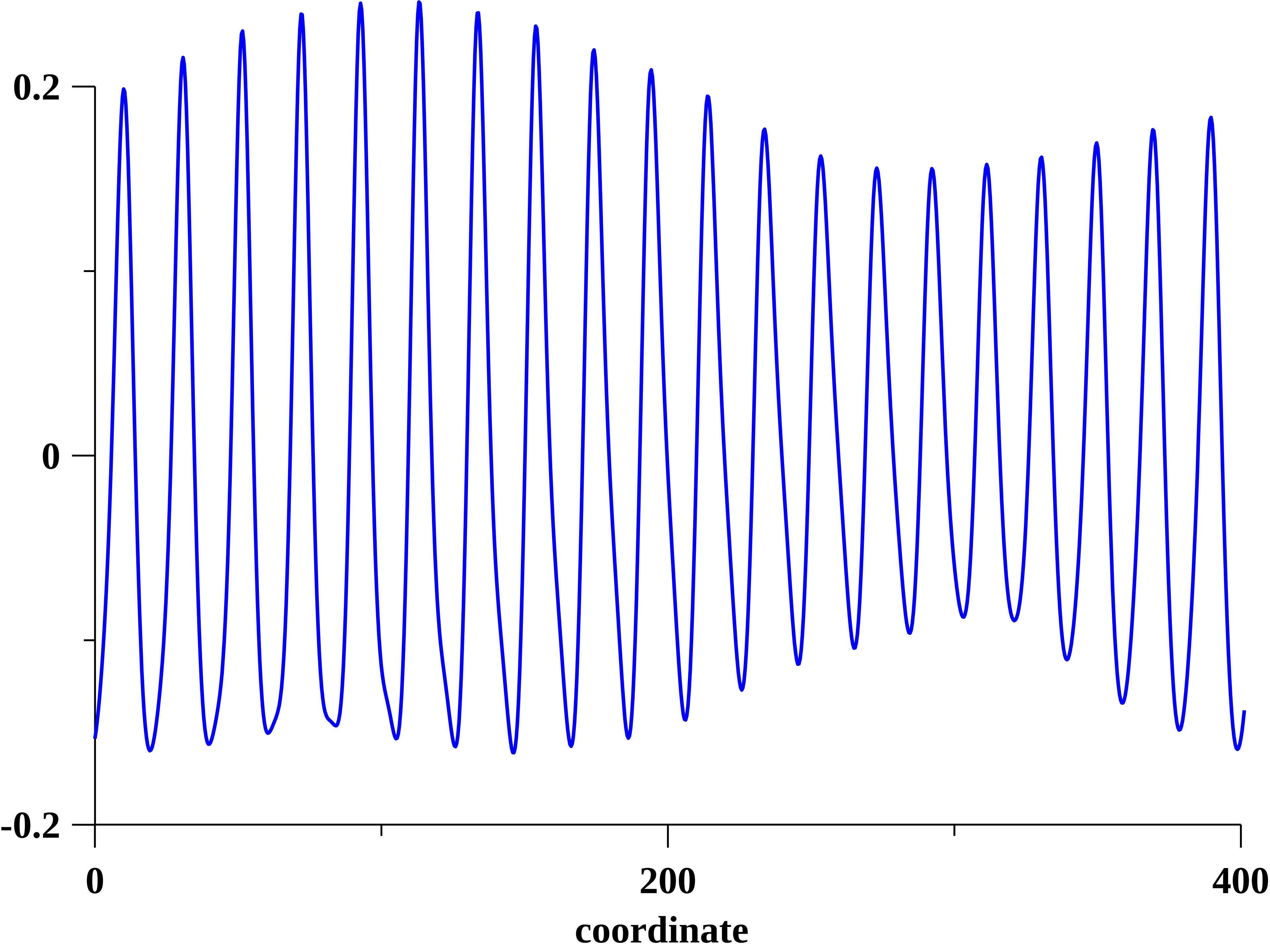

0.2
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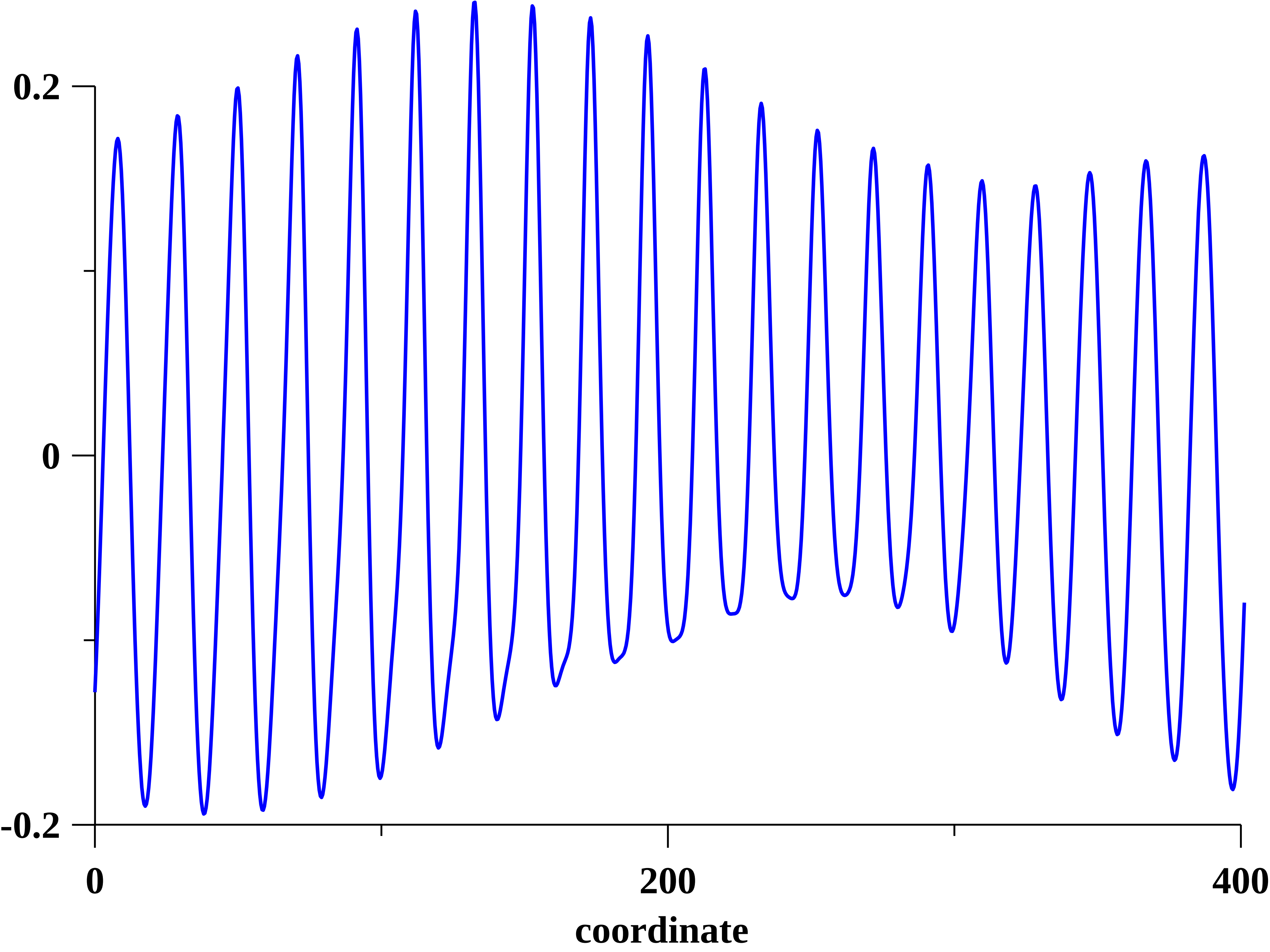
200
400
coordinate



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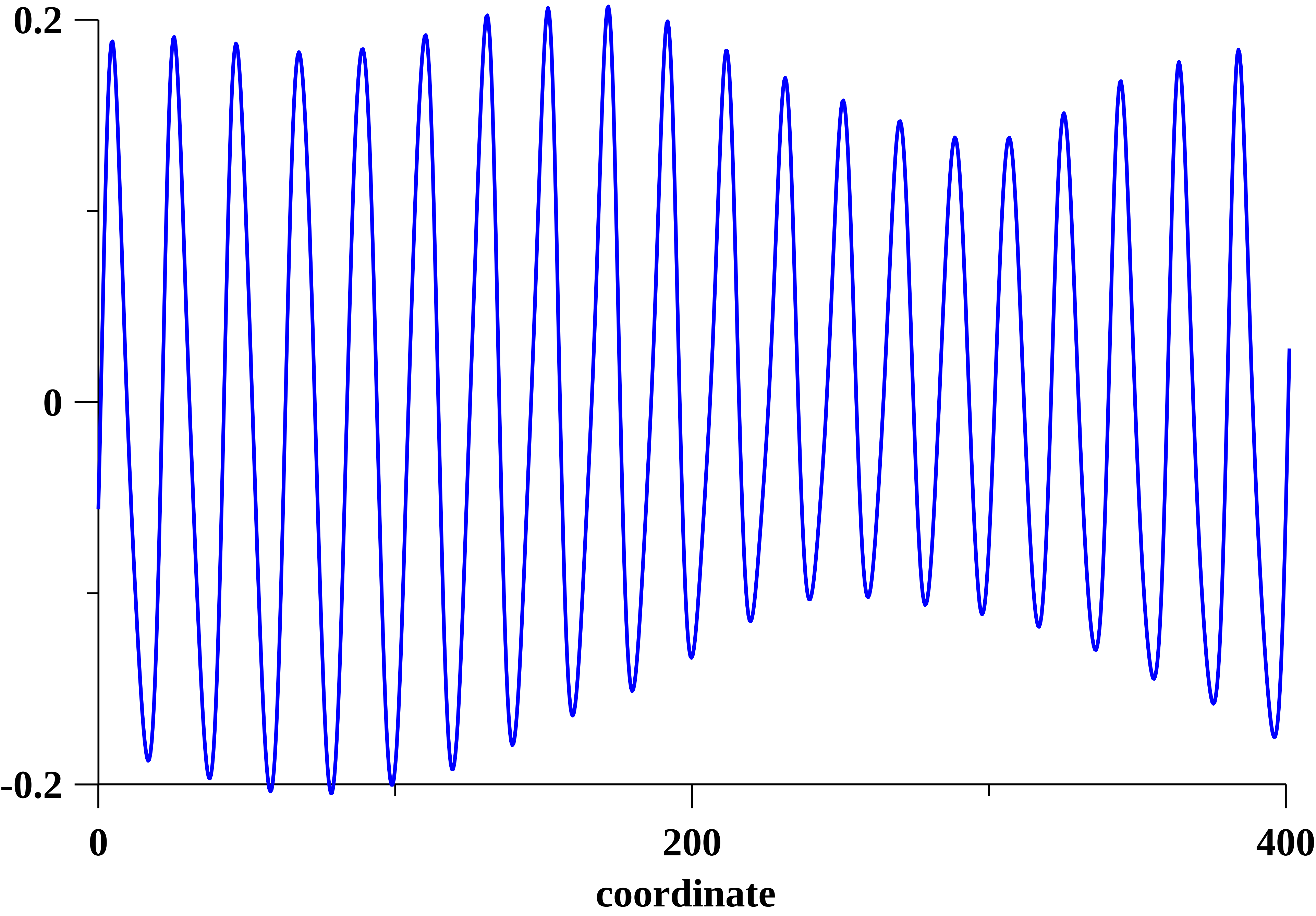
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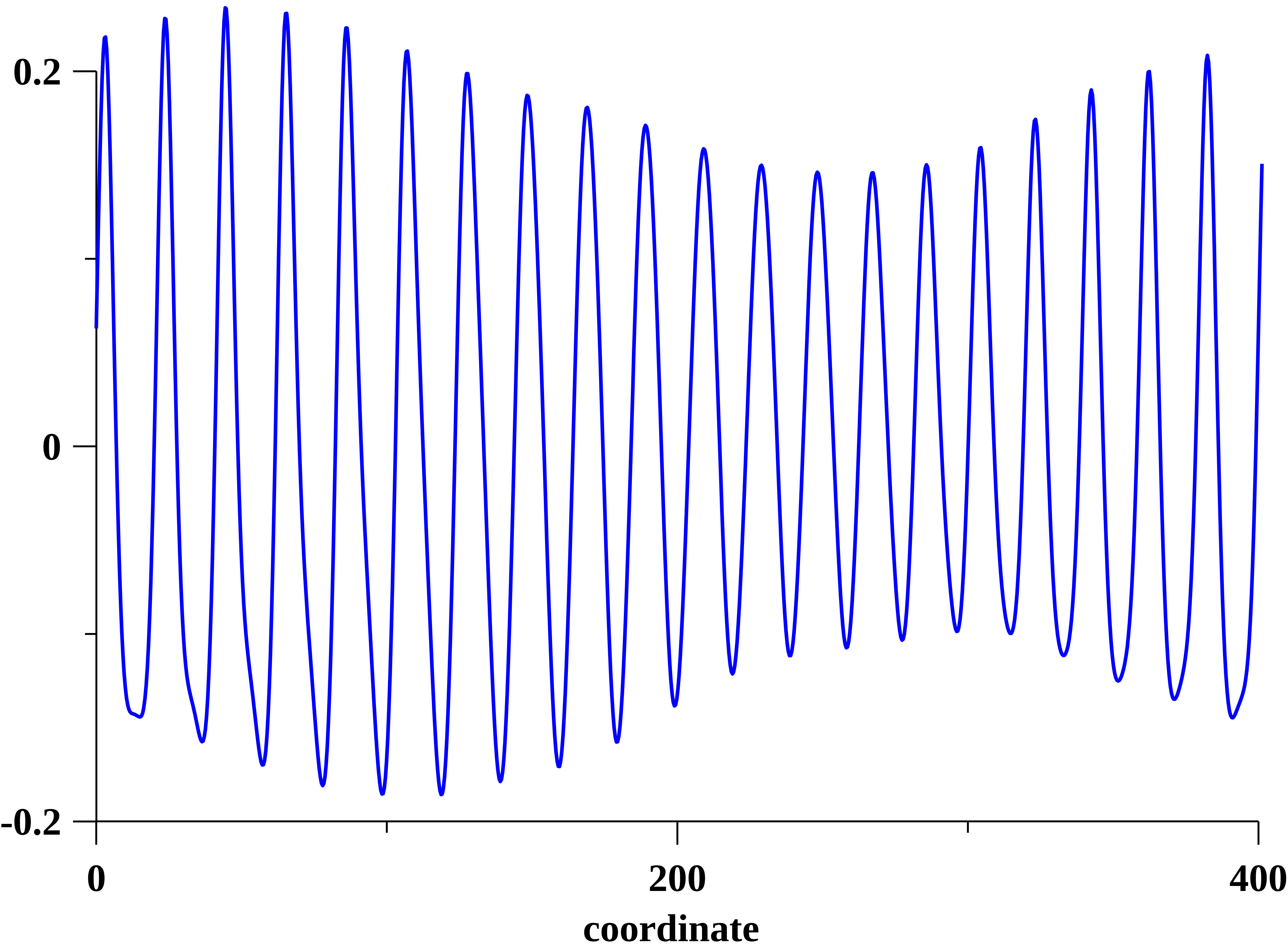

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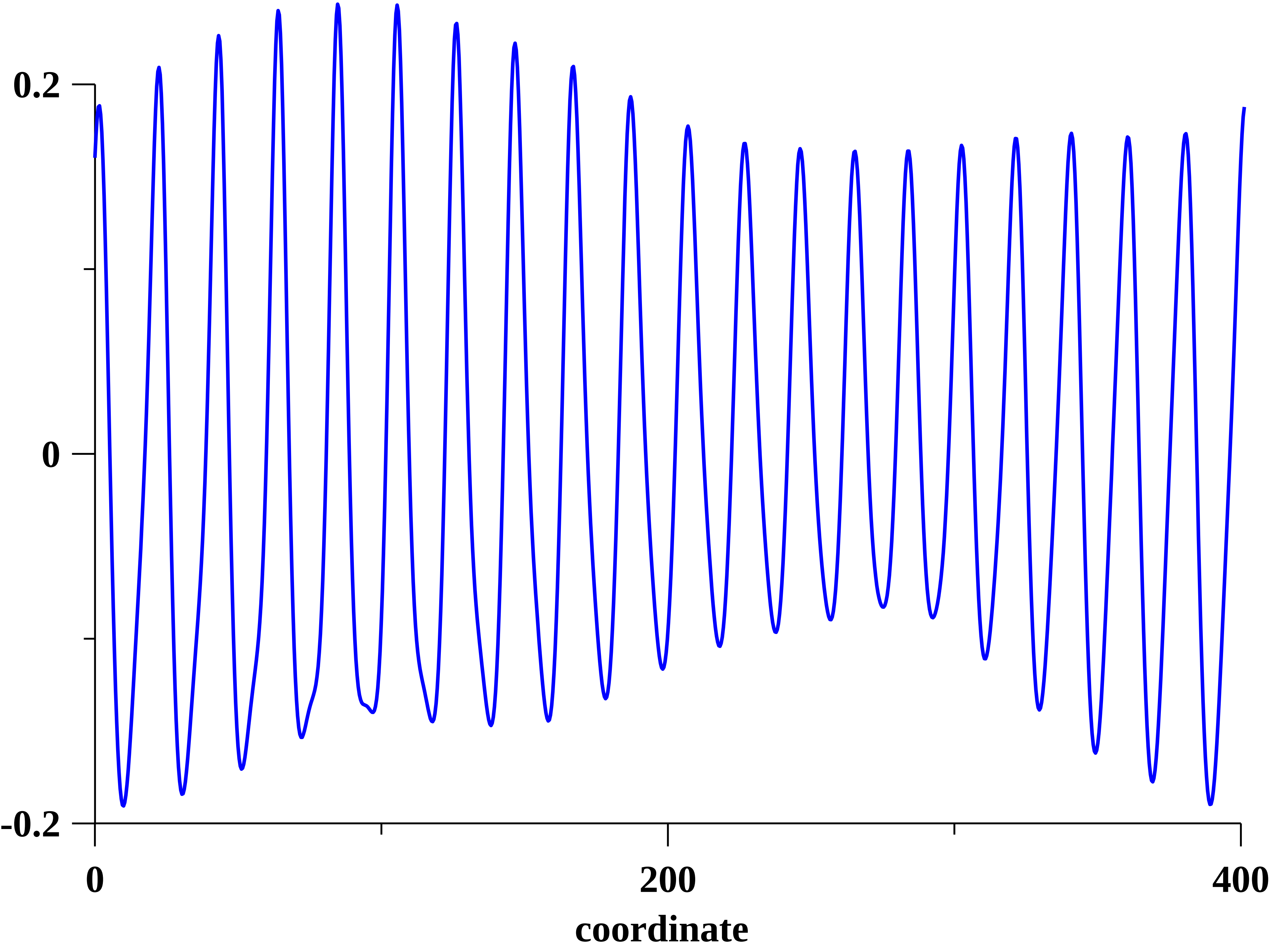

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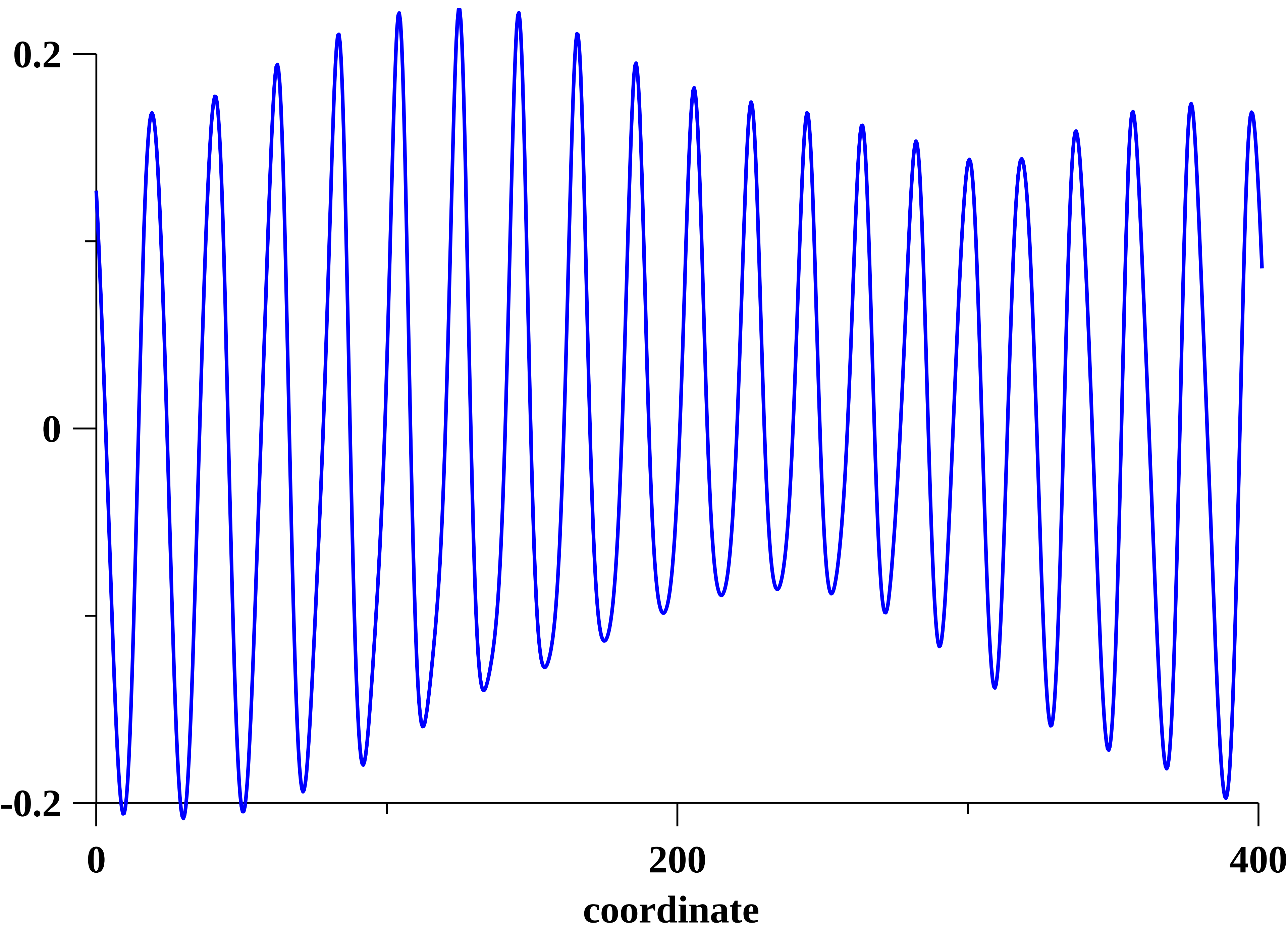

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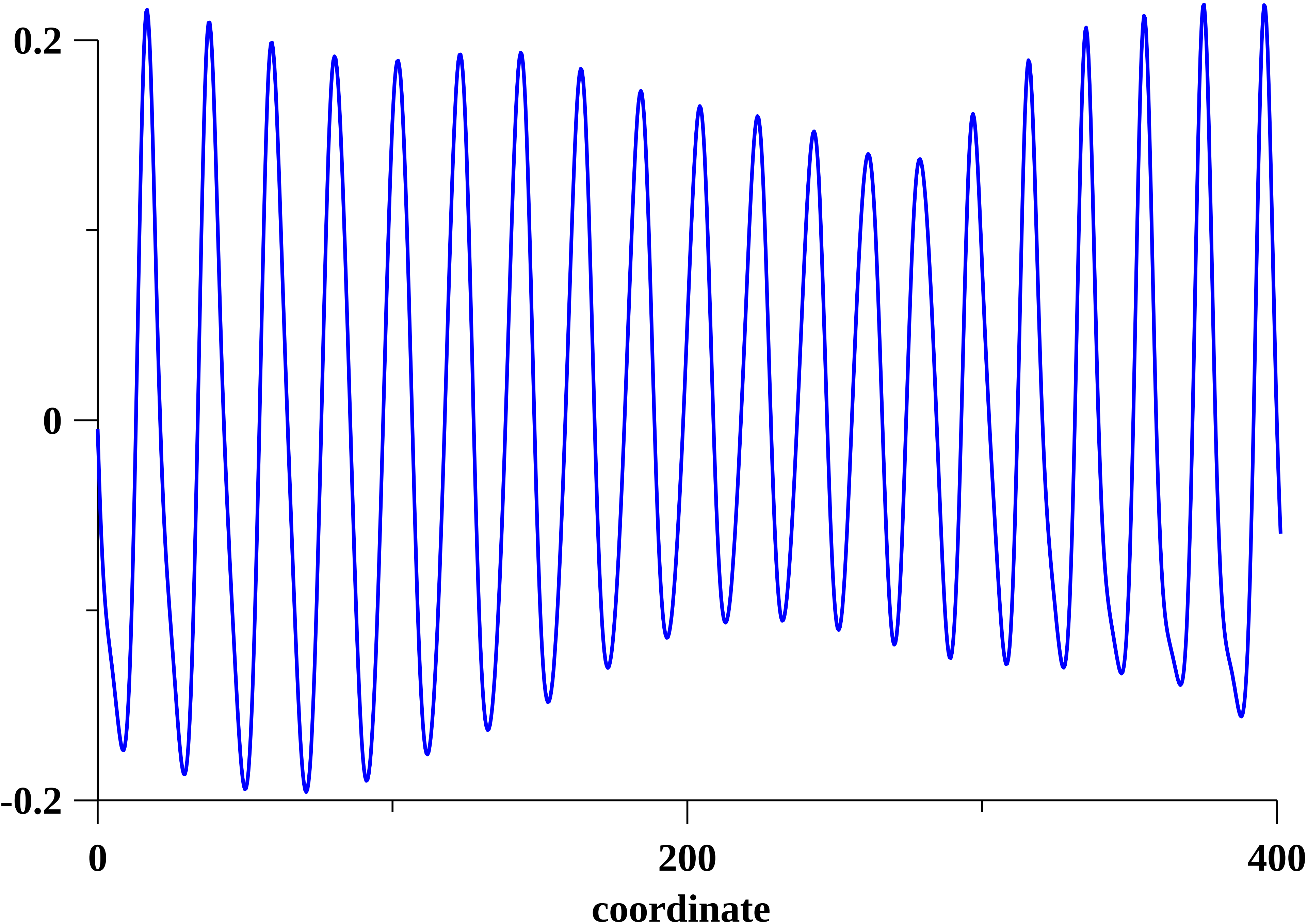
coordinate



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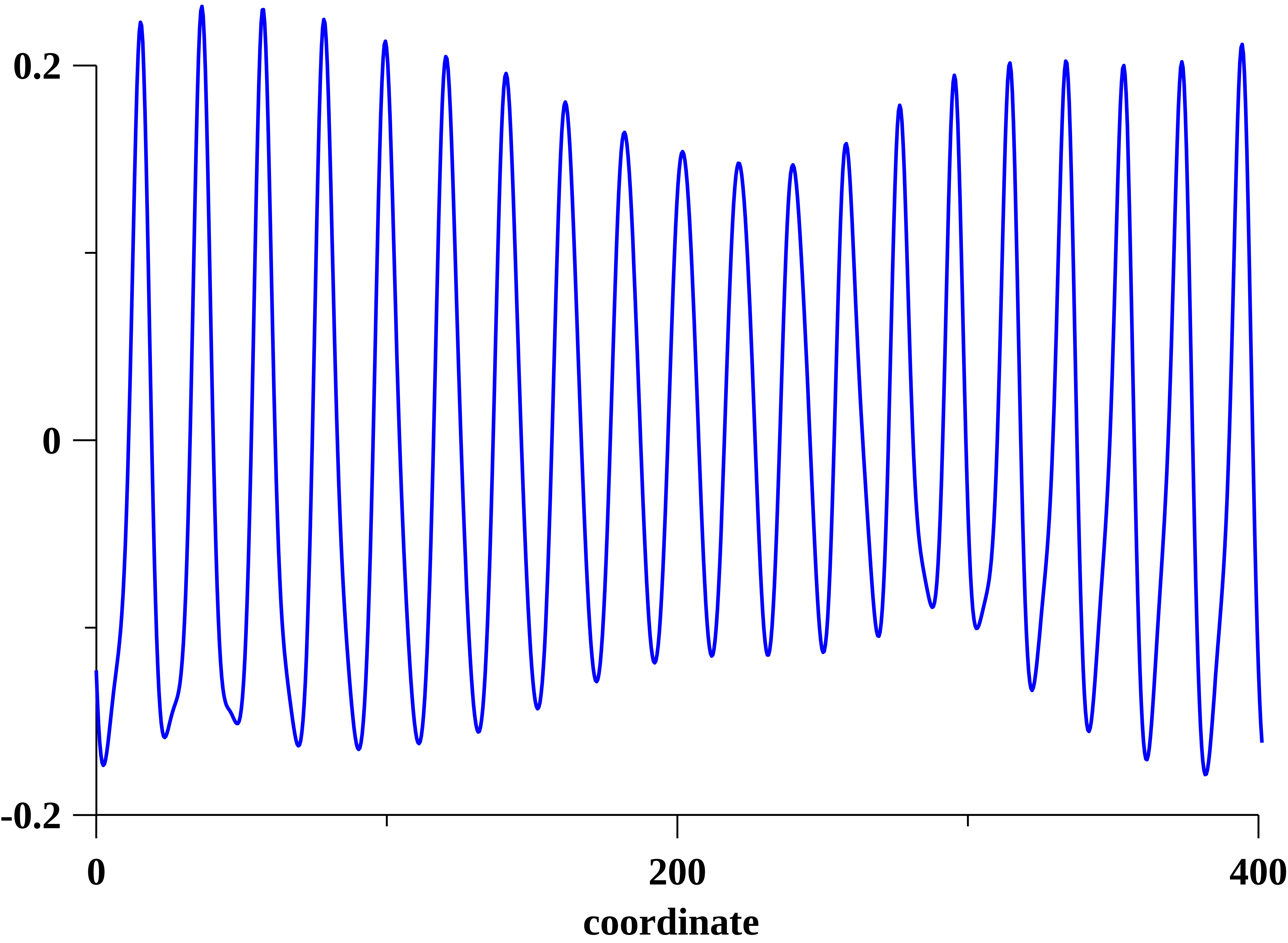
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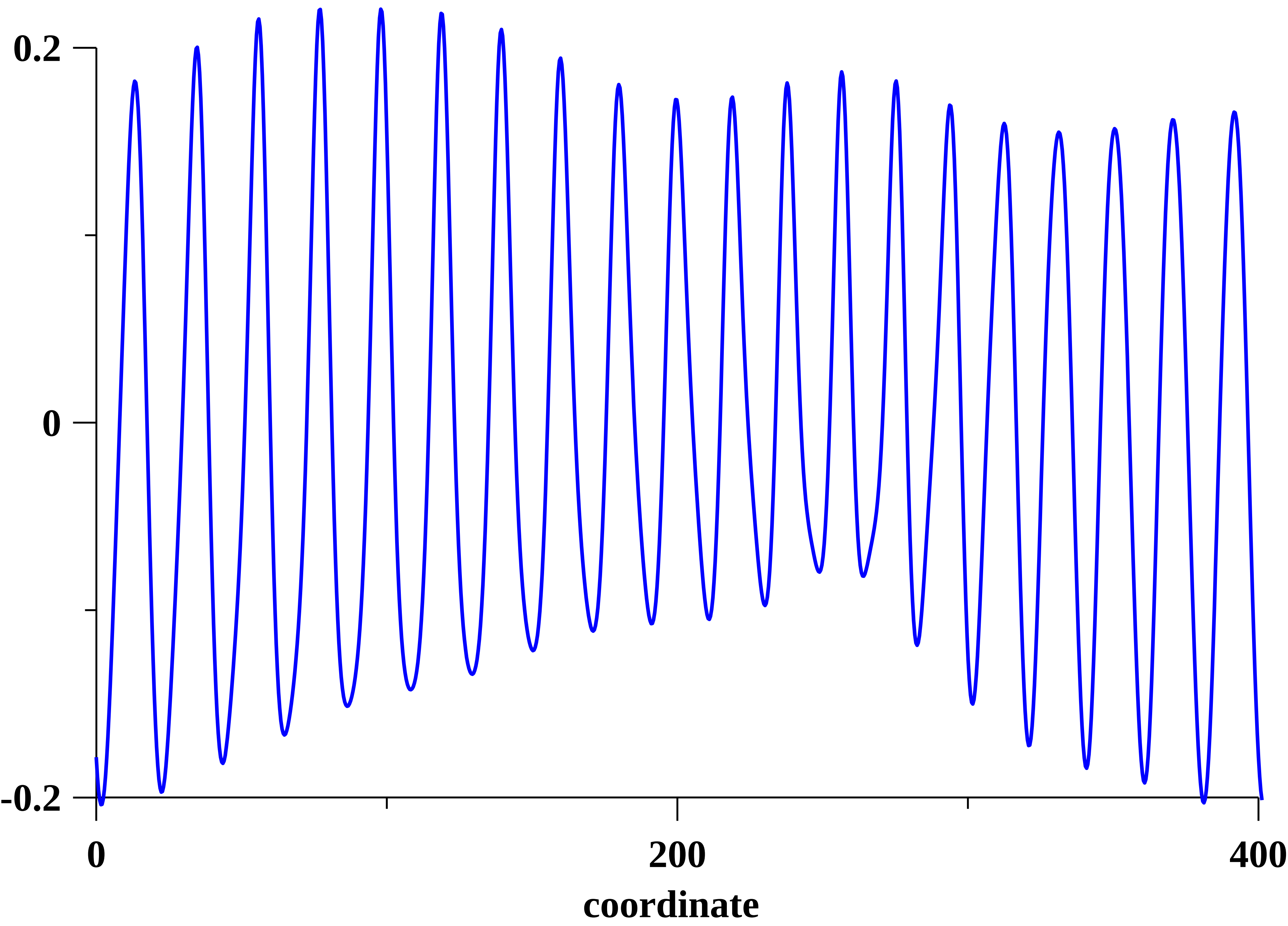

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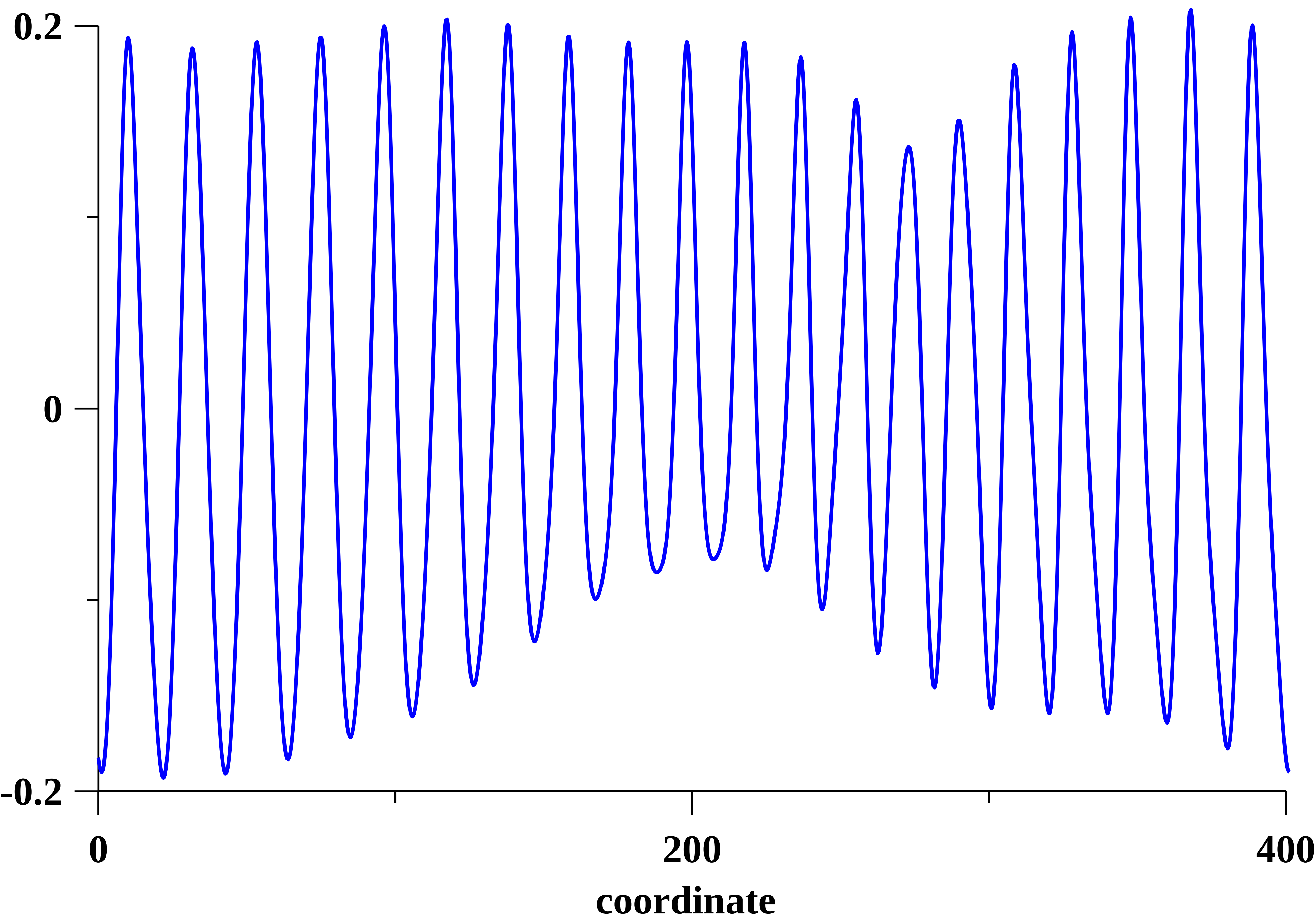

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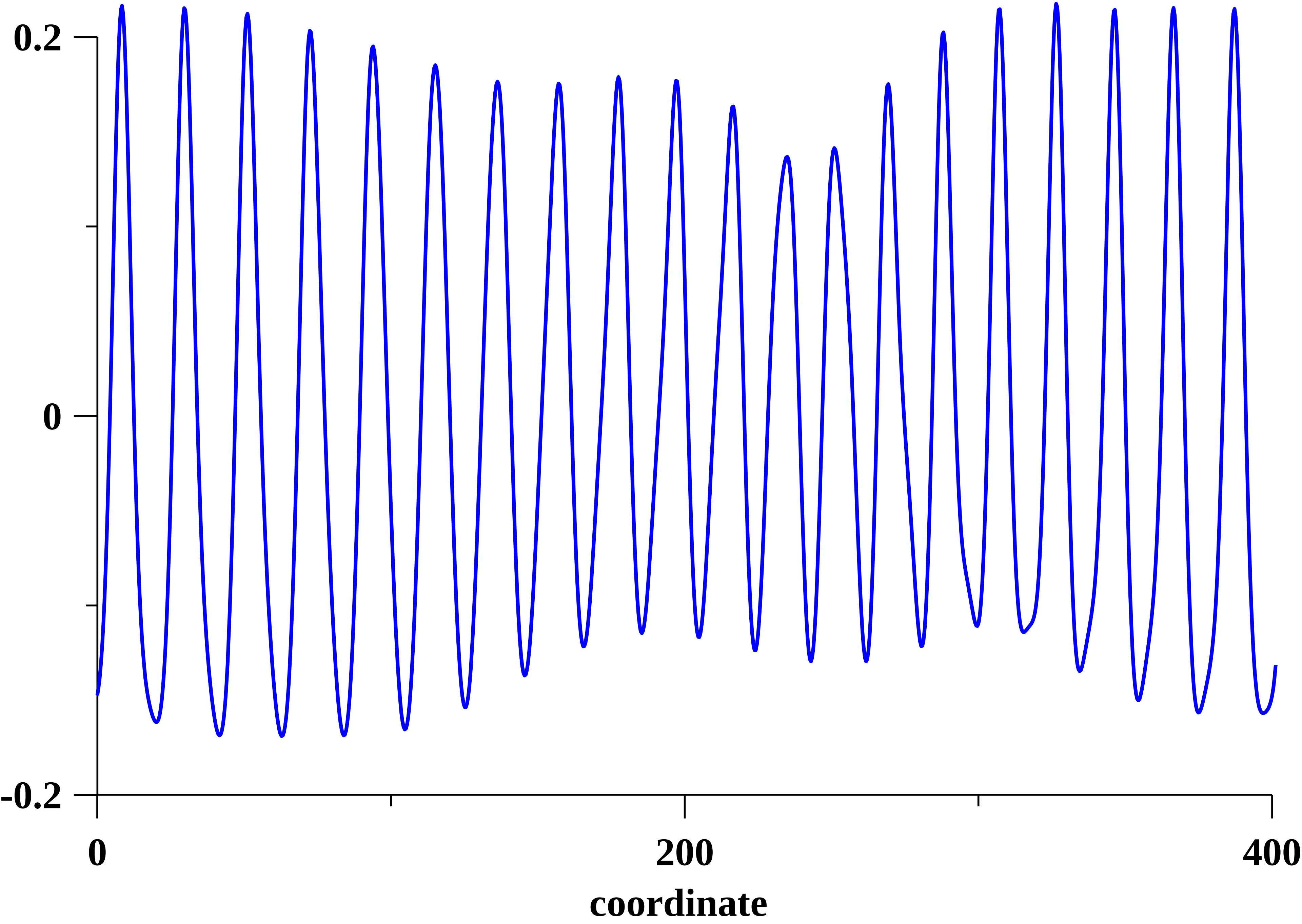

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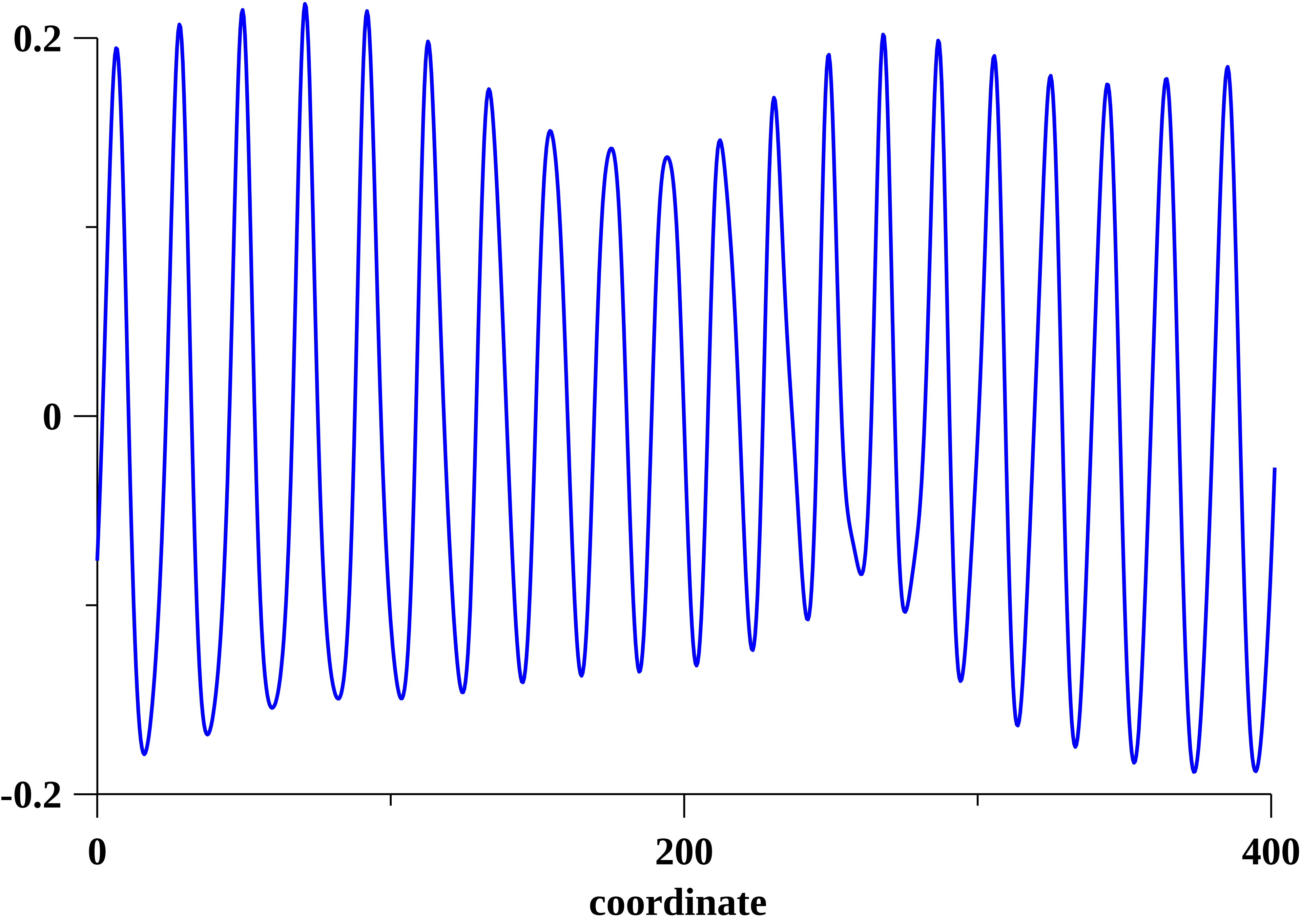

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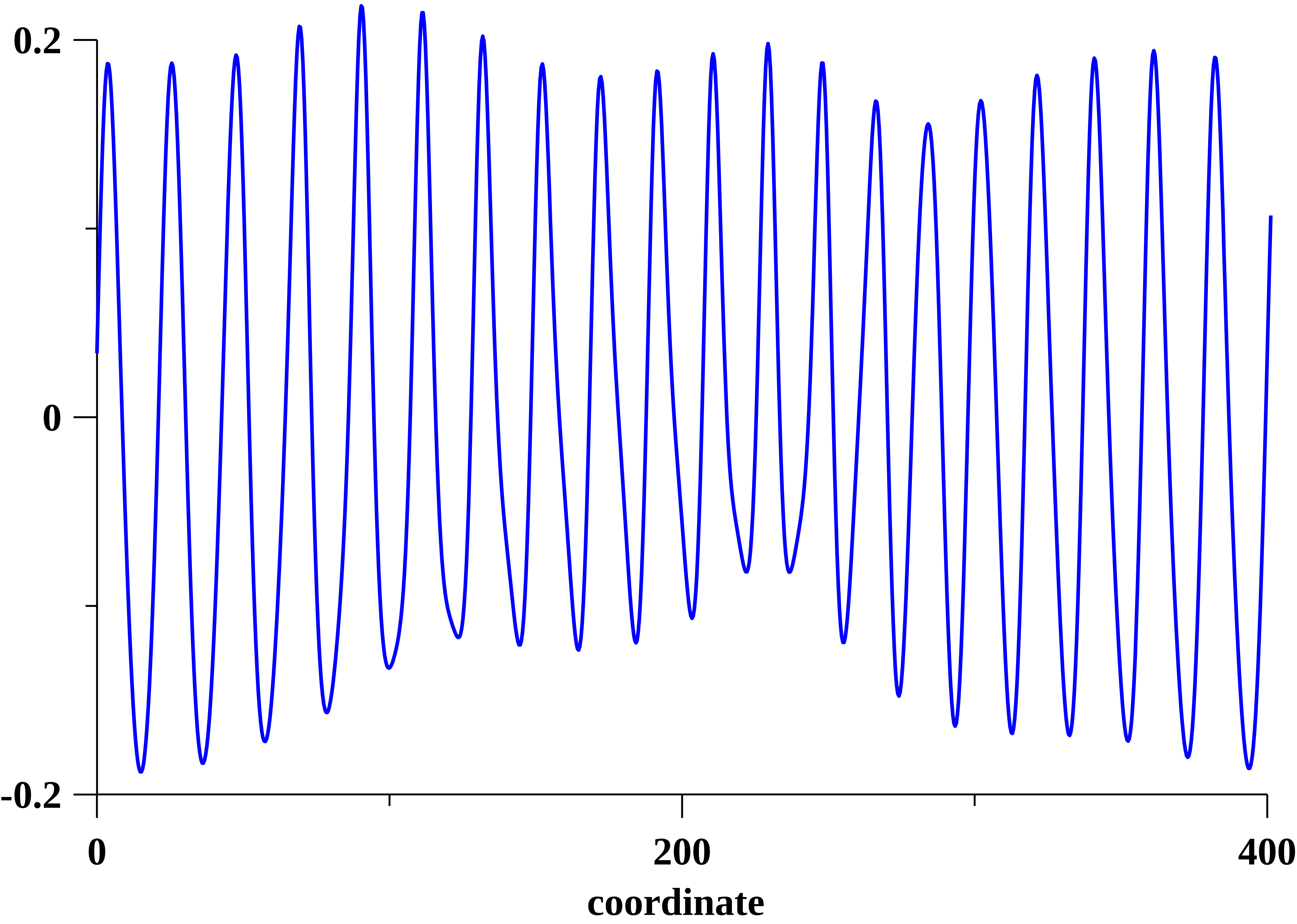
200
400
coordinate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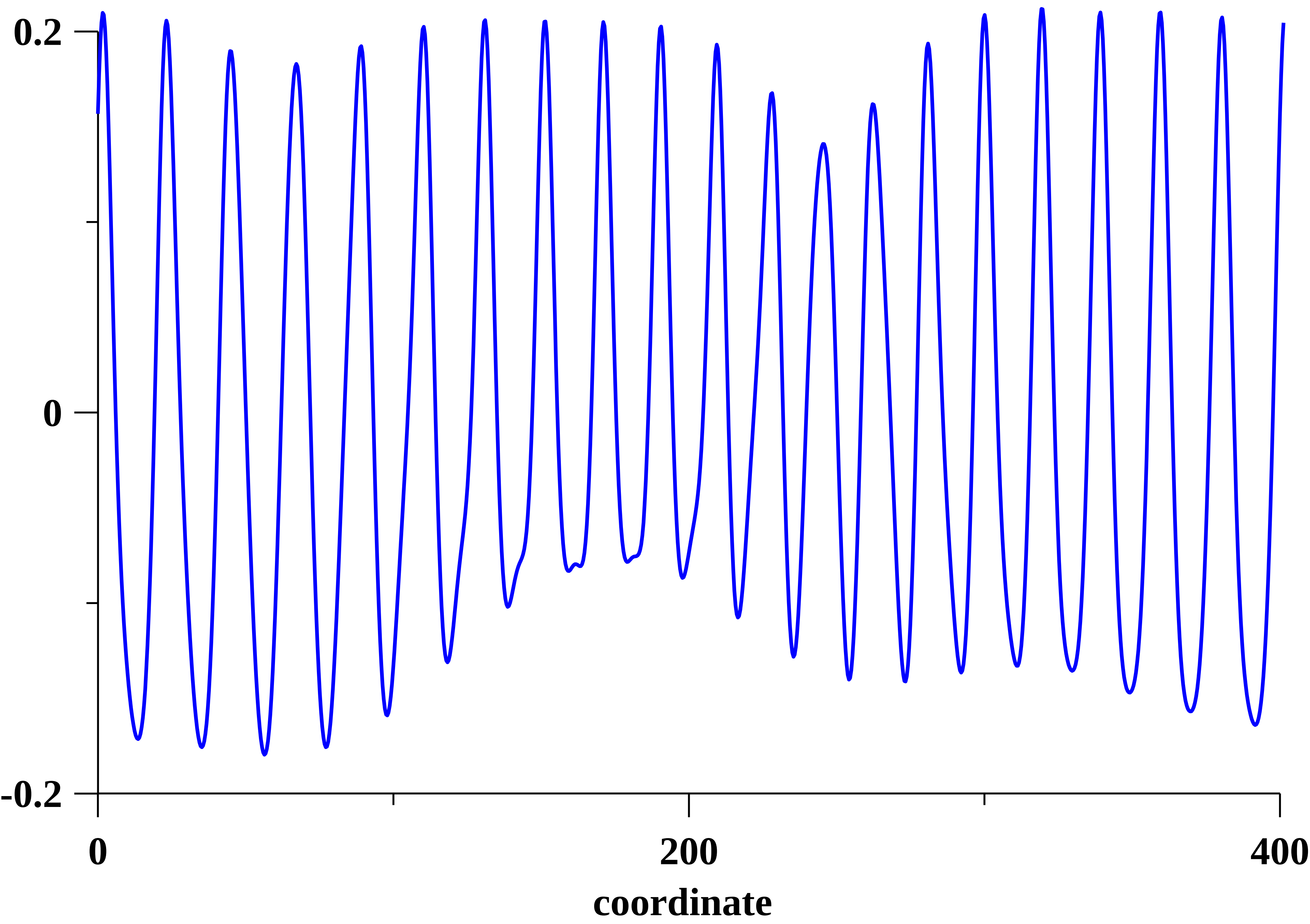
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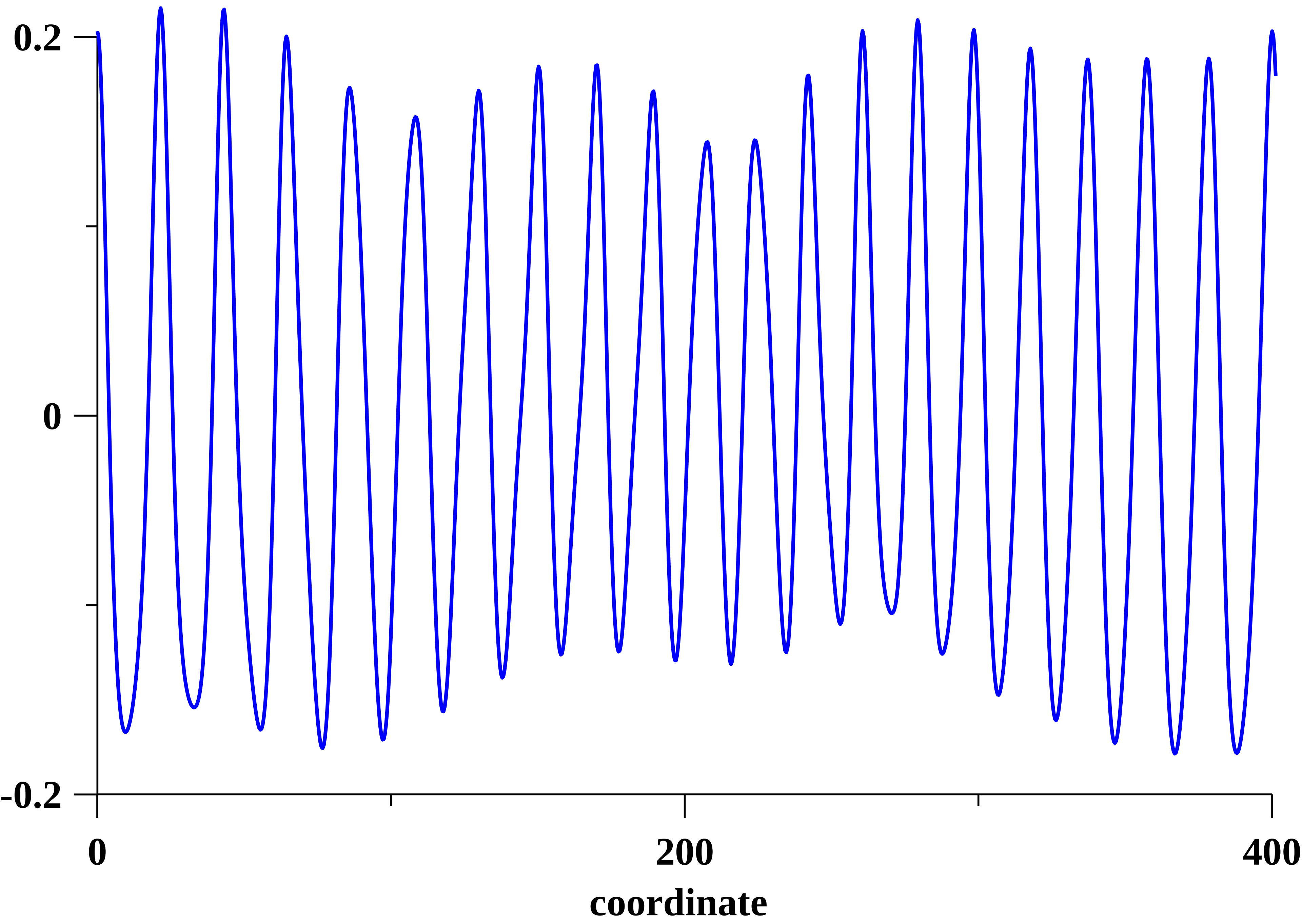
400
coordinate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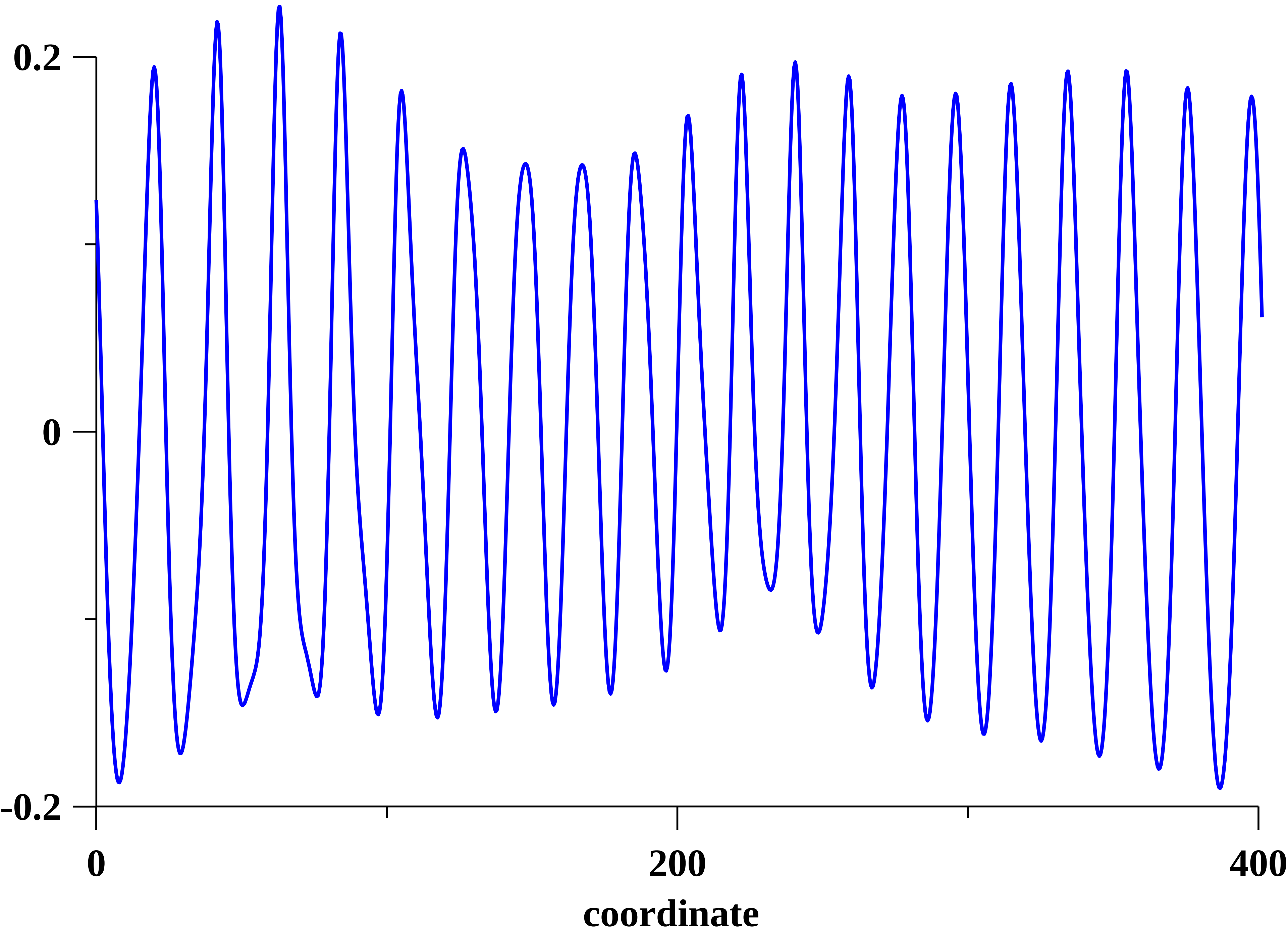

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200
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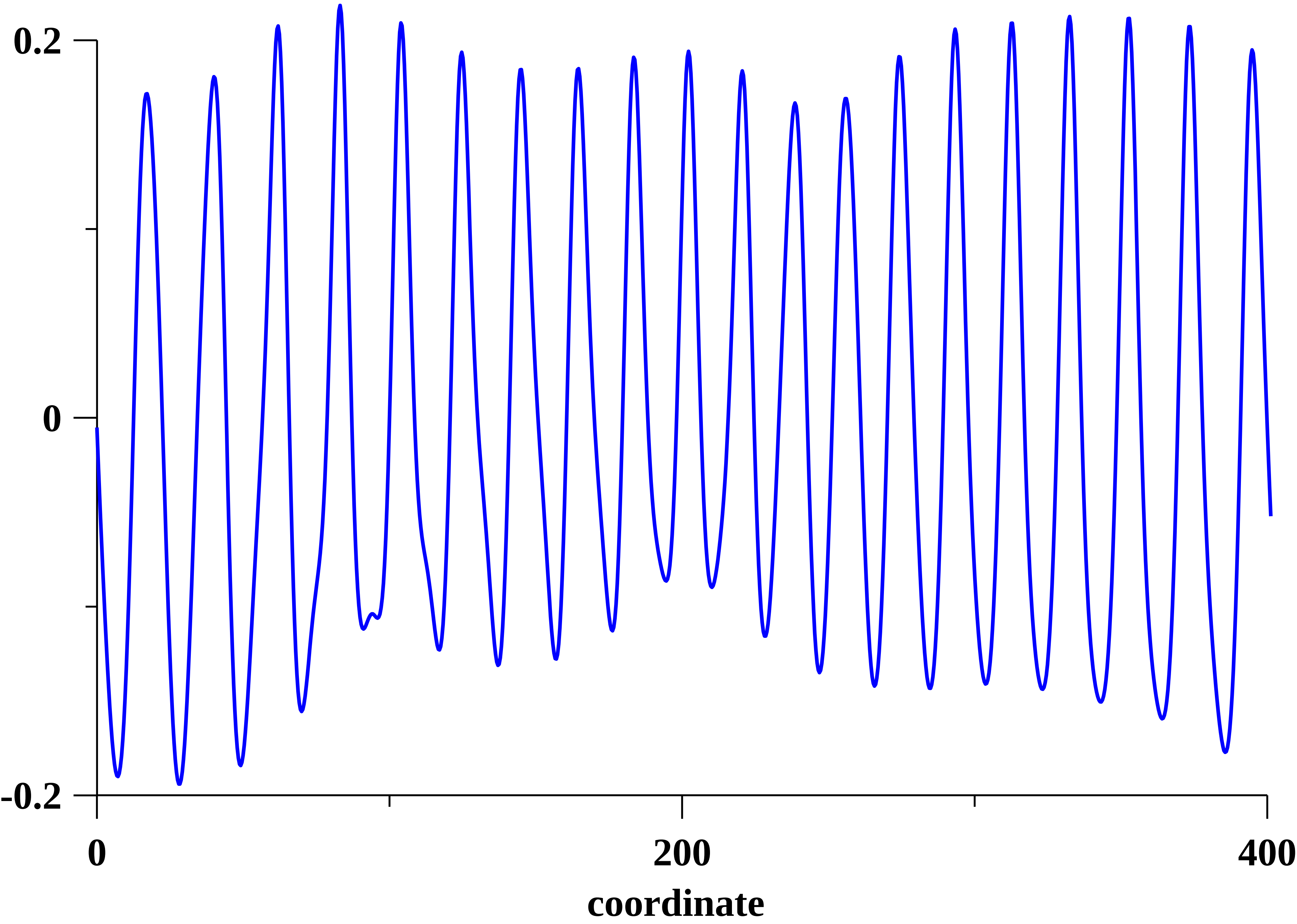
coordinate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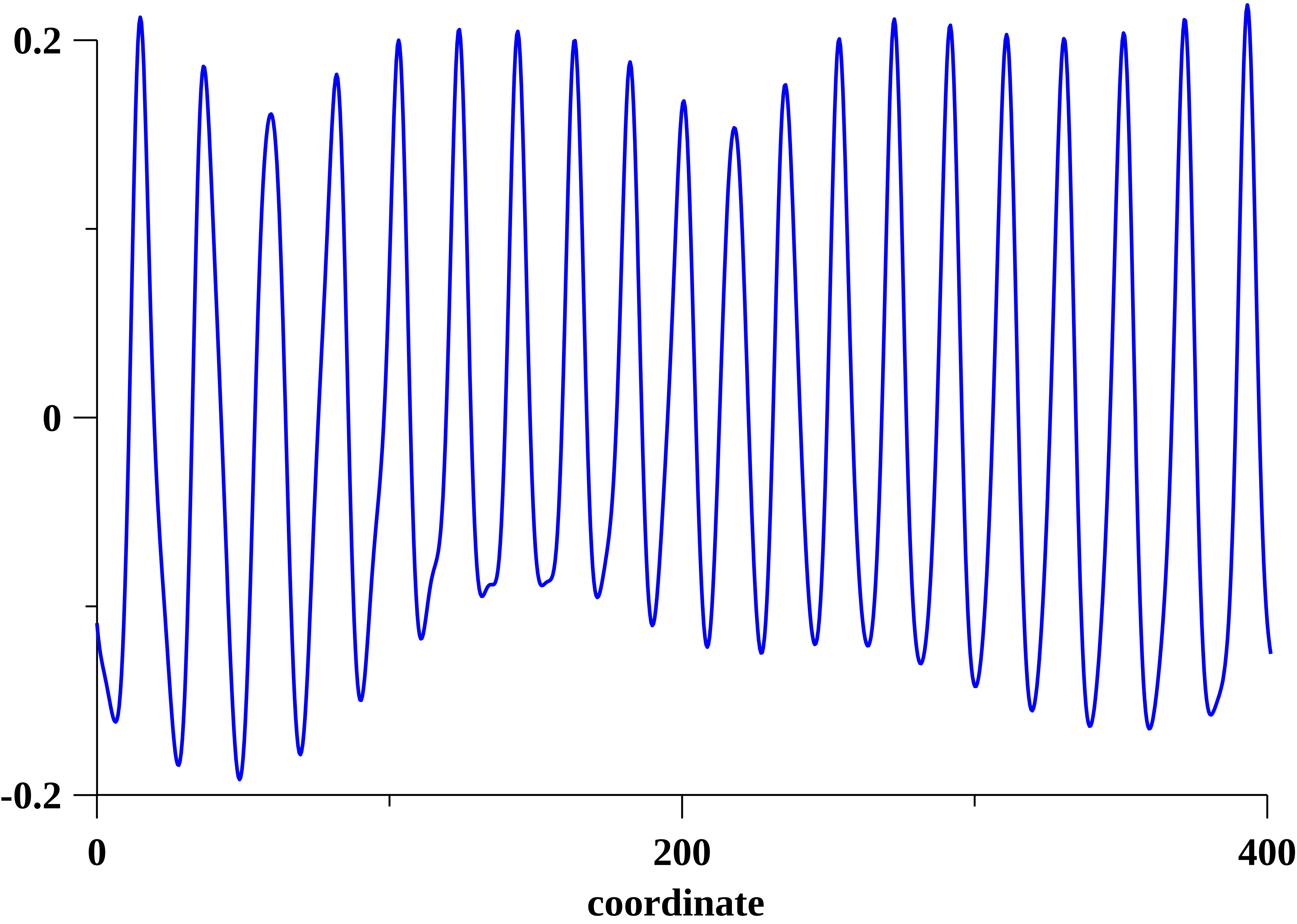
coordinate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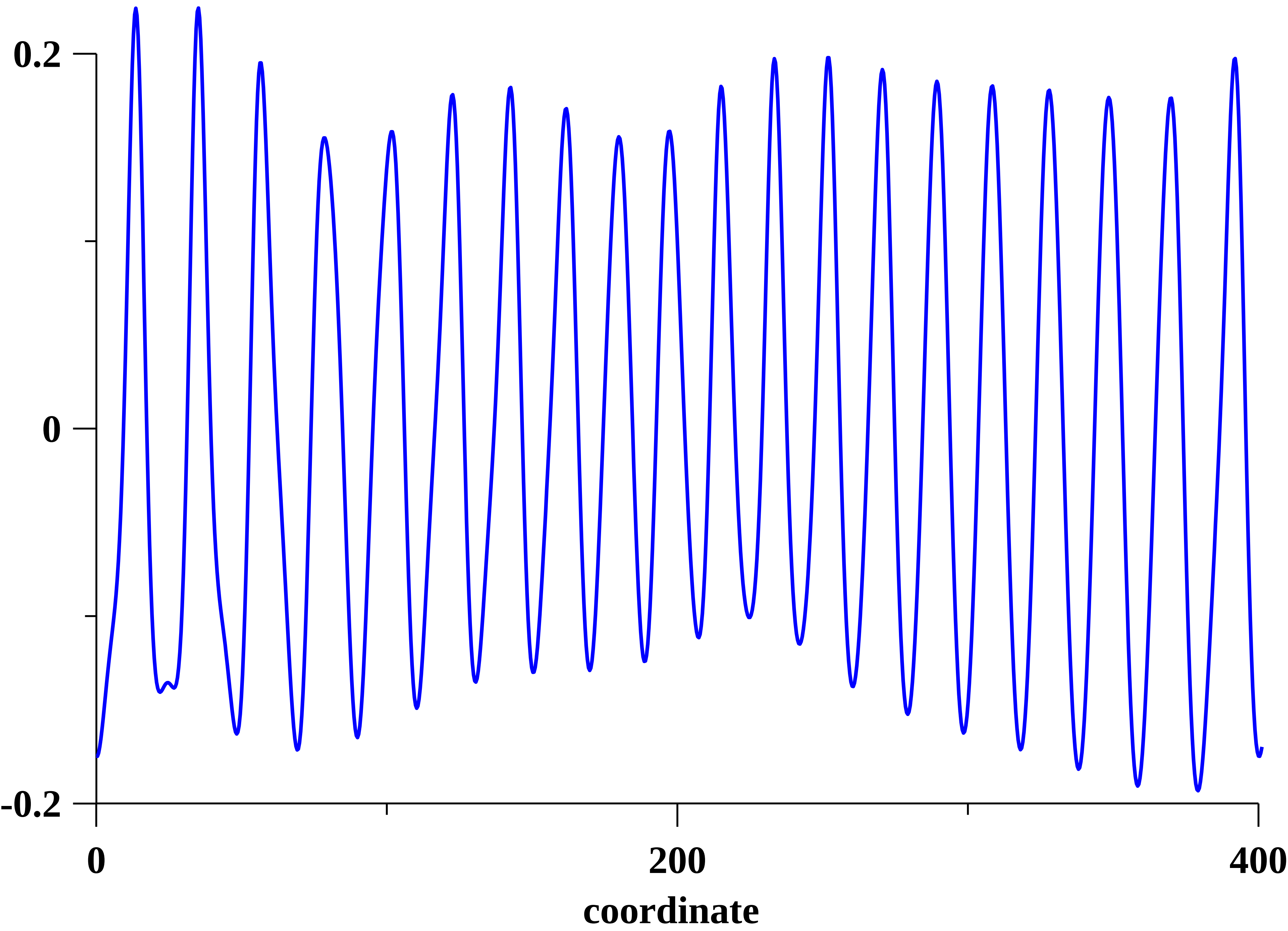
200
400
coordinate



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0
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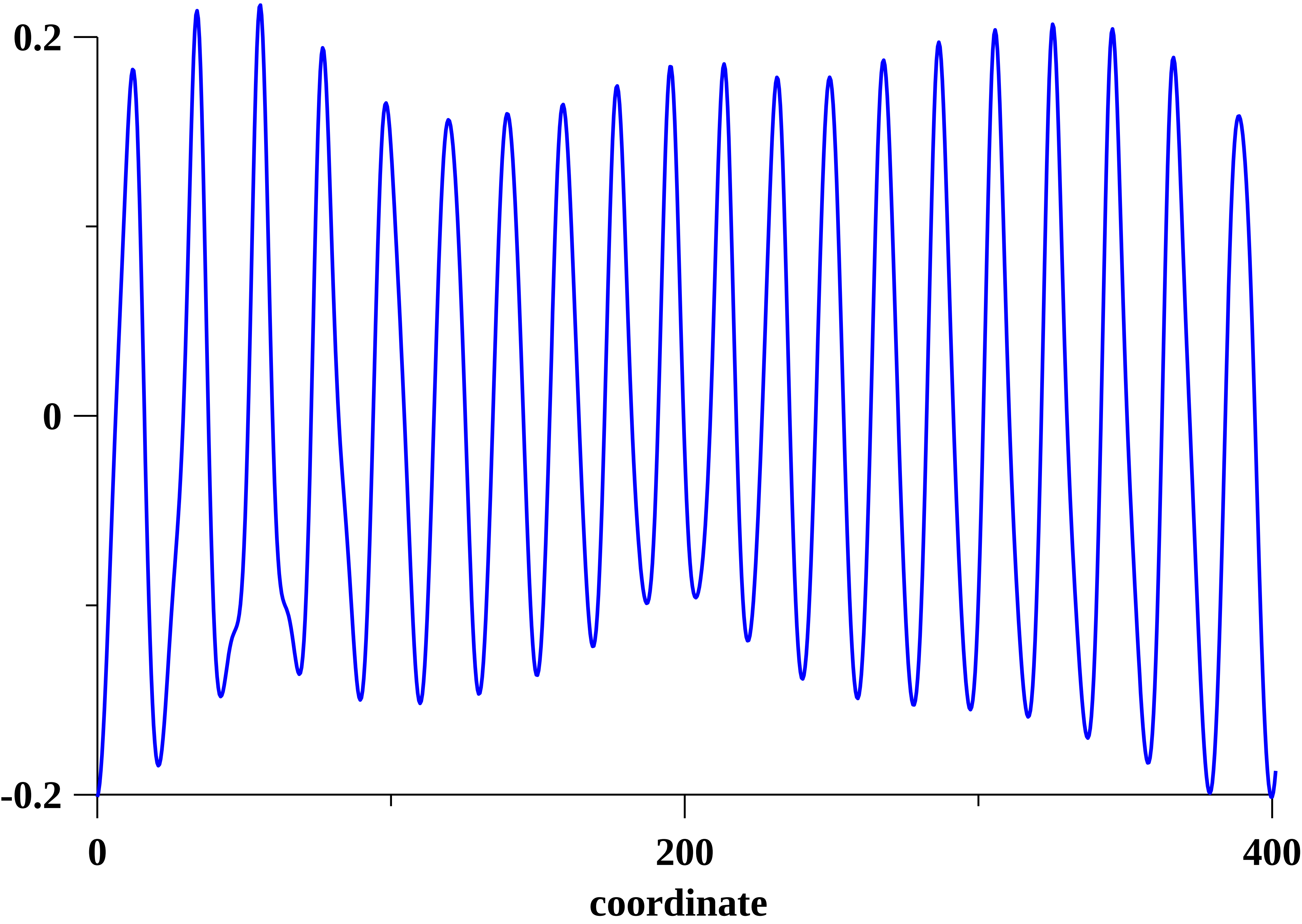
0
200
400
coordinate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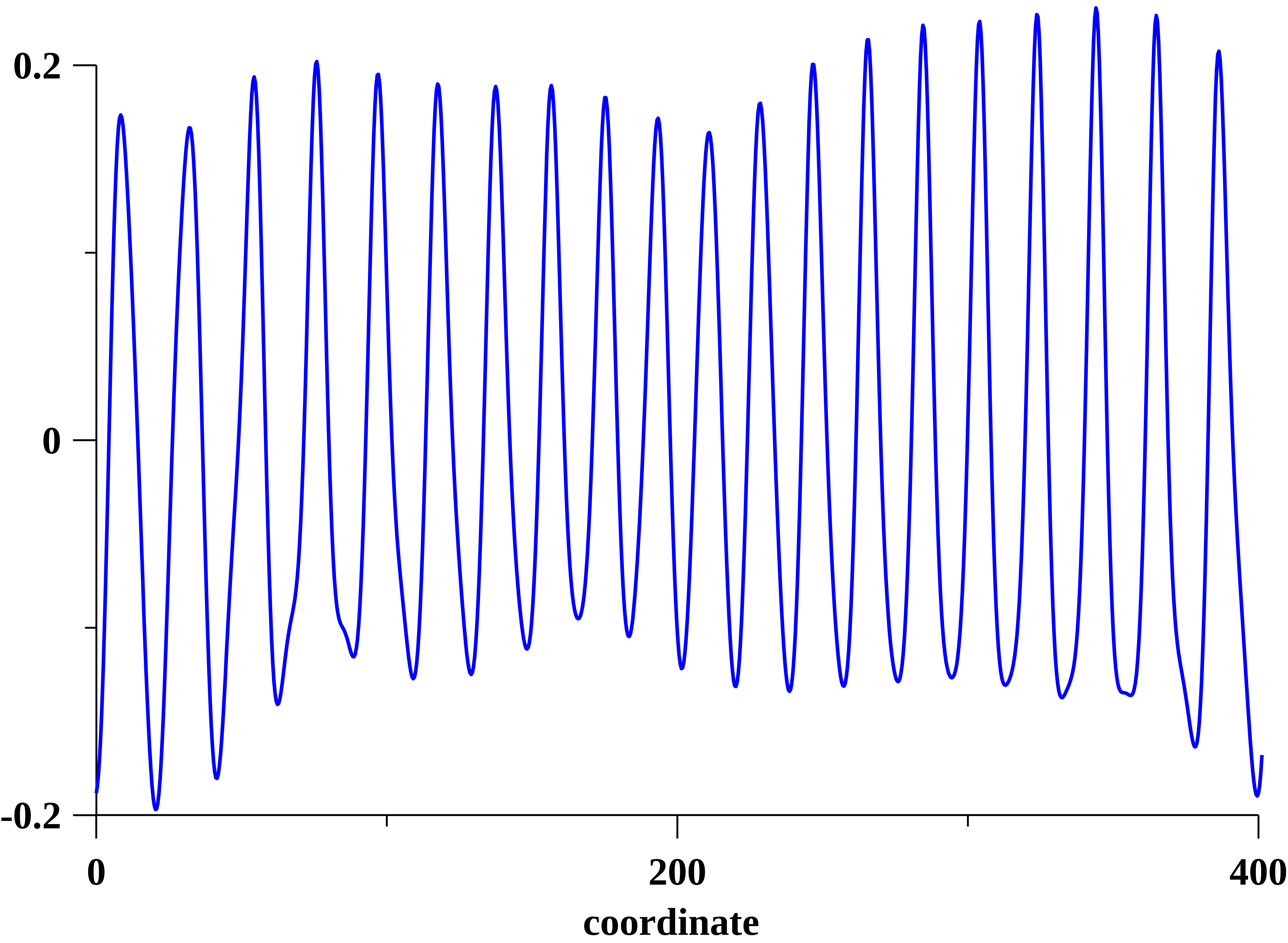
0
200
400
coordinate



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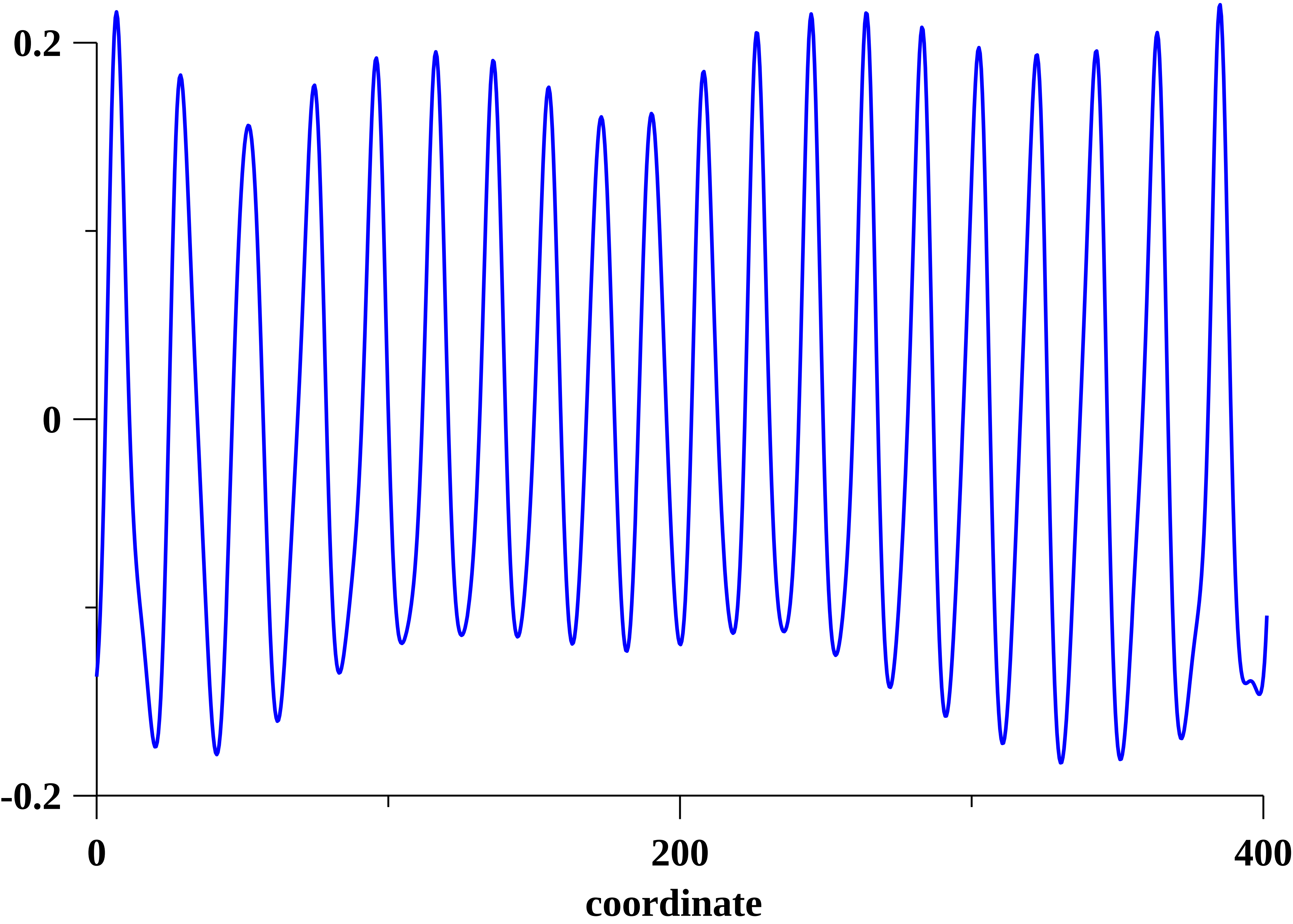
coordinate



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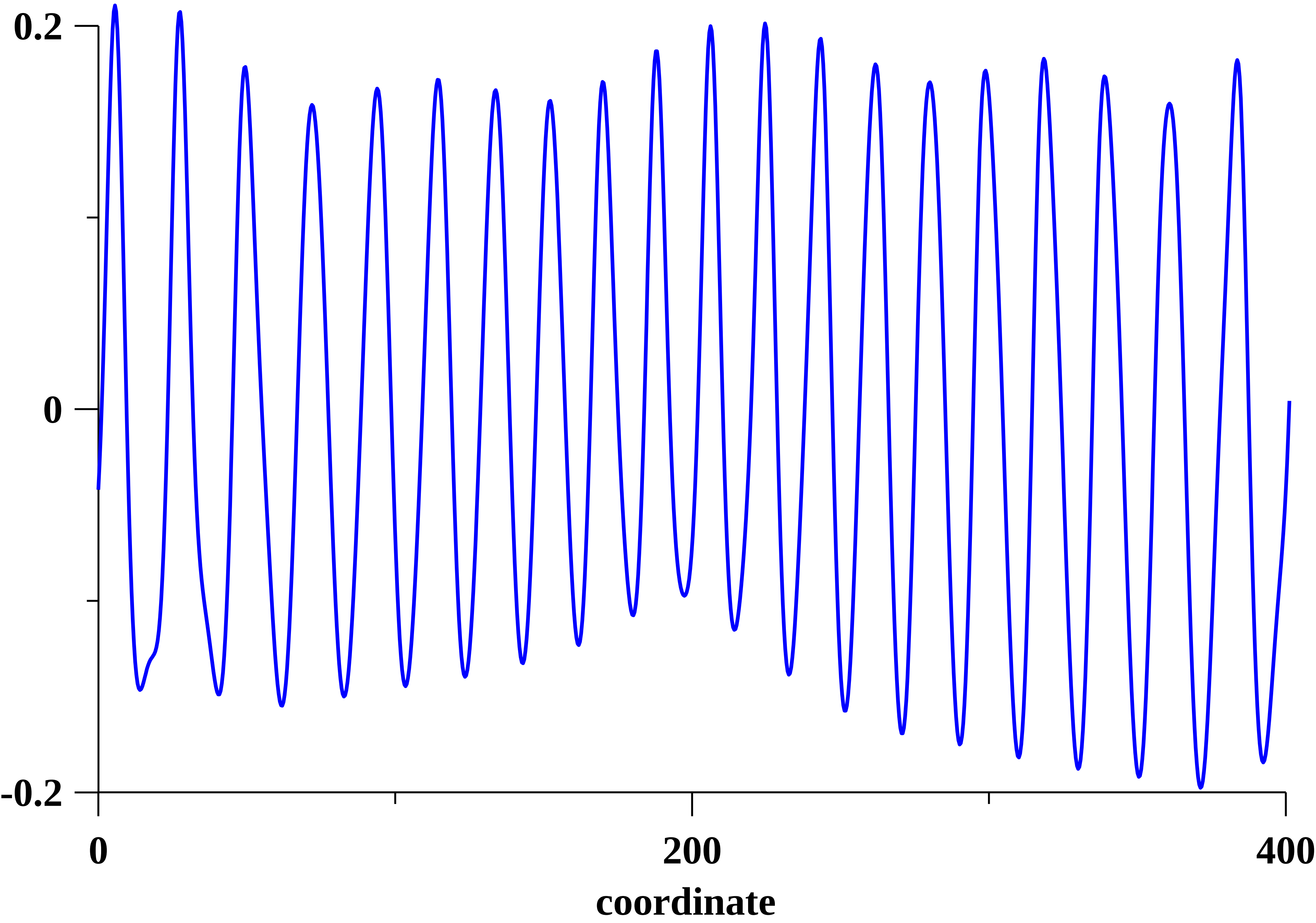
200
400
coordinate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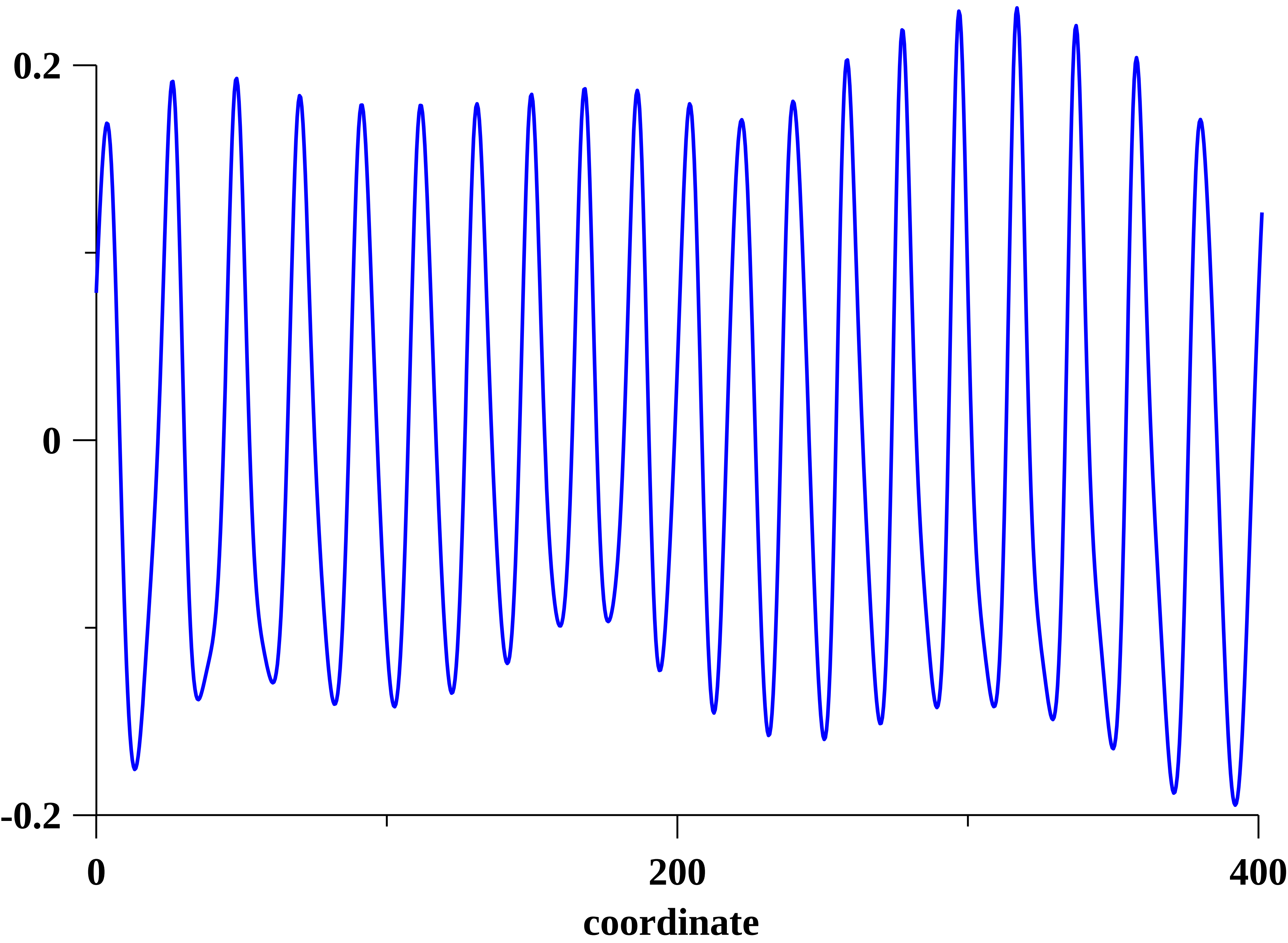
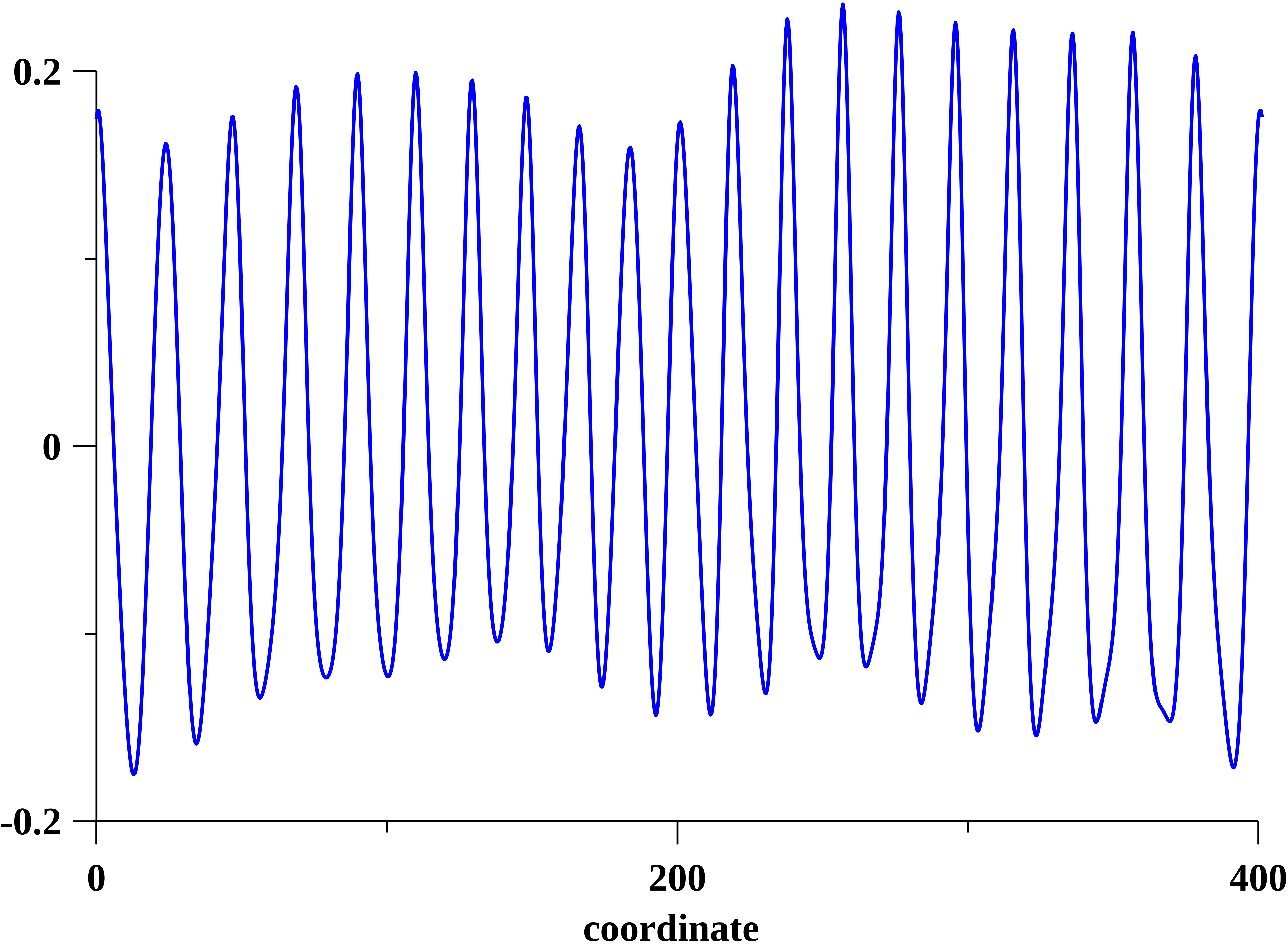
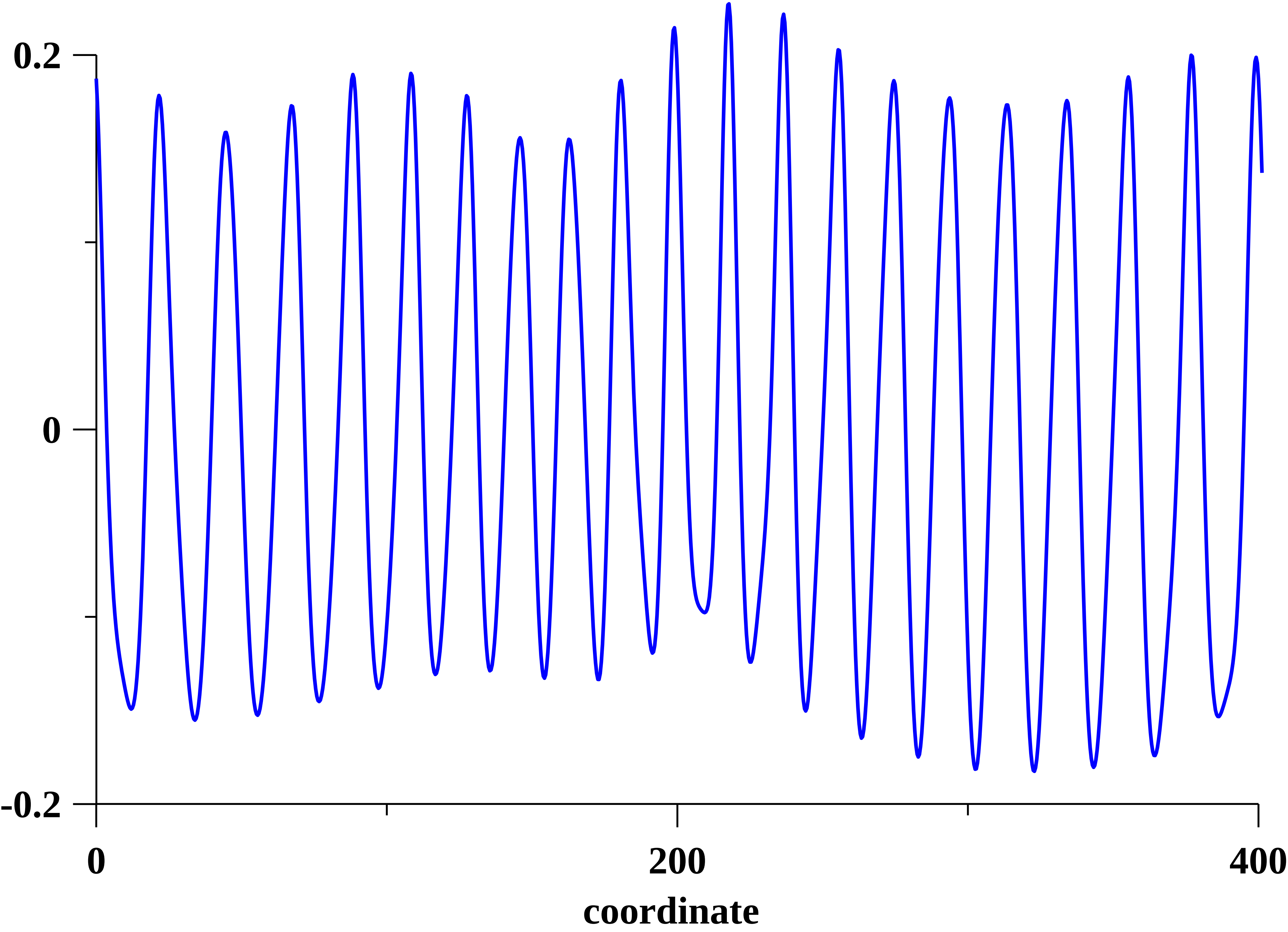
0
-0.2
0
200
400
coordinate



0.2
0
-0.2
0
200
400
coordinate



0.2
0
-0.2
0
200
400
coordinate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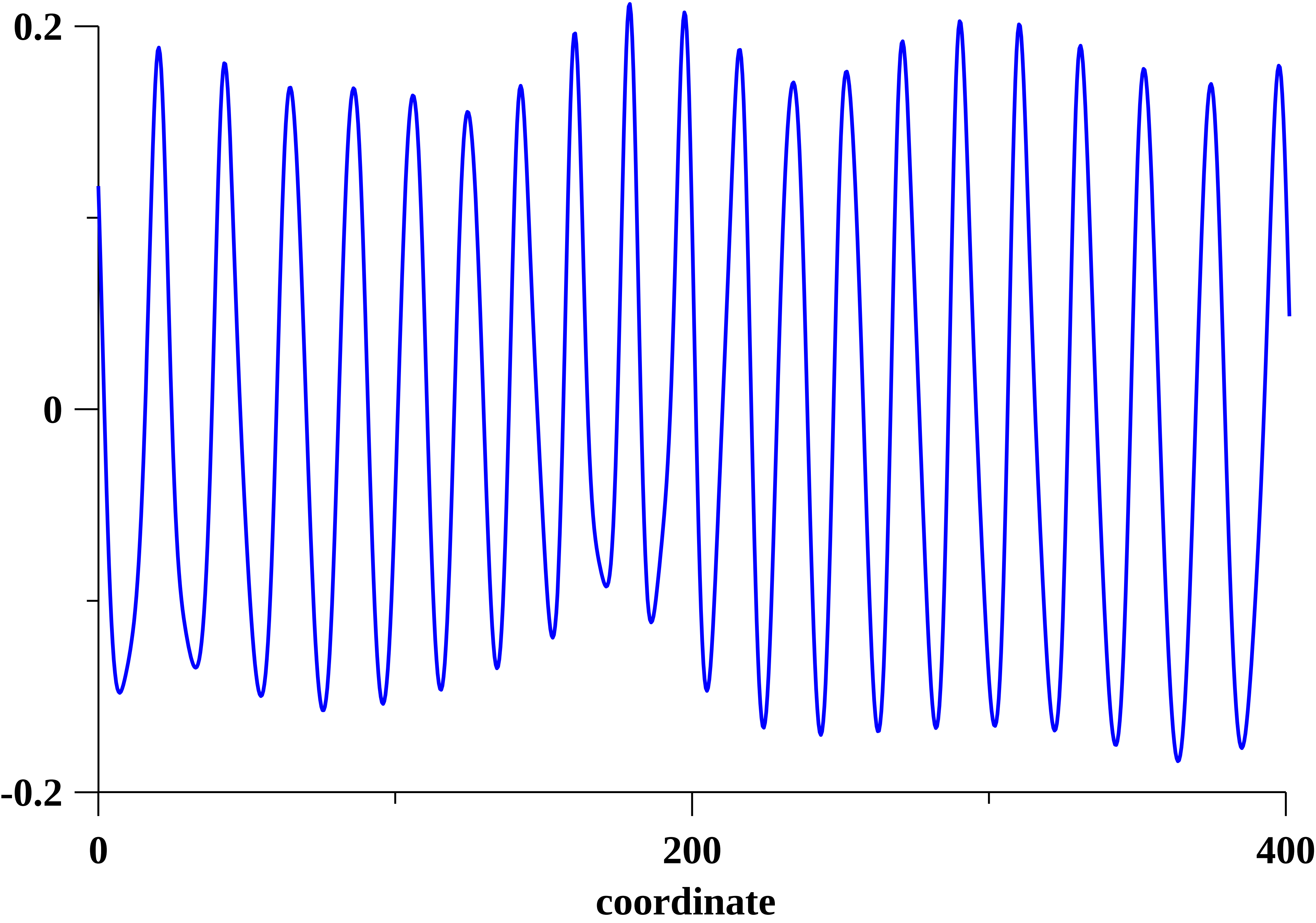
0
-0.2
0
200
400
coordinate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0
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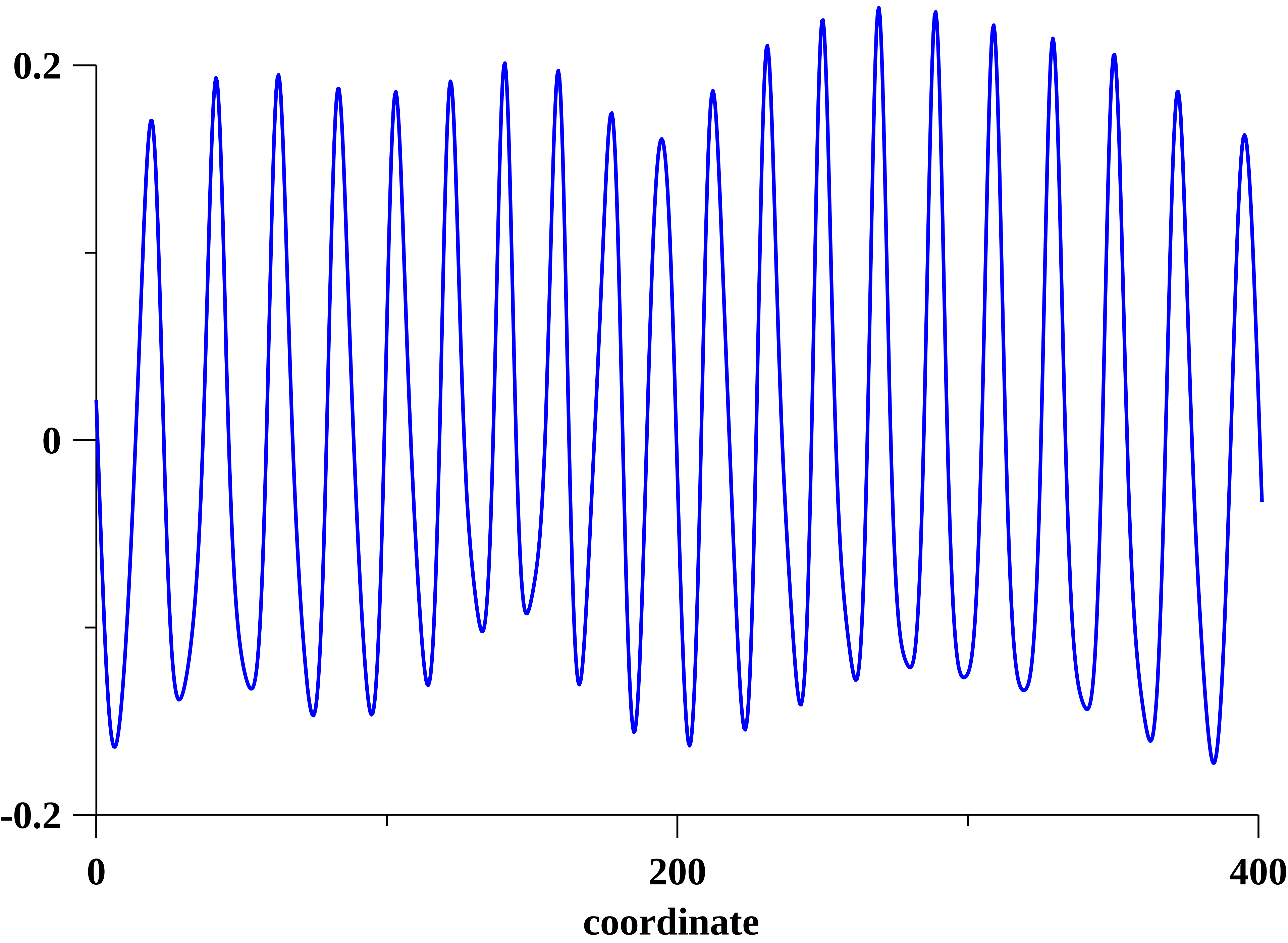
0
200
400
coordinate



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0
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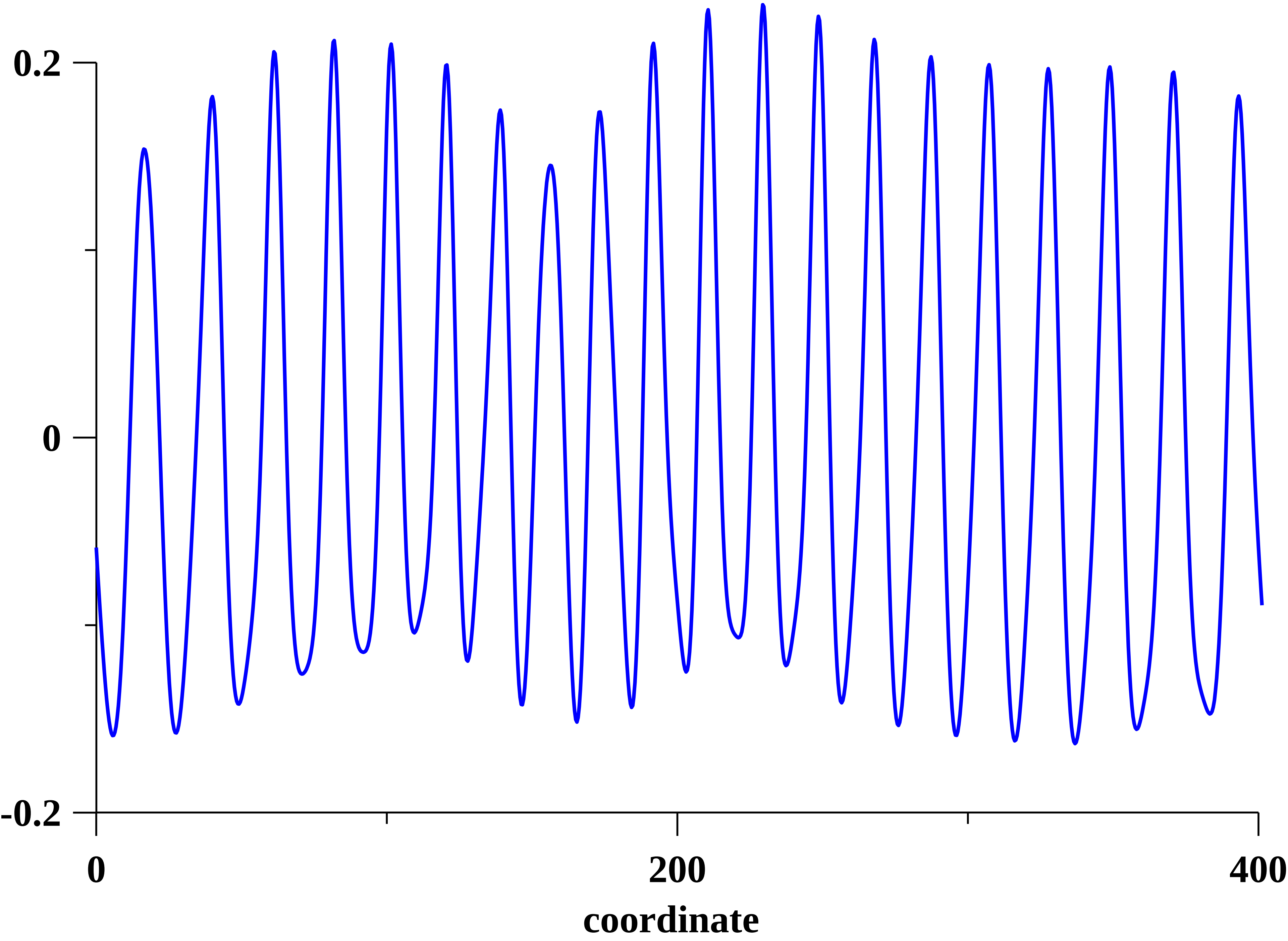
0
200
400
coordinate



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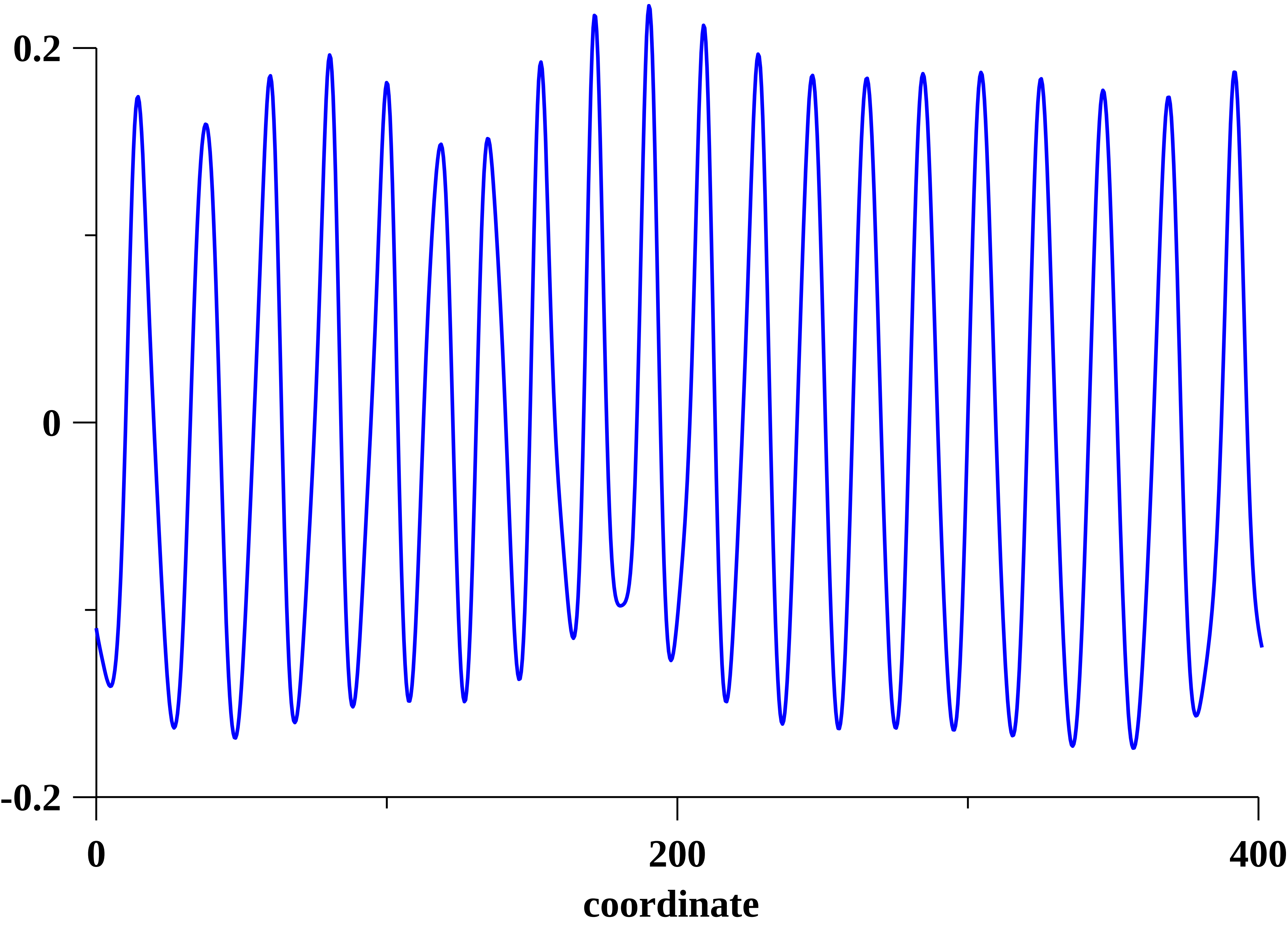
-0.2
0
200
400
coordinate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0
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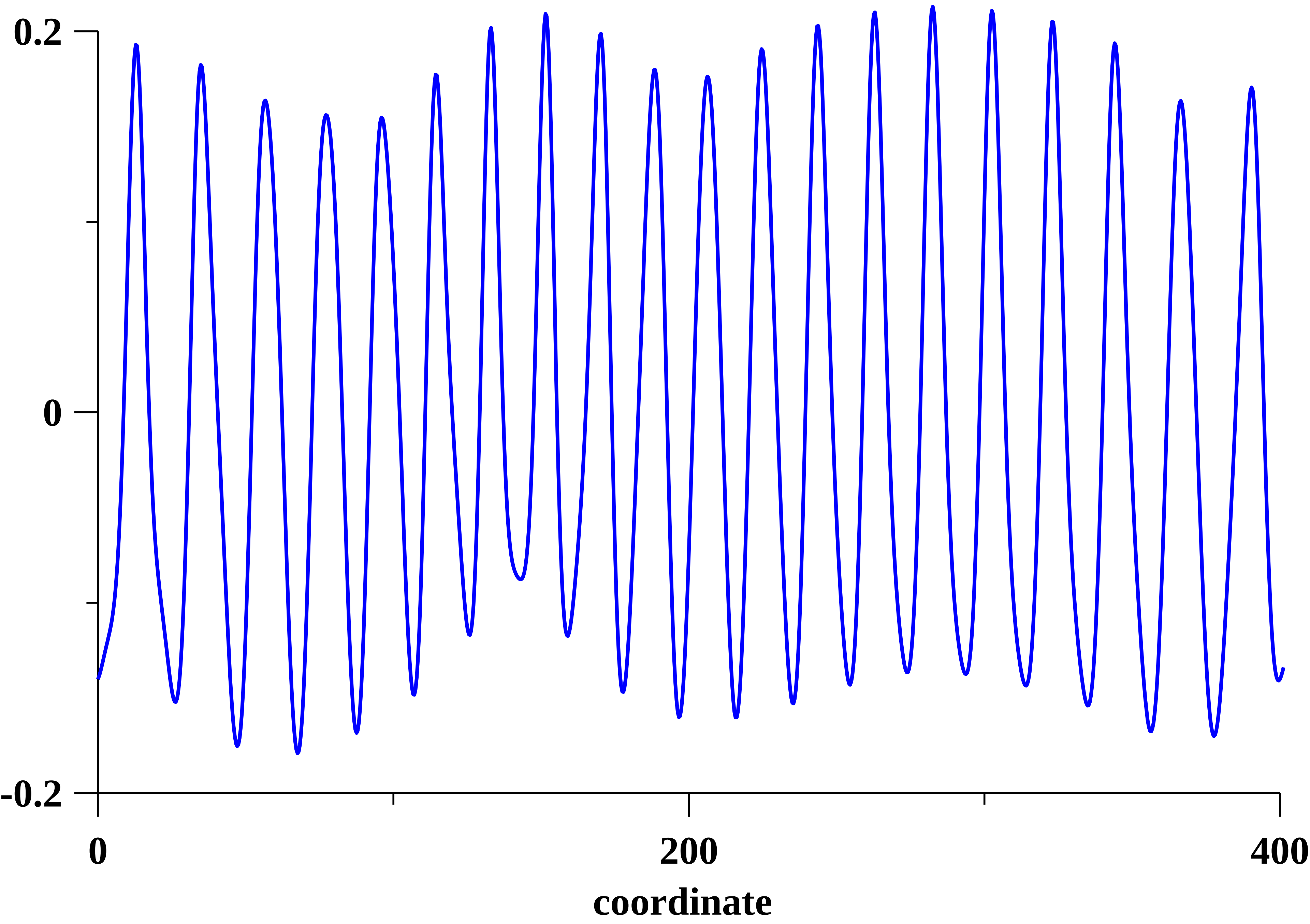
0
200
400
coordinate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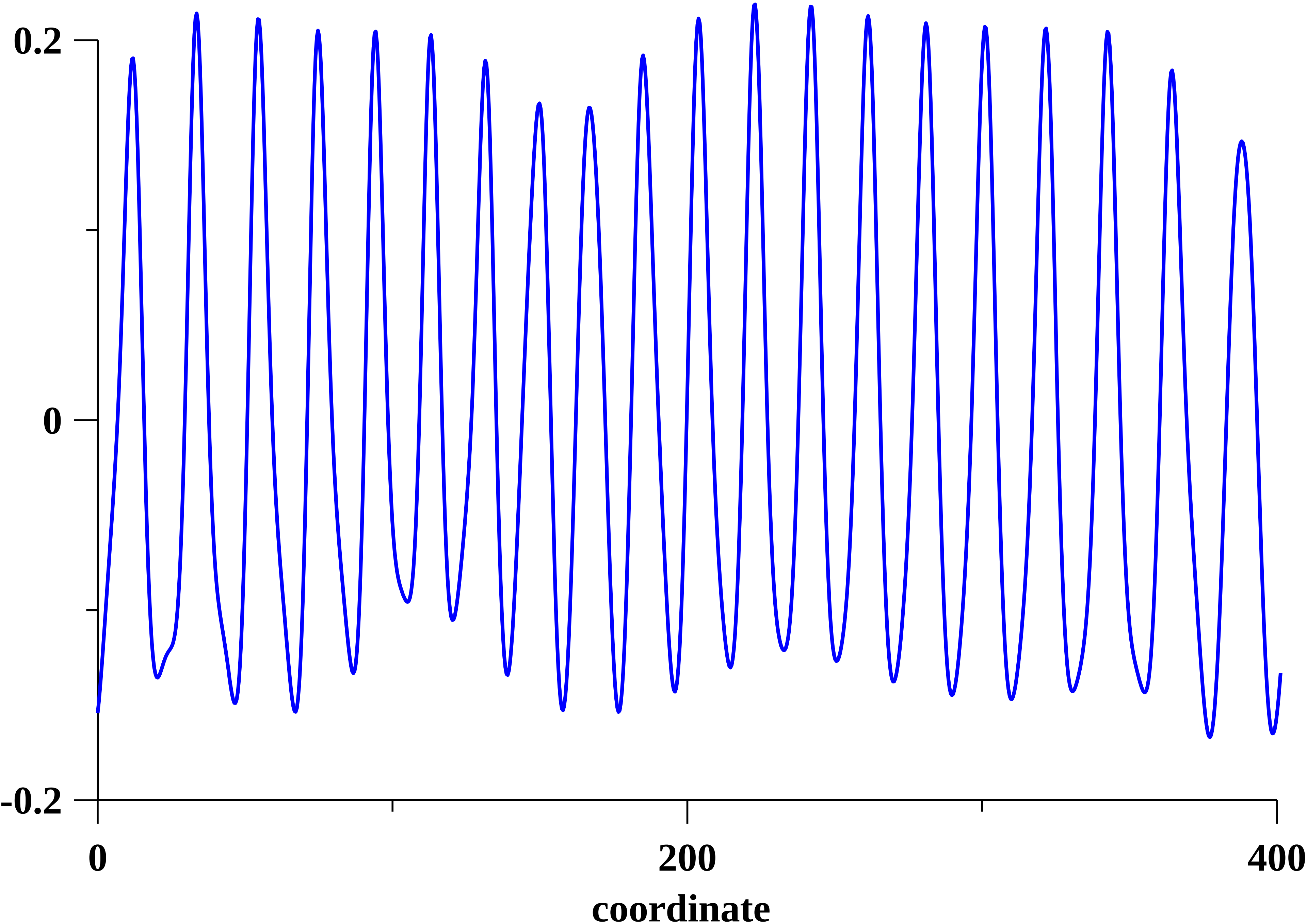
0
-0.2
0
200
400
coordinate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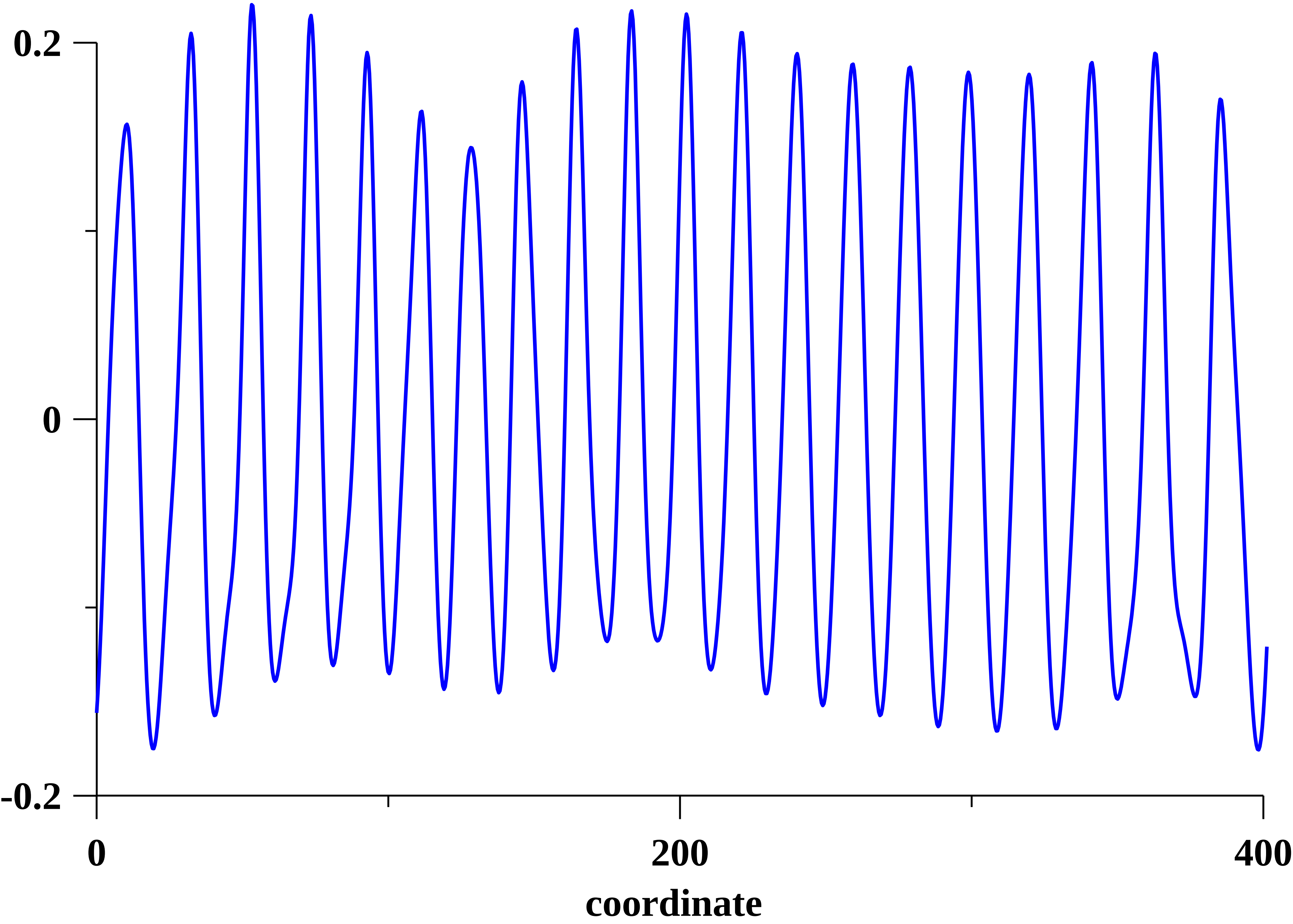
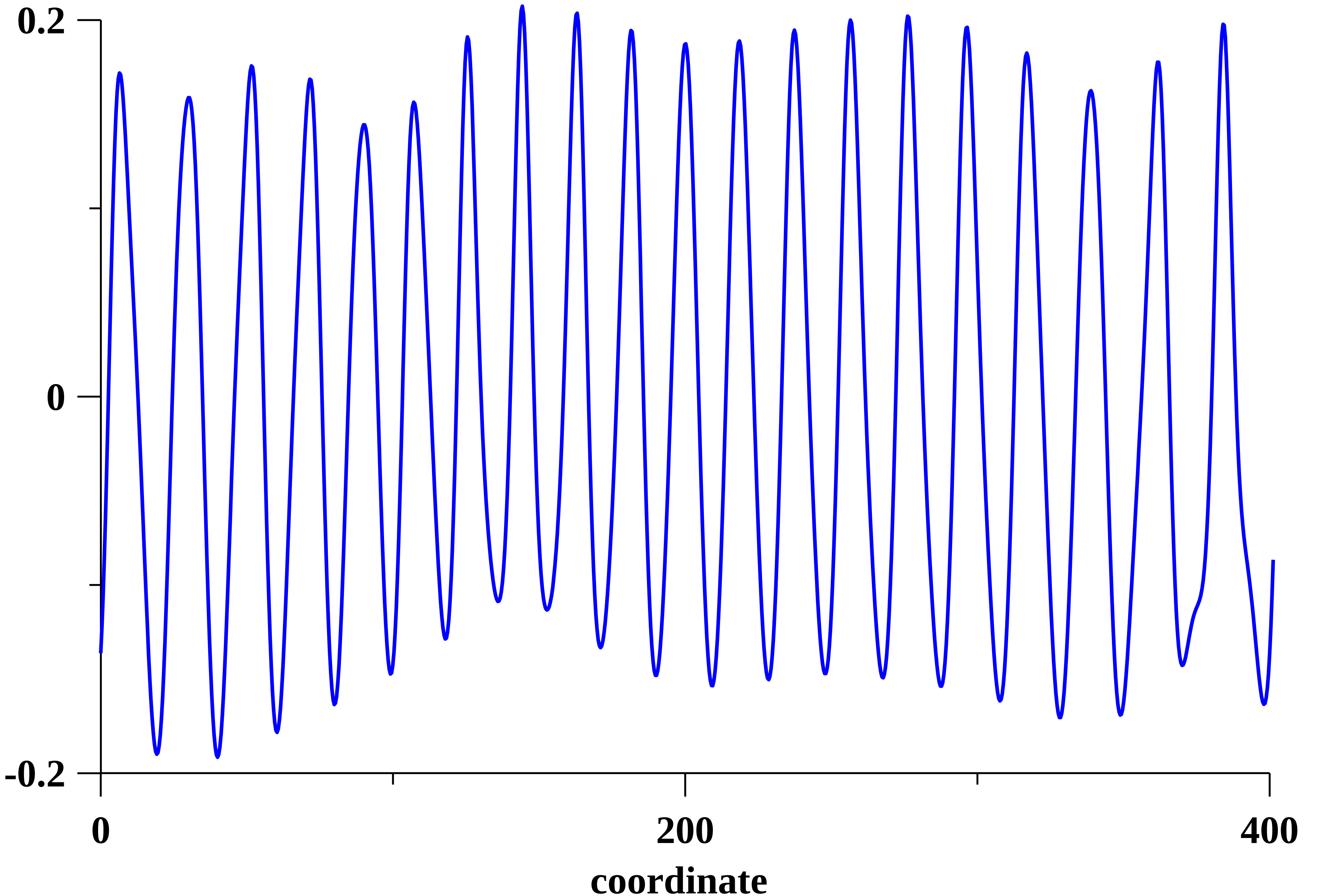
0
-0.2
0
200
400
coordinate



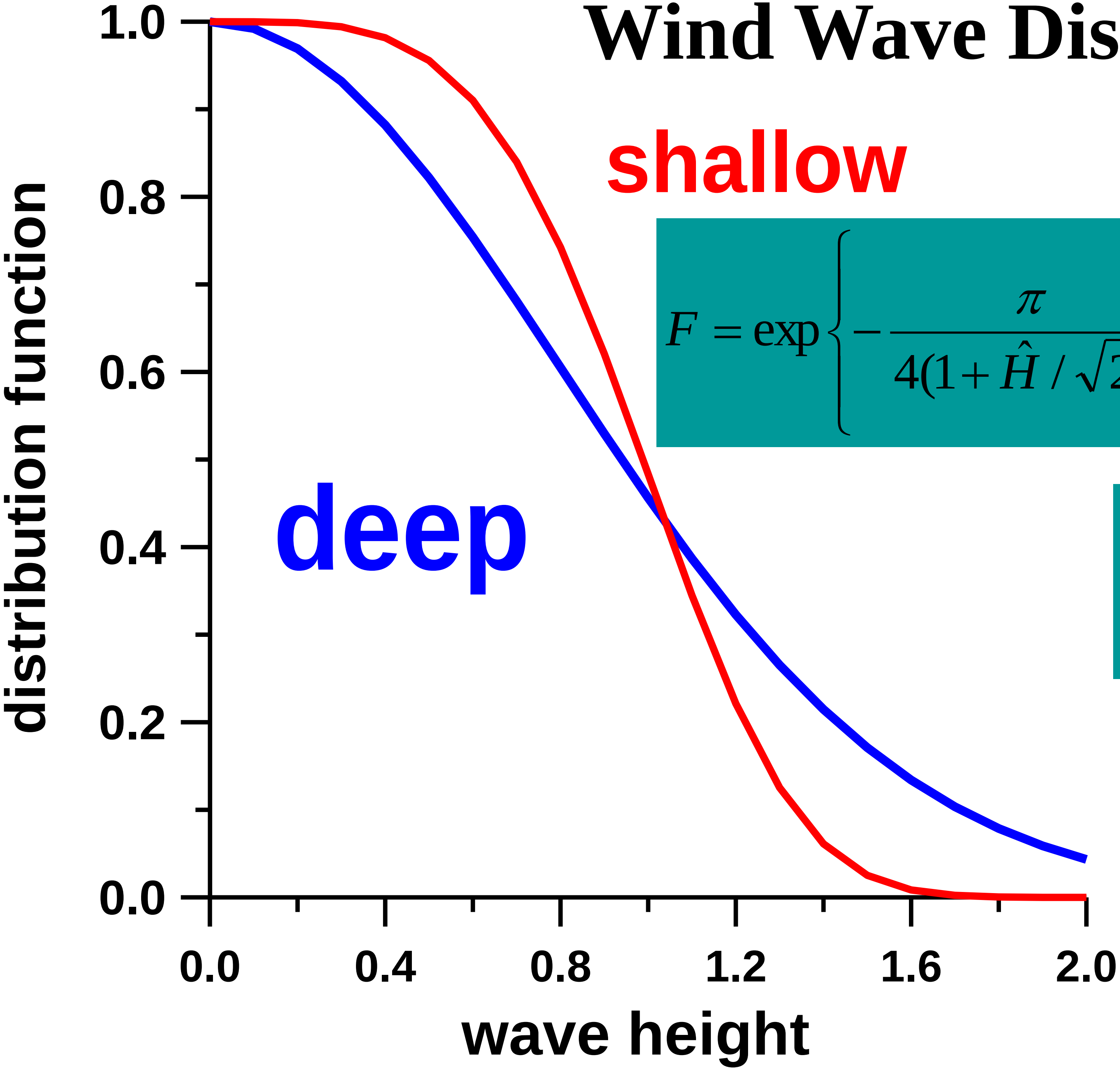
0.2
0
-0.2
0
200
400
coordinate

Demodulation: no freak wave

Recurrence

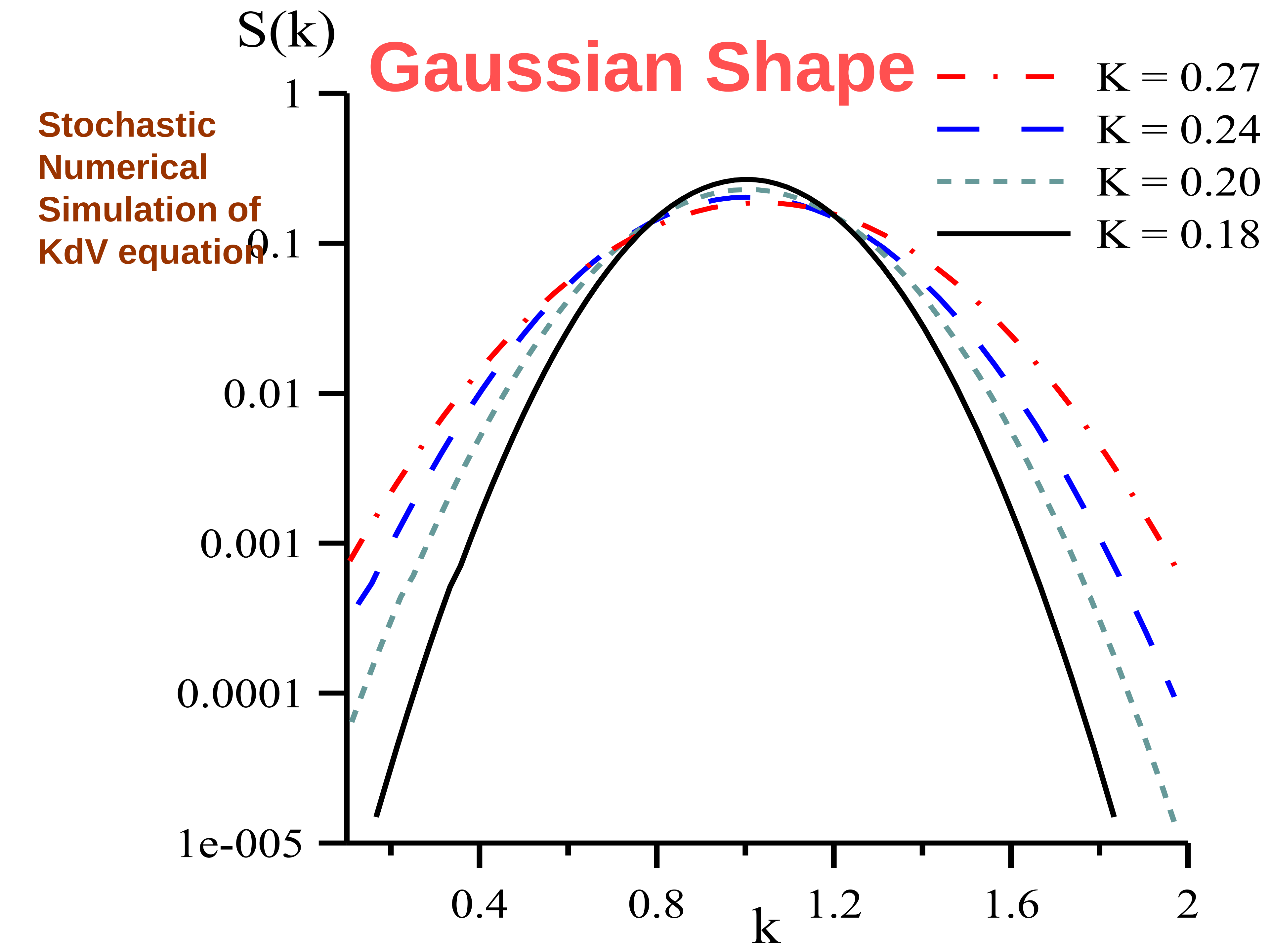


Wind Wave Distribution



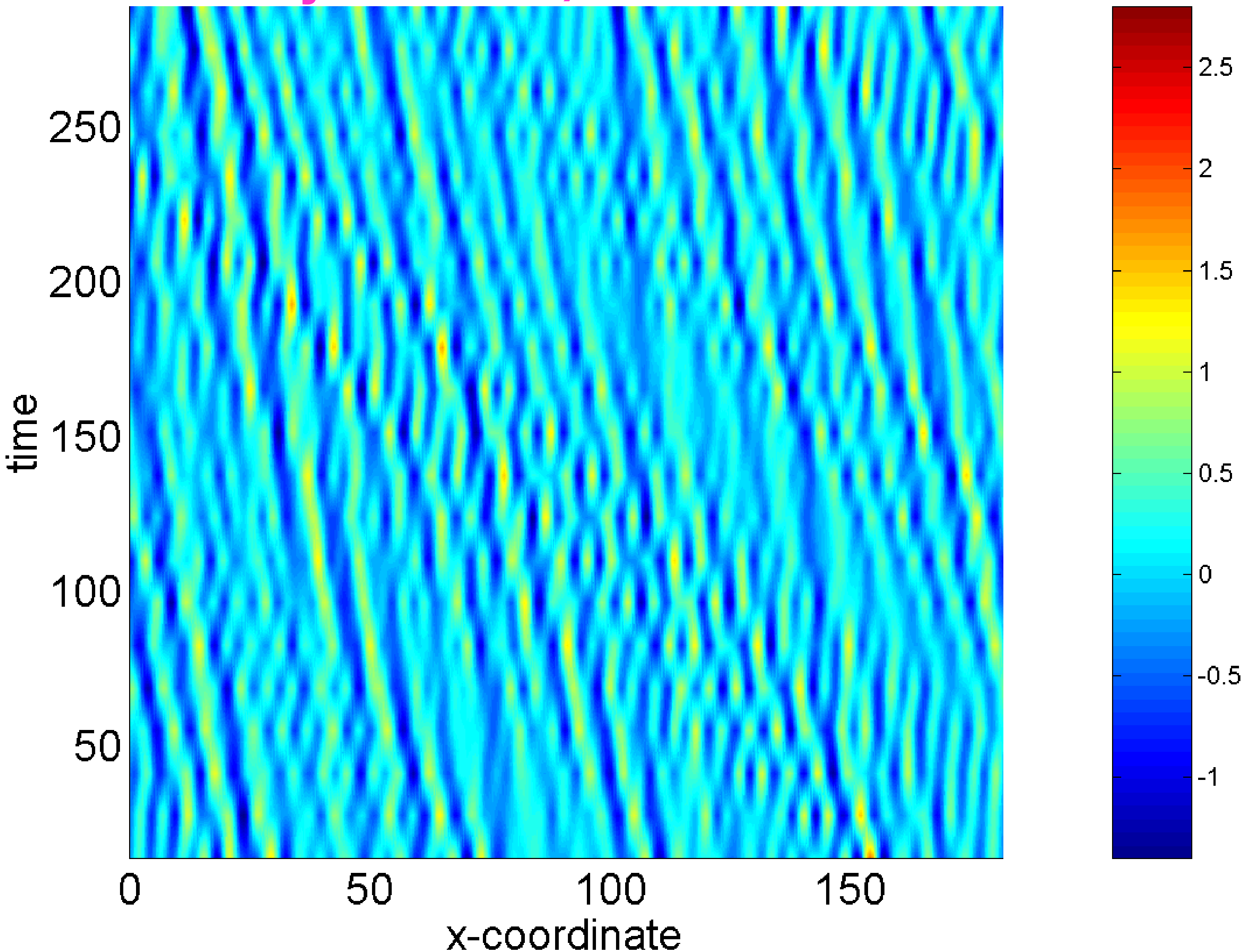
$$F = \exp\left\{-\frac{\pi}{4(1+\hat{H}/\sqrt{2\pi})}\left(\frac{H}{H_{mean}}\right)^{\frac{2}{1-\hat{H}}}\right\}$$

$$\hat{H} = \frac{H_{mean}}{h}$$

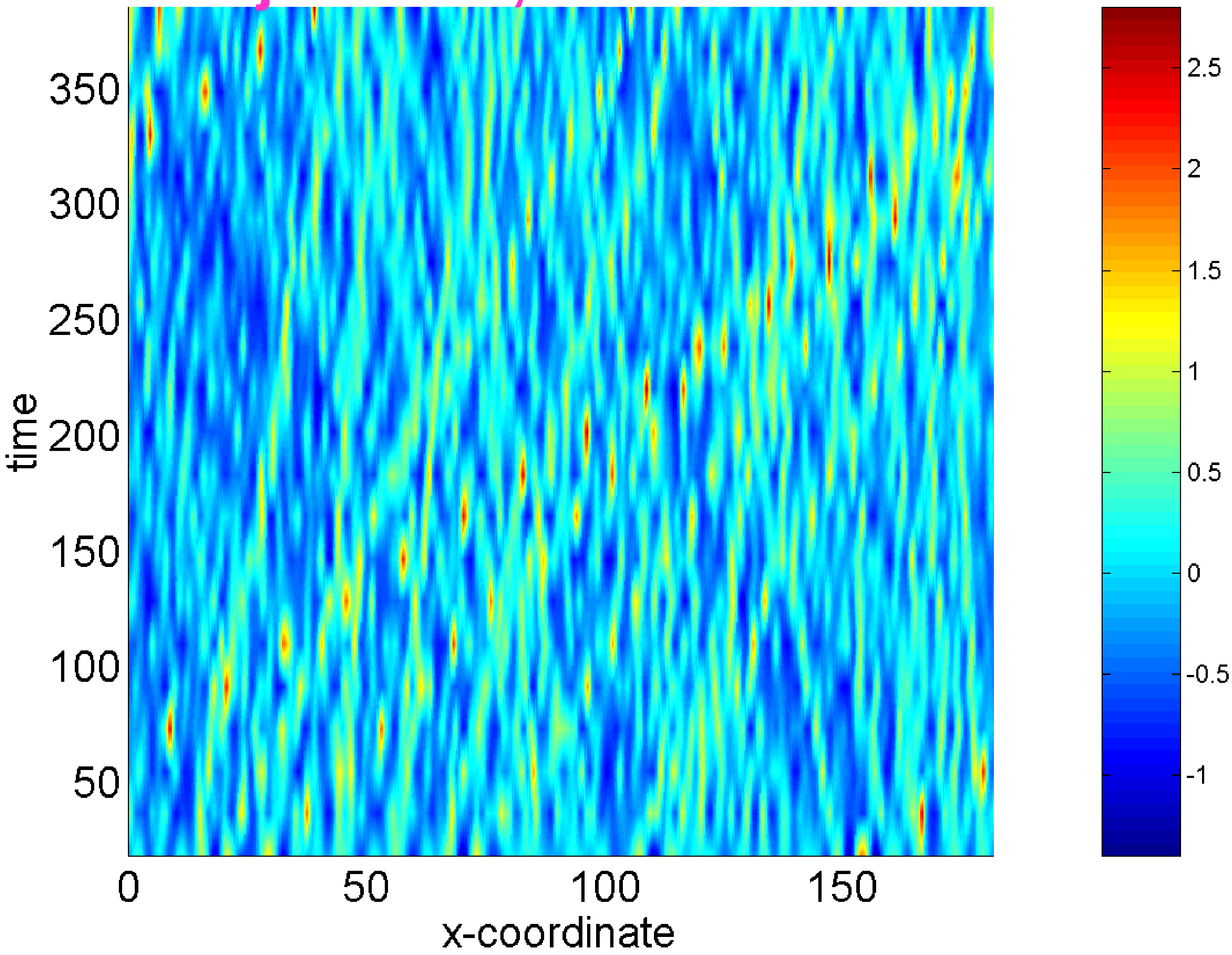


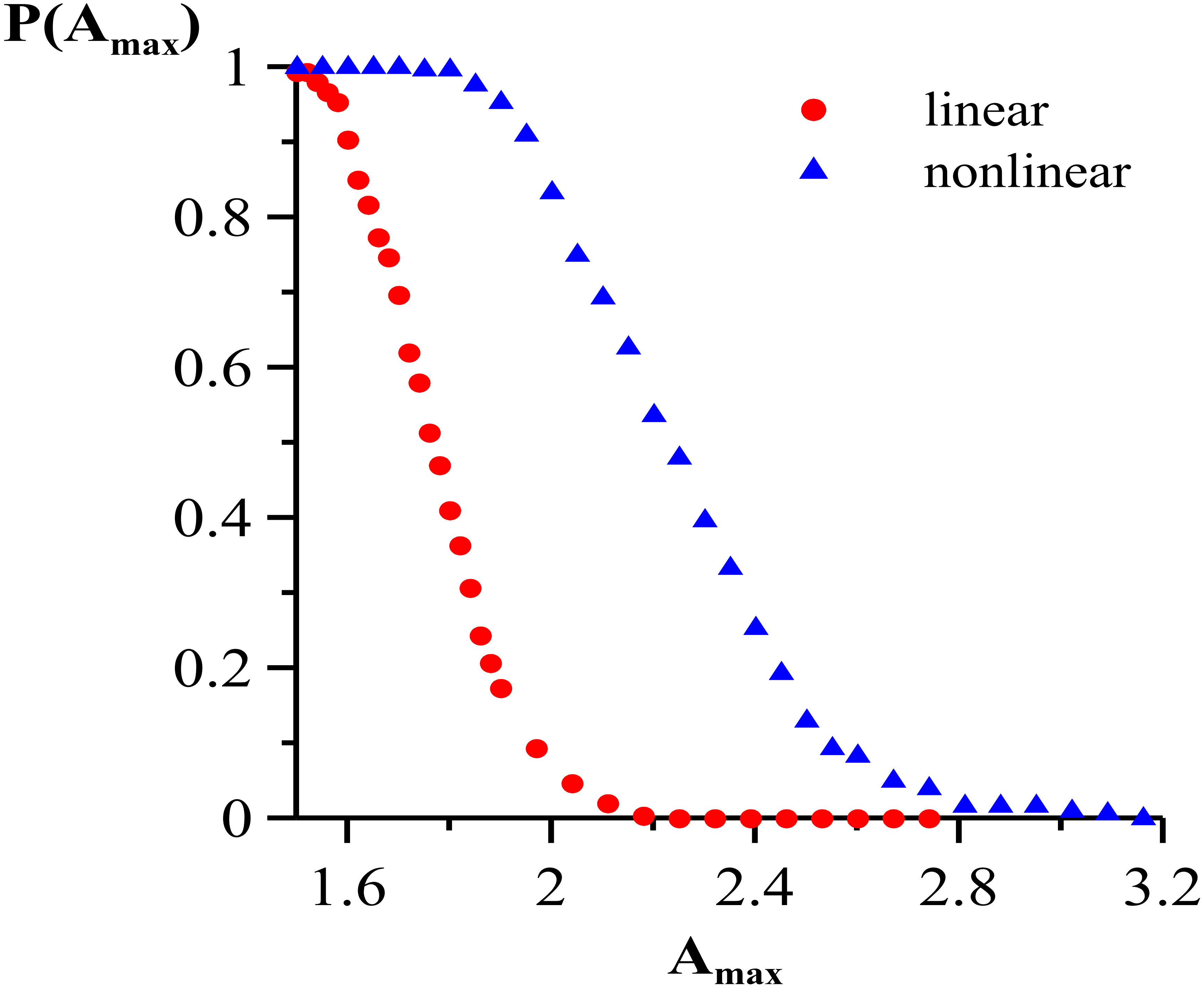
S(k)
Gaussian Shape
Stochastic
Numerical
Simulation of
KdV equation
K = 0.27
K = 0.24
K = 0.20
K = 0.18
1
0.1
0.01
0.001
0.0001
1e-005
0.4
0.8
1.2
1.6
2
k

Wave Trajectories, small Ur



Wave Trajectories, Ur = 1





P(A_max)
1
0.8
0.6
0.4
0.2
0
1.6
2
2.4
2.8
3.2
A_max
linear
nonlinear

Moment Calculations

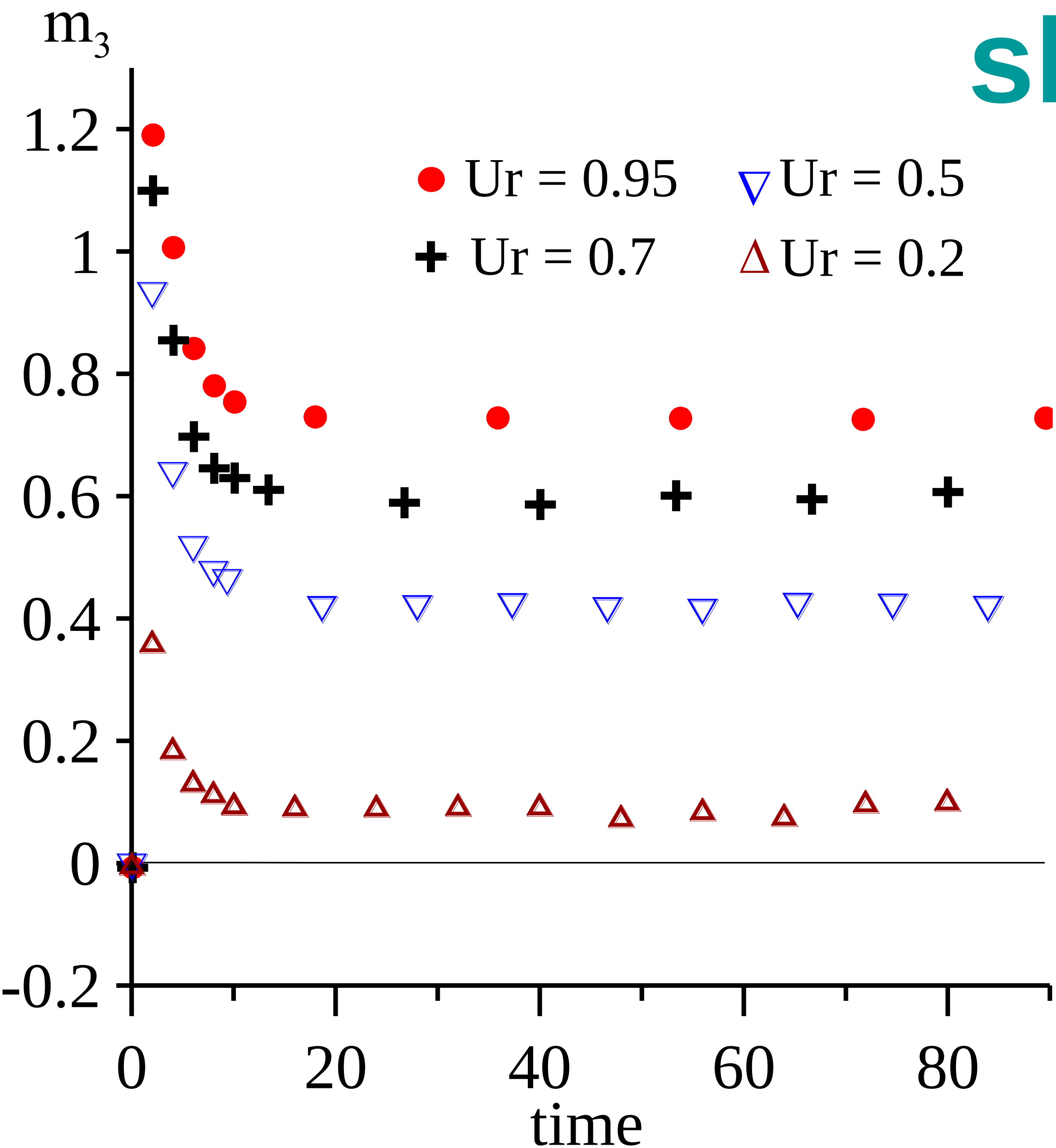
skewness

$$m_3 = \frac{M_3}{\sigma_0^3}$$

kurtosis

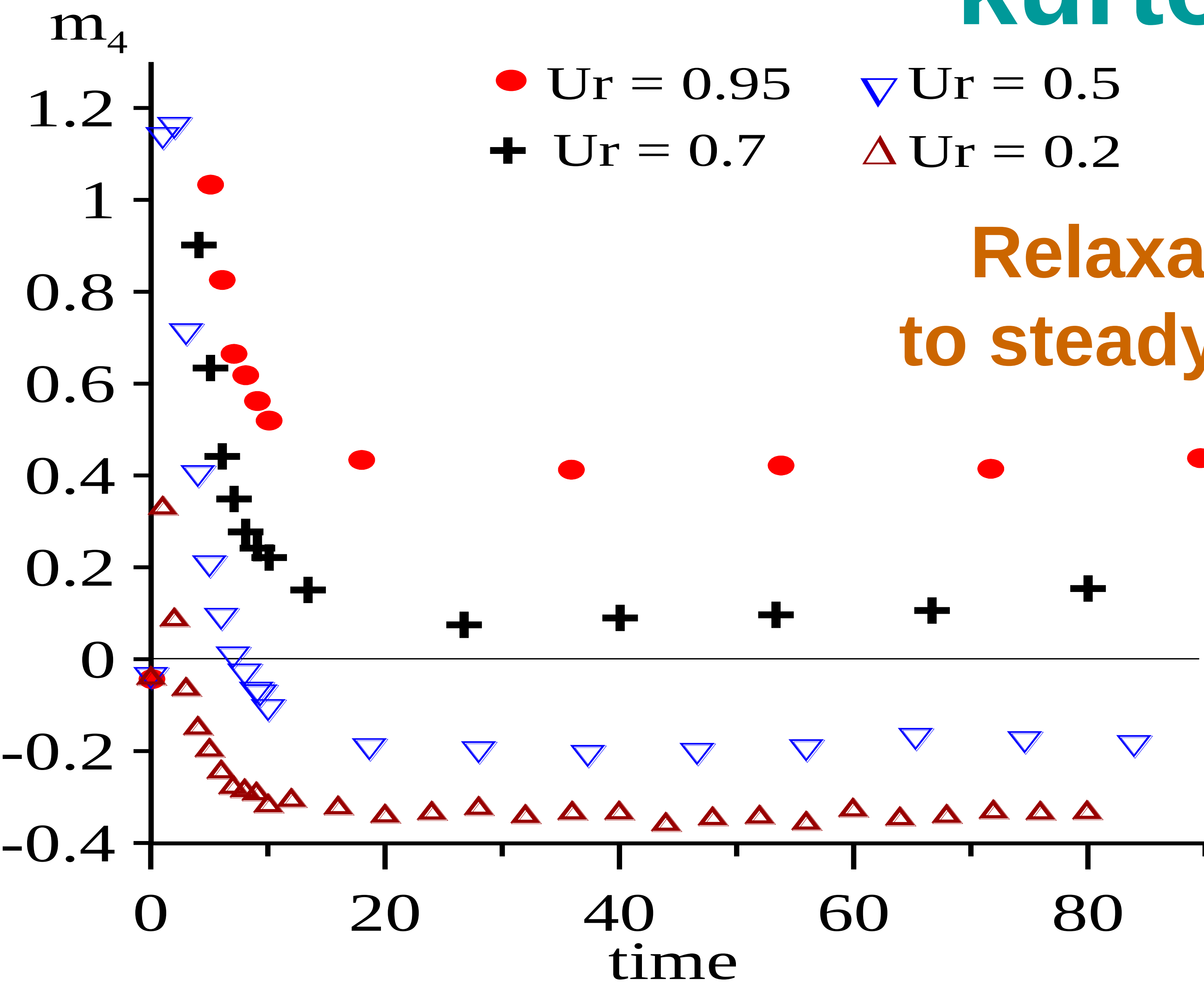
$$m_4 = \frac{M_4}{s_0^4} - 3$$

skewness



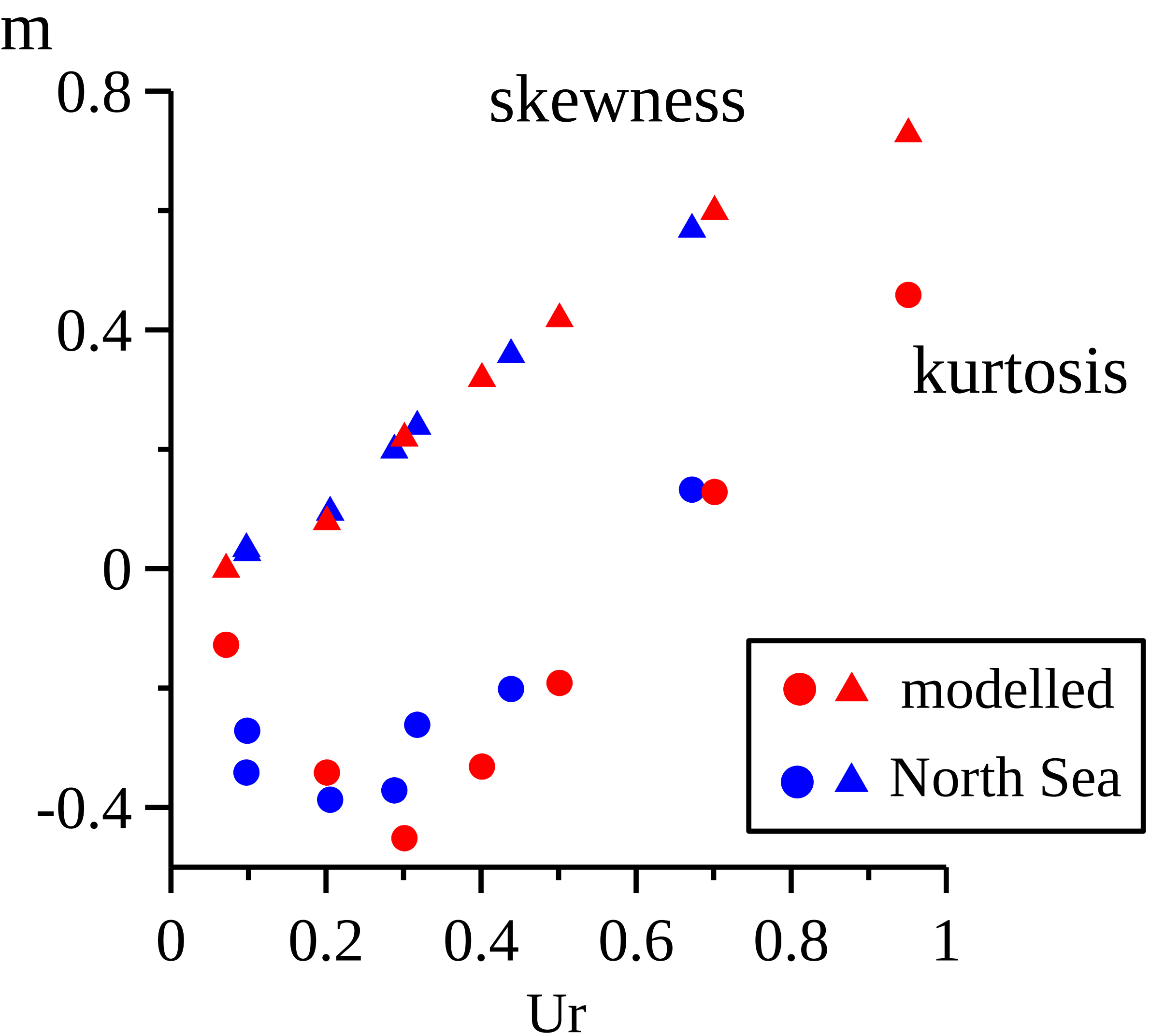
Relaxation to steady state

kurtosis



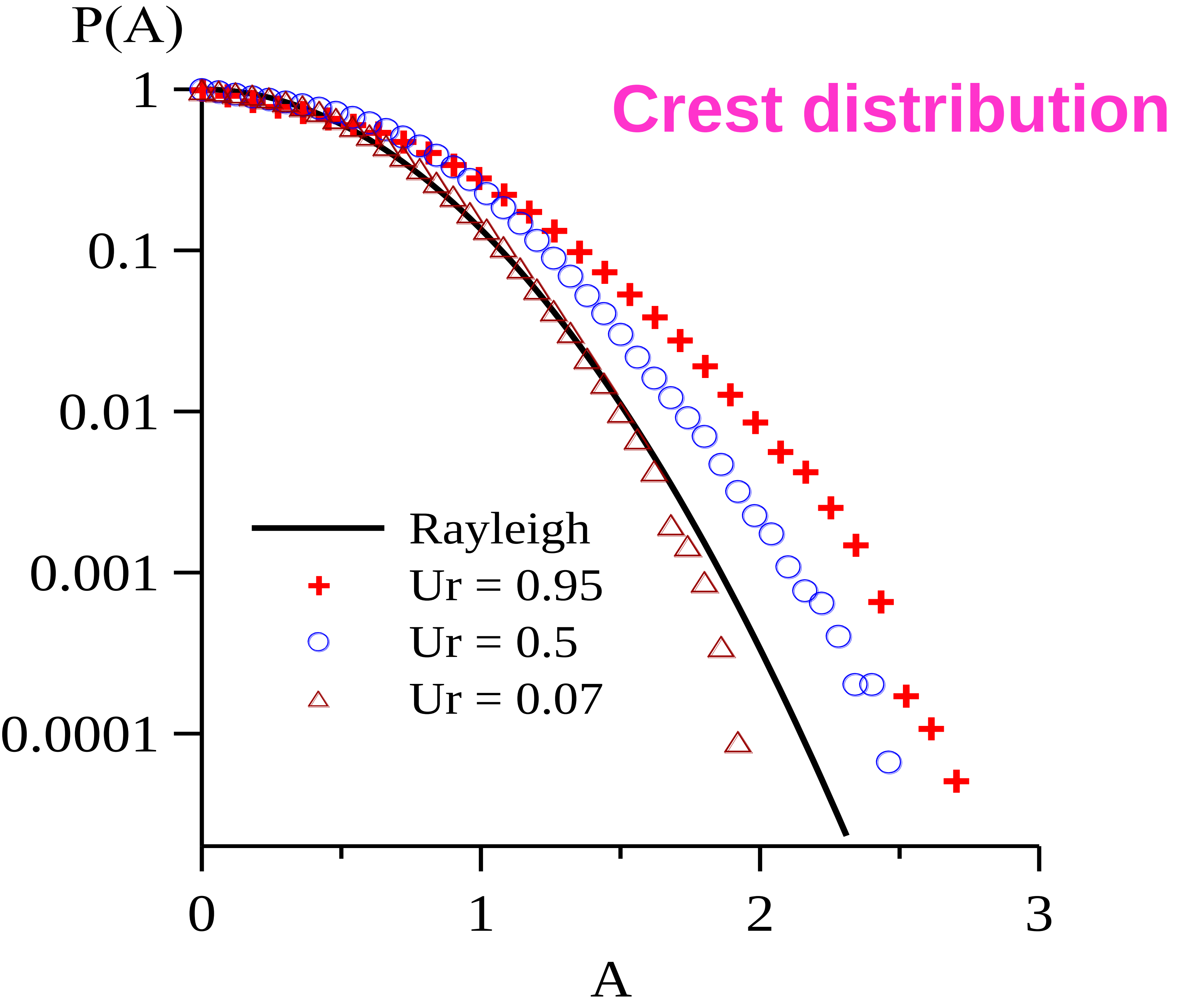
Relaxation to steady state

Universal curves

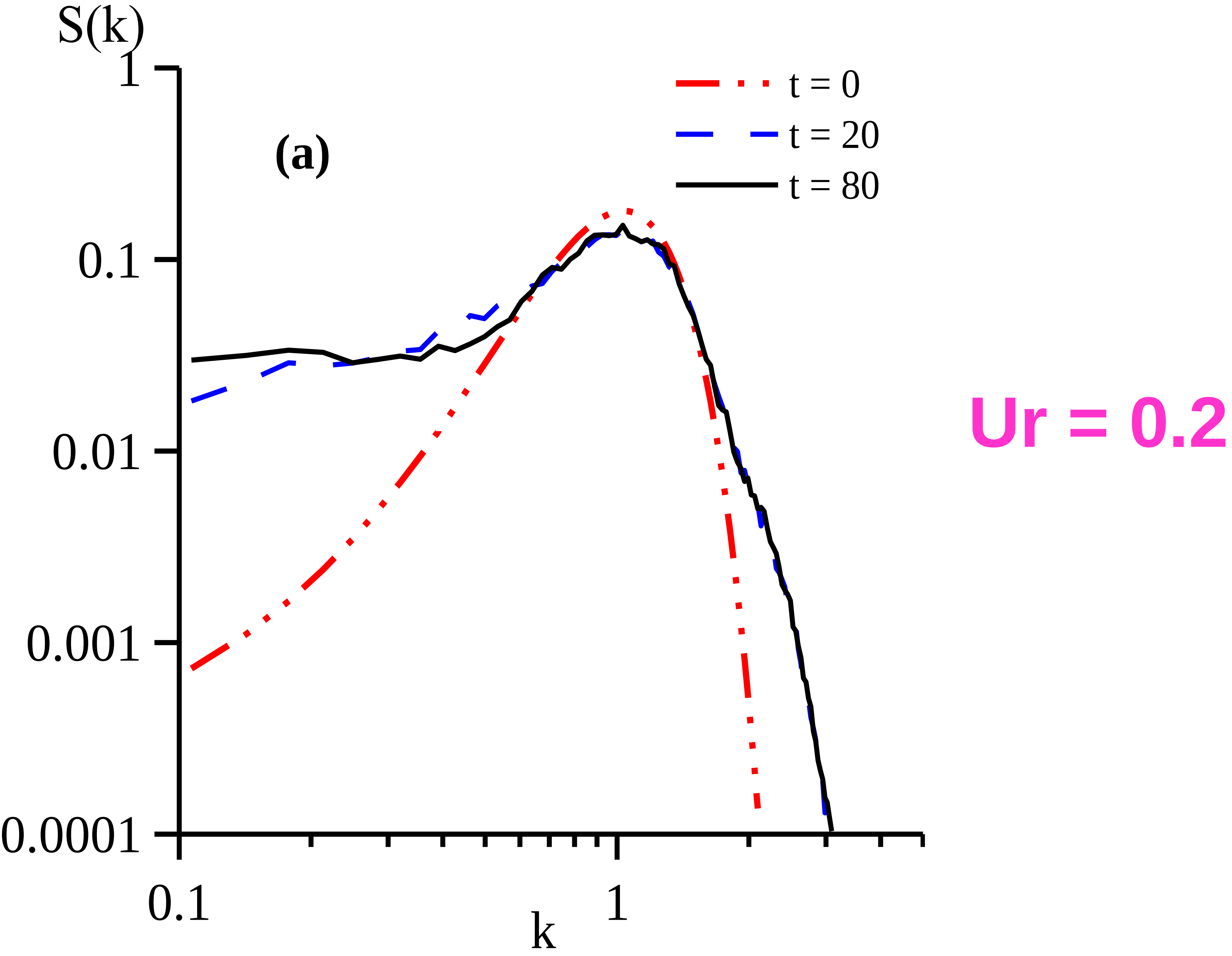


Pelinovsky, E., and Sergeeva, A. Numerical modeling of the KdV random wave field. *European Journal of Mechanics – B/Fluid*. 2006, vol. 25, 425-434.

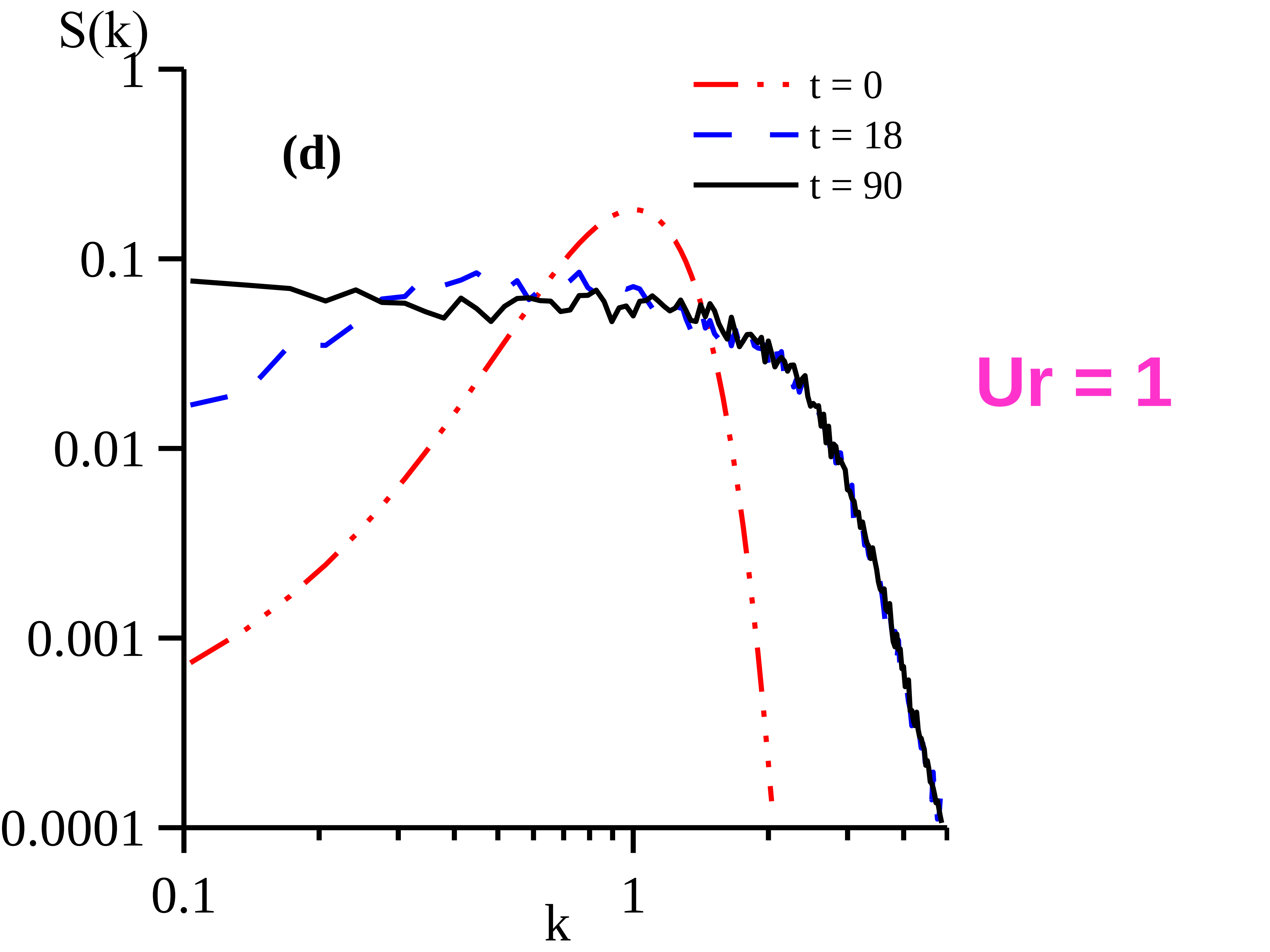
Sergeeva, A., Pelinovsky, E., and Talipova T. Nonlinear Random Wave Field in Shallow Water: Variable Korteweg – de Vries Framework. *Natural Hazards and Earth System Science*, 2011, vol. 11, 323-330



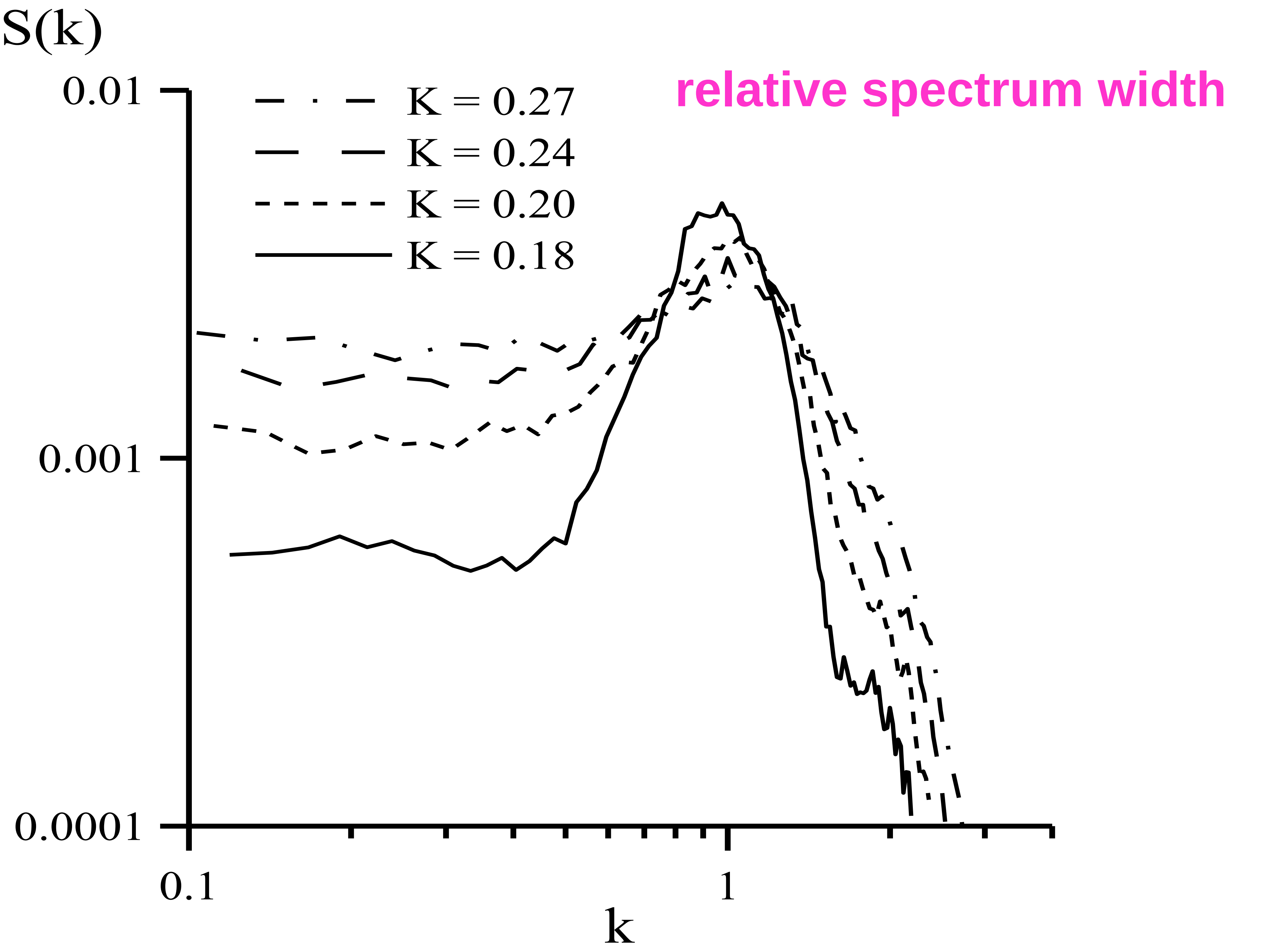
P(A)
1
0.1
0.01
0.001
0.0001
Crest distribution
Rayleigh
Ur = 0.95
Ur = 0.5
Ur = 0.07
0
1
2
3
A

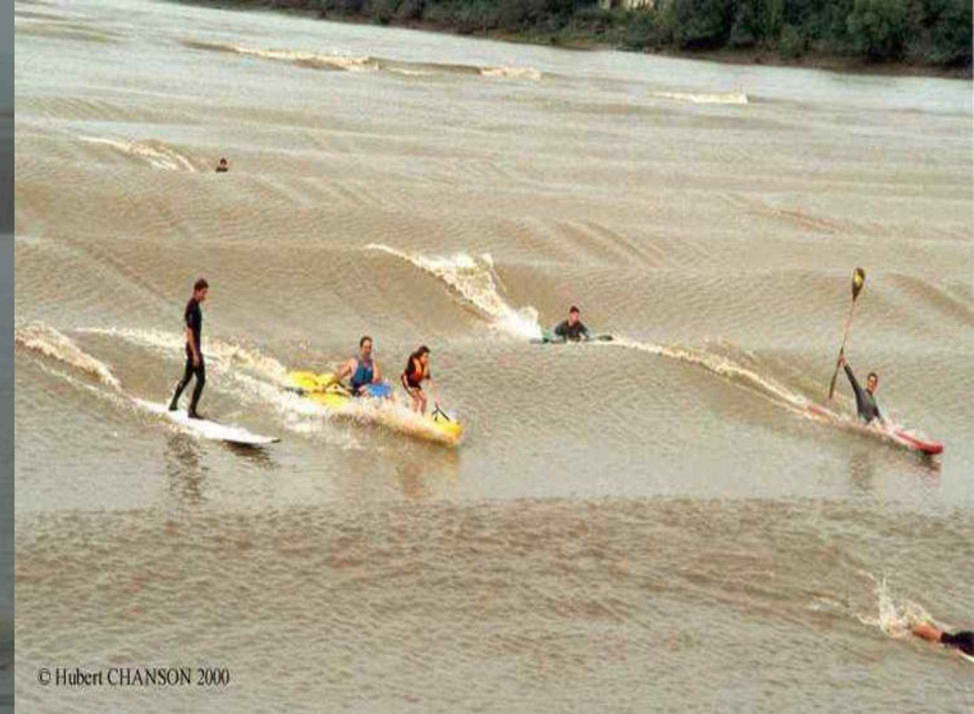


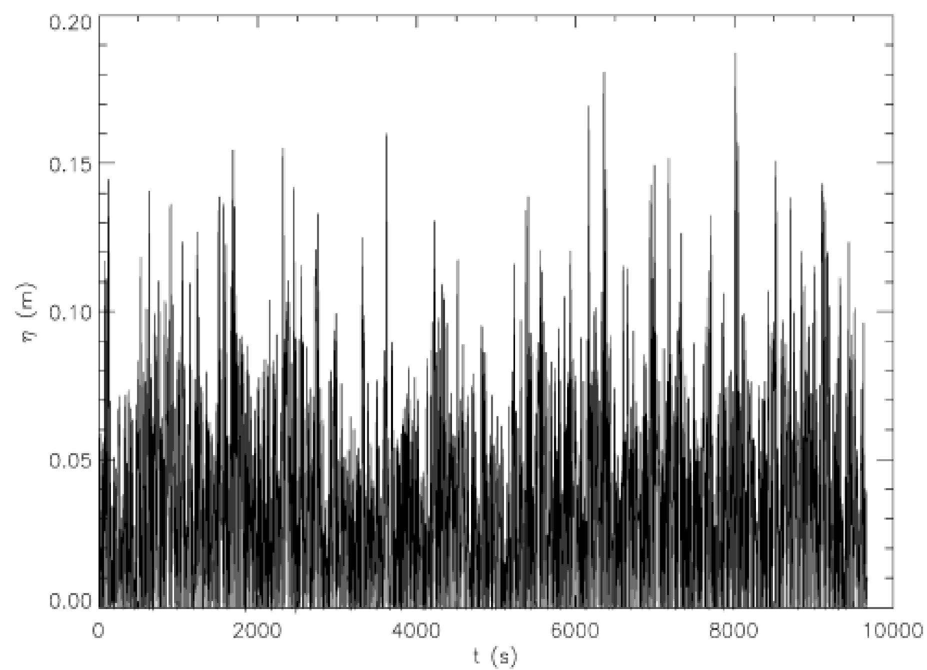
S(k)
1
0.1
0.01
0.001
0.0001
(a)
t = 0
t = 20
t = 80
0.1
1
k
Ur = 0.2



S(k)
1
0.1
0.01
0.001
0.0001
(d)
t = 0
t = 18
t = 90
Ur = 1
0.1
1
k


S(k)
0.01
0.001
0.0001
relative spectrum width
K = 0.27
K = 0.24
K = 0.20
K = 0.18
0.1
1
k


© Hubert CHANSON 2000


0.20
0.15
0.10
0.05
0.00
η (m)
0
2000
4000
6000
8000
10000
t (s)

Soliton Turbulence

V.E. Zakharov, Kinetic equation for solitons, Soviet JETP 60 (1971) 993-1000
A. **Salupere, P. Peterson, and J. Engelbrecht, Long-time behaviour of soliton ensembles. Part 1 – Emergence of ensembles, Chaos, Solitons and Fractals, 14 (2002) 1413-1424.**

- **Salupere, P. Peterson, and J. Engelbrecht, Long-time behaviour of soliton ensembles. Part 2 – Periodical patterns of trajectorism, Chaos, Solitons and Fractals 15 (2003a) 29-40.**
- **Salupere, P. Peterson, and J. Engelbrecht, Long-time behavior of soliton ensembles, Mathematics and Computers in Simulation 62 (2003b) 137-147.**
- **Salupere, G.A. Maugin, J. Engelbrecht, and J. Kalda, On the KdV soliton formation and discrete spectral analysis, Wave Motion 123 (1996) 49-66.**

M. Brocchini and R.Gentile. Modelling the run-up of significant wave groups. Continental Shelf Research, 2001, vol. 21, 1533-1550.

Statistical Characteristics of Solitons: No Interaction – Linear Approach

$$u(x,t) = \sum_{i=1}^{N} A_i \operatorname{sech}^2\left[K_i\left(x - 4K_i^2 t - x_i\right)\right]$$

$$A_i = 2K_i^2$$

- **Soliton amplitudes and phases are random and statistically independent**

- **Phases are uniformly distributed in domain -L/2 < x < L/2**

Moments of N-Soliton "Linear" Ensembles

$$< u > = \frac{4N}{L} < K > = \frac{2\sqrt{2}N}{L} < A^{1/2} >$$

$$\sigma \approx \sqrt{\frac{16N}{3L} < K^3 >} = \sqrt{\frac{8N}{3\sqrt{2}L} < A^{3/2} >}$$

$$Sk \approx \frac{2\sqrt{3}}{5}\sqrt{\frac{L}{N}}\frac{< K^5 >}{\left[< K^3 >\right]^{3/2}} = \frac{2^{3/4}\sqrt{3}}{5}\sqrt{\frac{L}{N}}\frac{< A^{5/2} >}{\left[< A^{3/2} >\right]^{3/2}}$$

$$Ku \approx \frac{18L}{35N}\frac{< K^7 >}{< K^6 >} = \frac{18\sqrt{2}}{35}\frac{L}{N}\frac{< A^{7/2} >}{< A^3 >}$$

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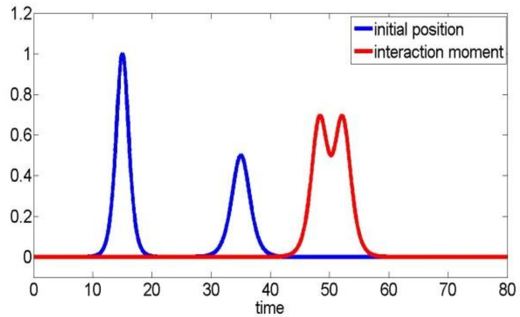
Two-soliton interaction as an elementary act of soliton turbulence in integrable systems

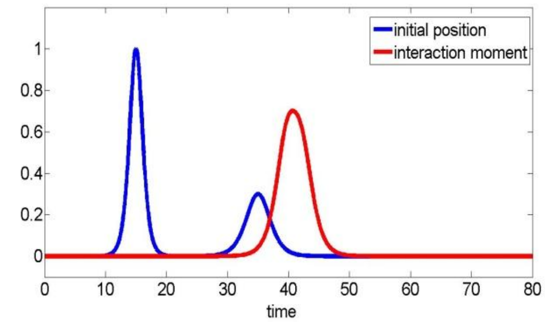
E.N. Pelinovsky [a,b], E.G. Shurgalina [b,c], A.V. Sergeeva [b,c], T.G. Talipova [b,c], G.A. El [d,*], R.H.J. Grimshaw [d]

$$\frac{A_1}{A_2} < 2.8$$

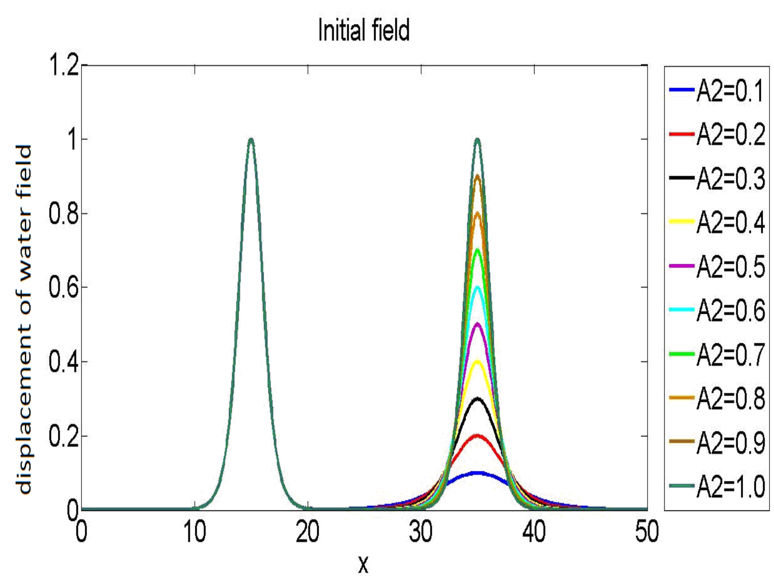


«Exchange»

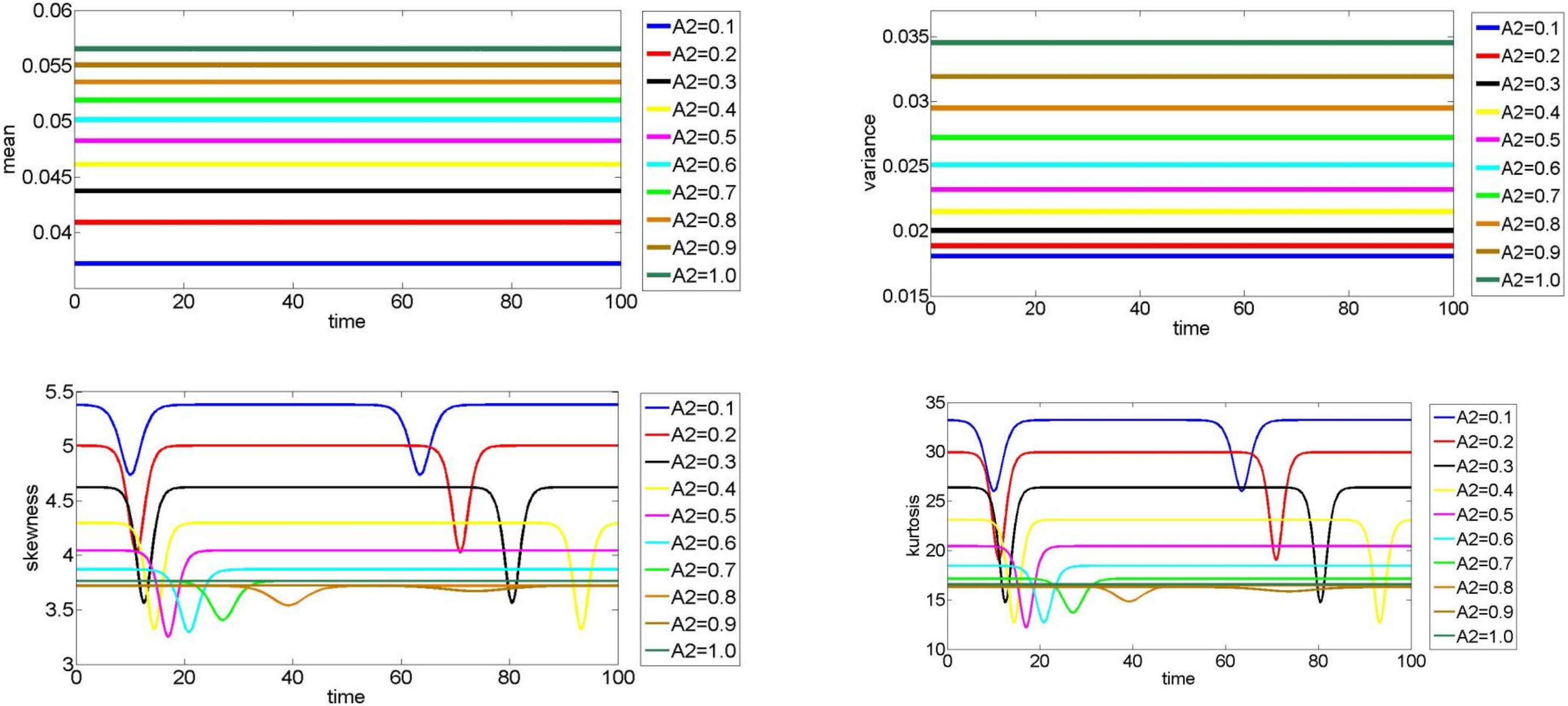
$$\frac{A_1}{A_2} > 2.8$$



«Overtake»



Analysis for various ratio of soliton amplitudes

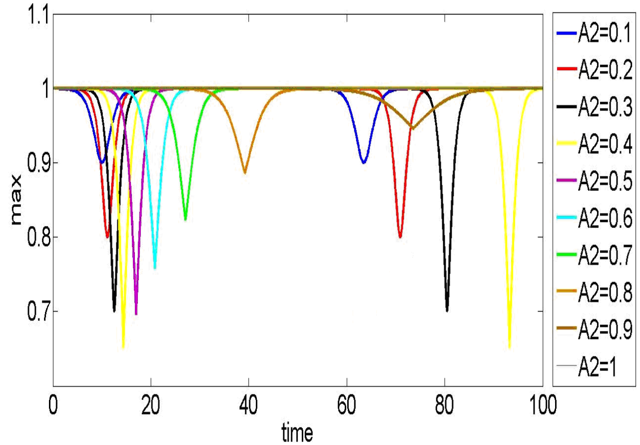


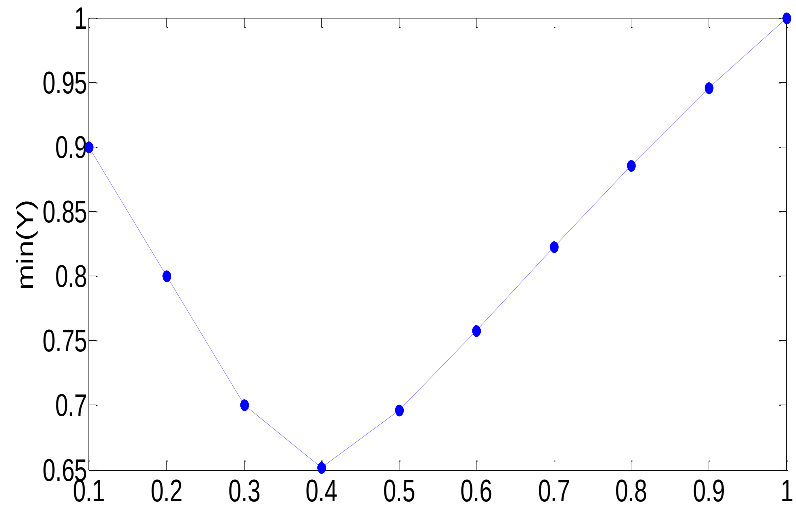
Two “first” moments (mean value and dispersion) are constant *(KdV invariants)*

“Skewness” and “Kurtosis” are reduced when solitons interact

Extreme characteristics versus A2/A1

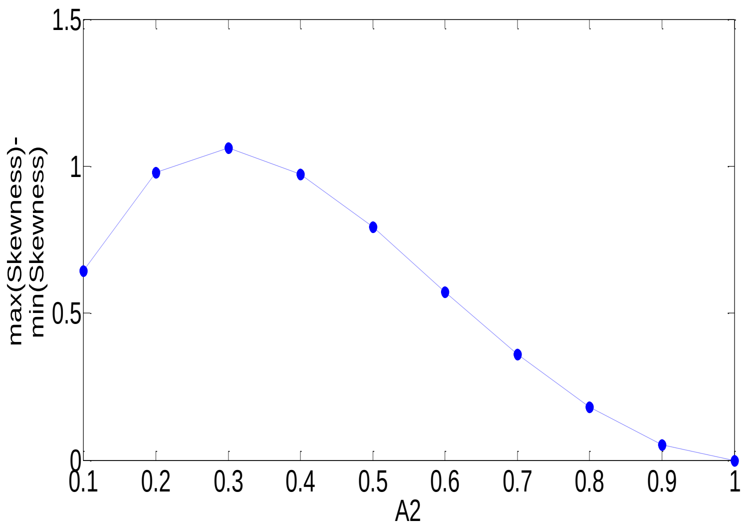
Wave amplitude at collision time



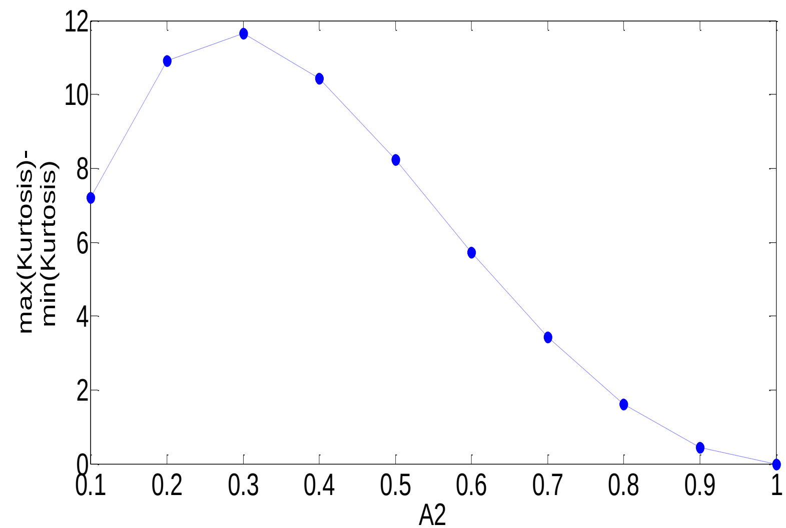


Not
the same point
A1/A2 = 2.8!

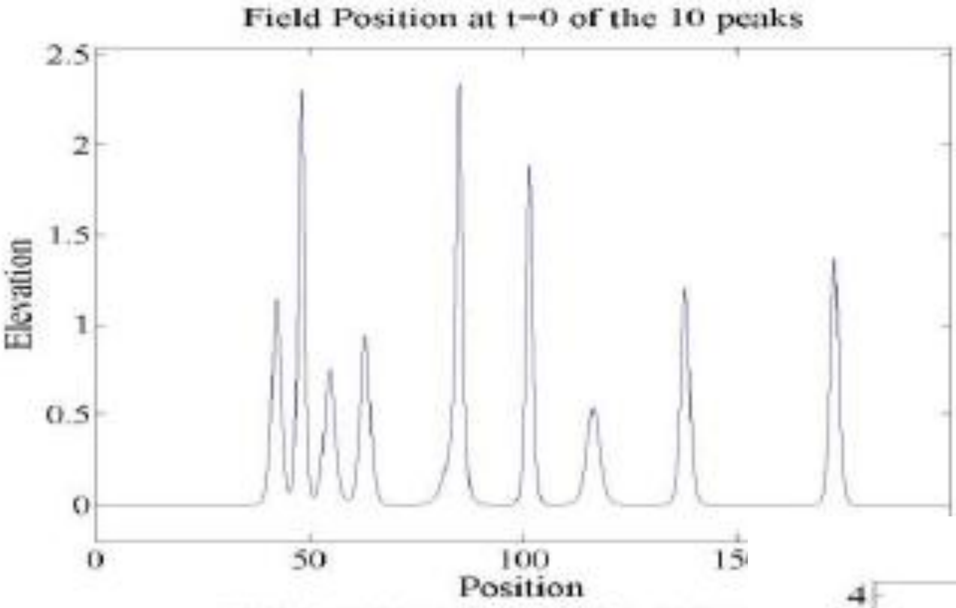
Skewness difference

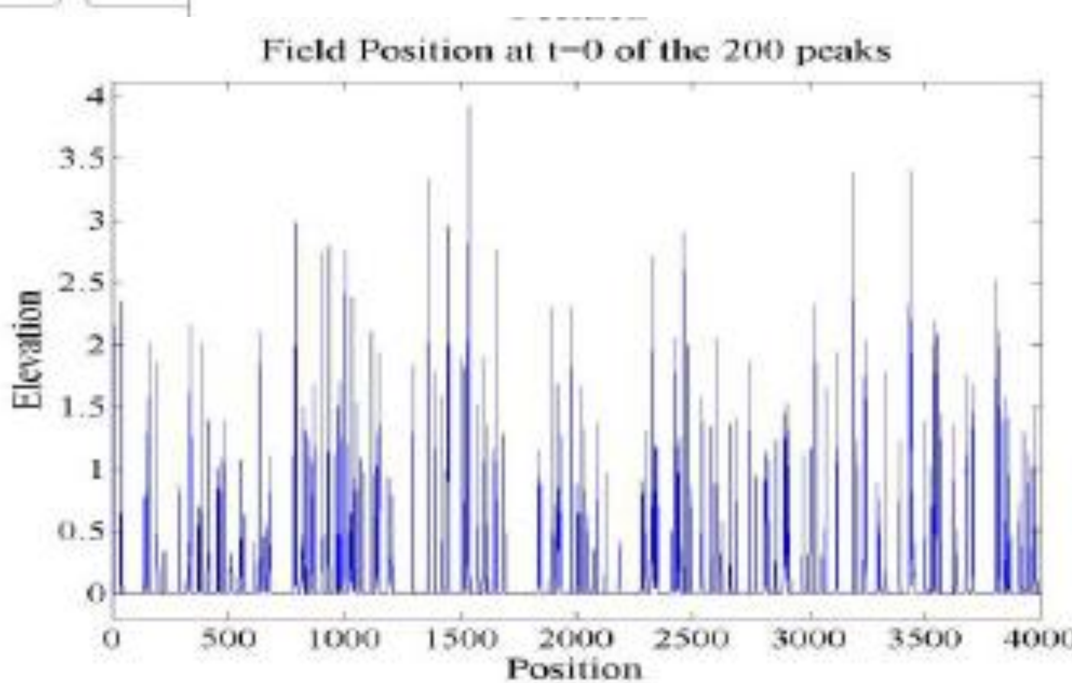


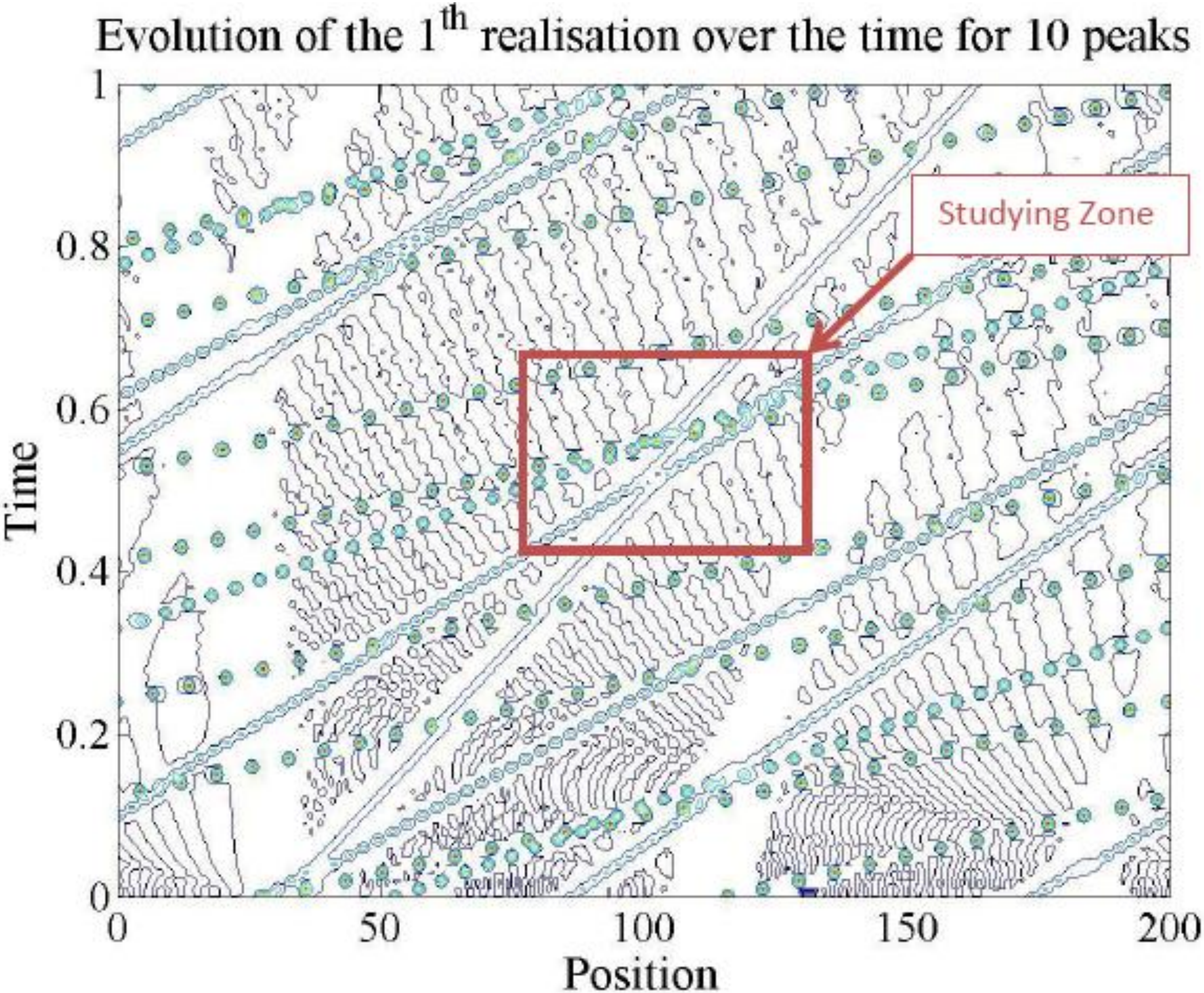
Kurtosis difference

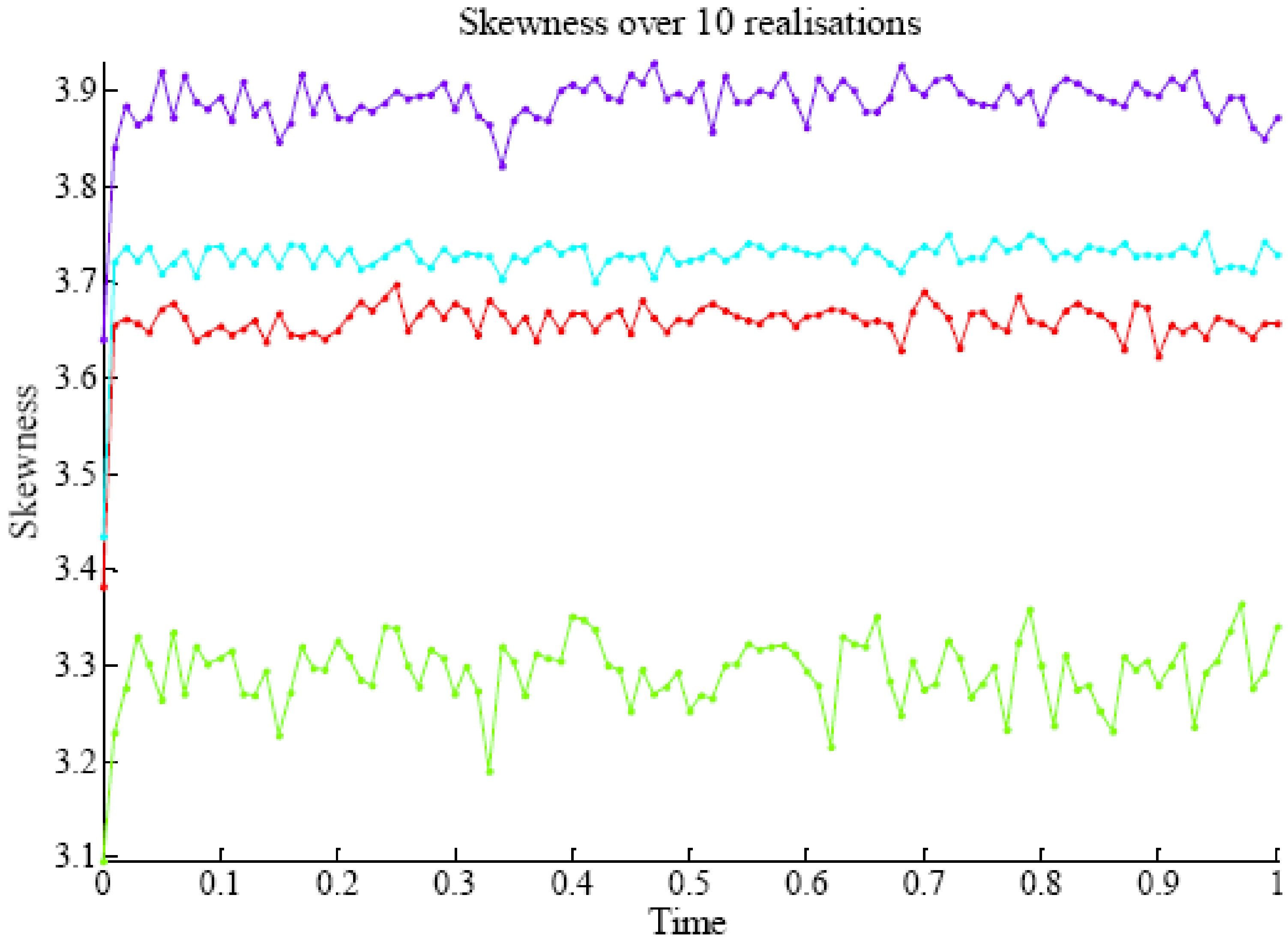


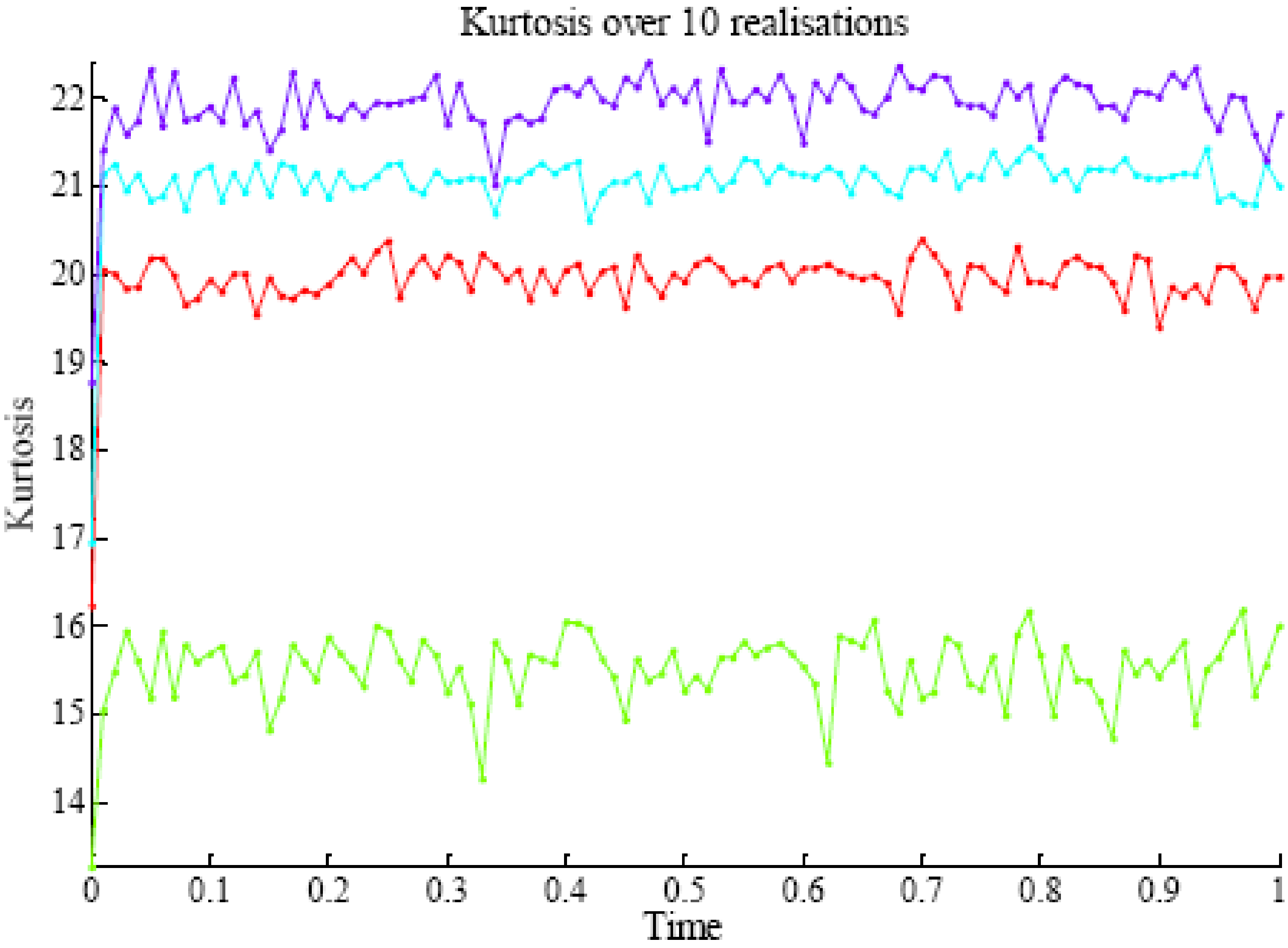
Numerical Simulation within KdV equation






Evolution of the 1th realisation over the time for 10 peaks
Studying Zone
Time
Position
0
0.2
0.4
0.6
0.8
1
0
50
100
150
200

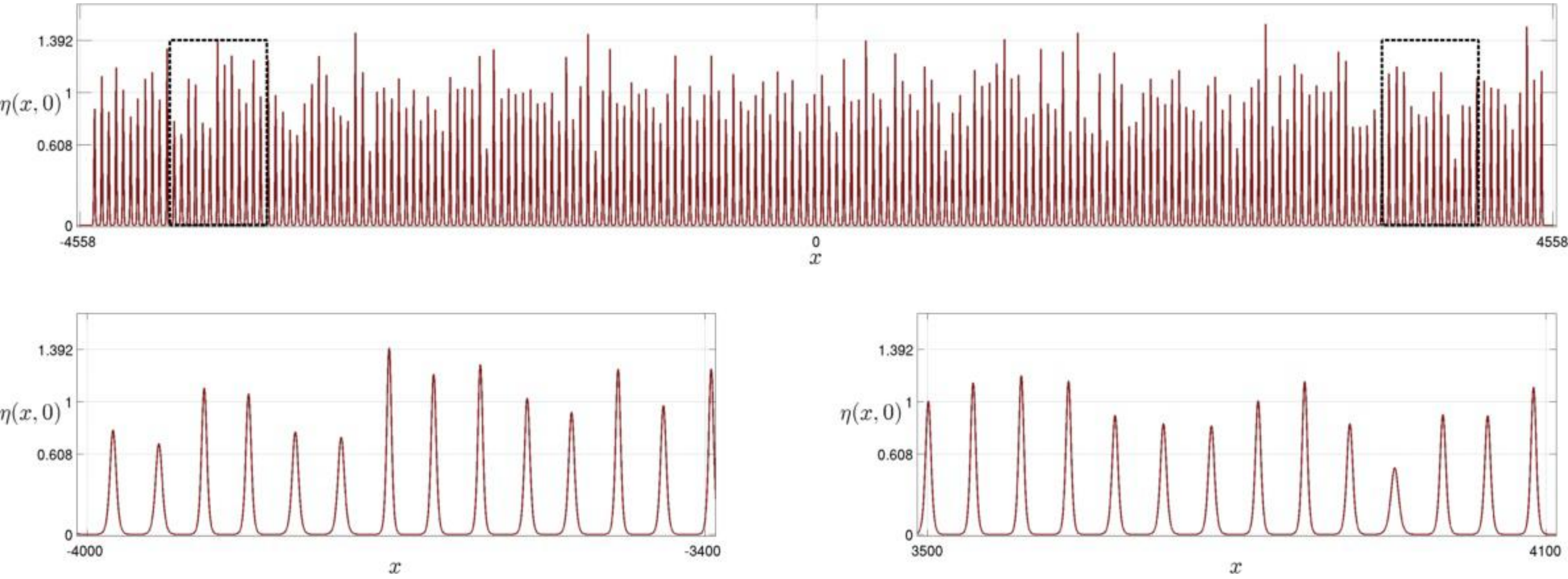

Skewness over 10 realisations
Skewness
3.1
3.2
3.3
3.4
3.5
3.6
3.7
3.8
3.9
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Time



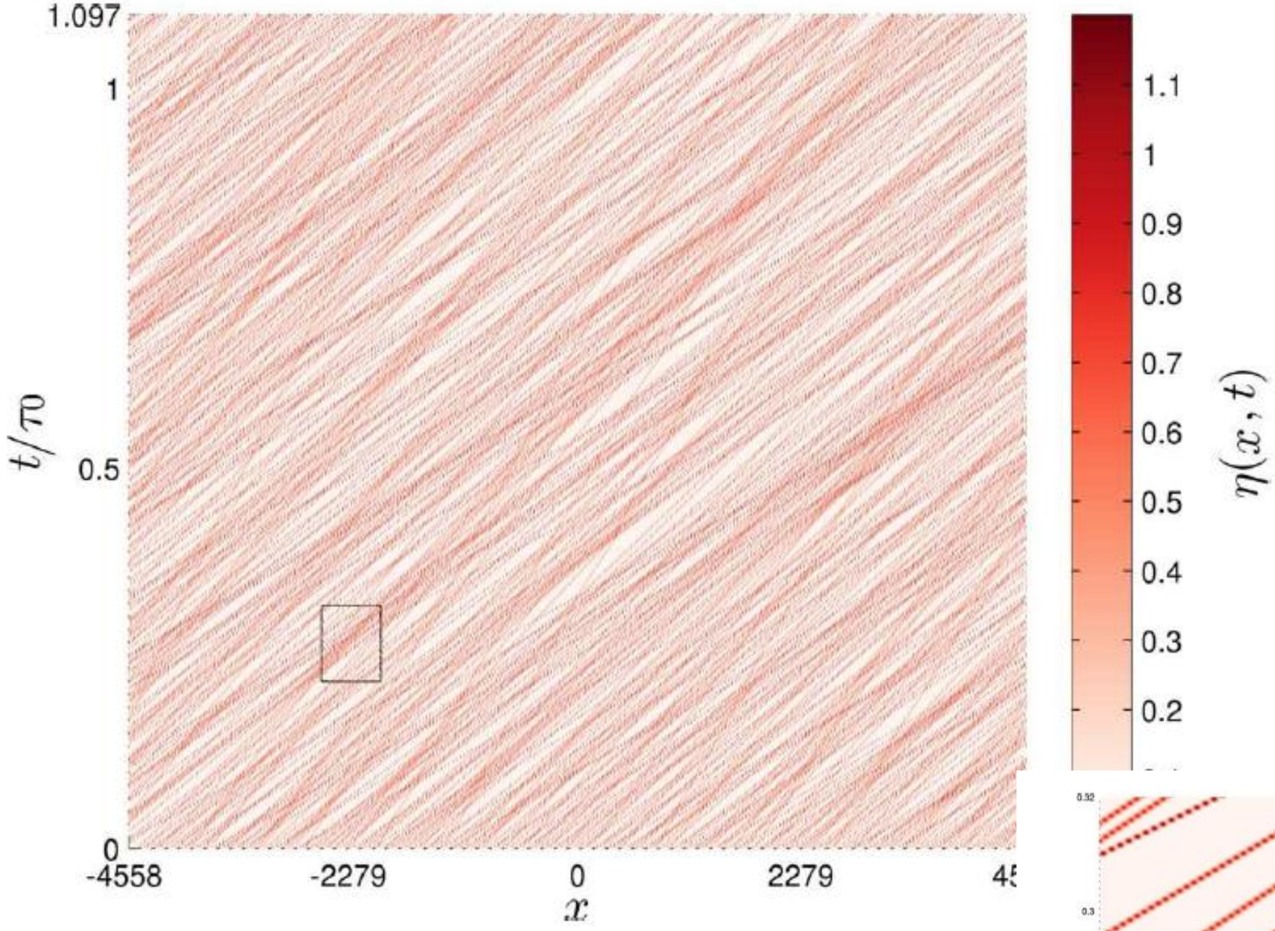
Kurtosis over 10 realisations
Kurtosis
Time
22
21
20
19
18
17
16
15
14
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

NUMERICAL SIMULATION OF A SOLITONIC GAS IN SOME INTEGRABLE AND NON-INTEGRABLE MODELS

DENYS DUTYKH AND EFIM PELINOVSKY*

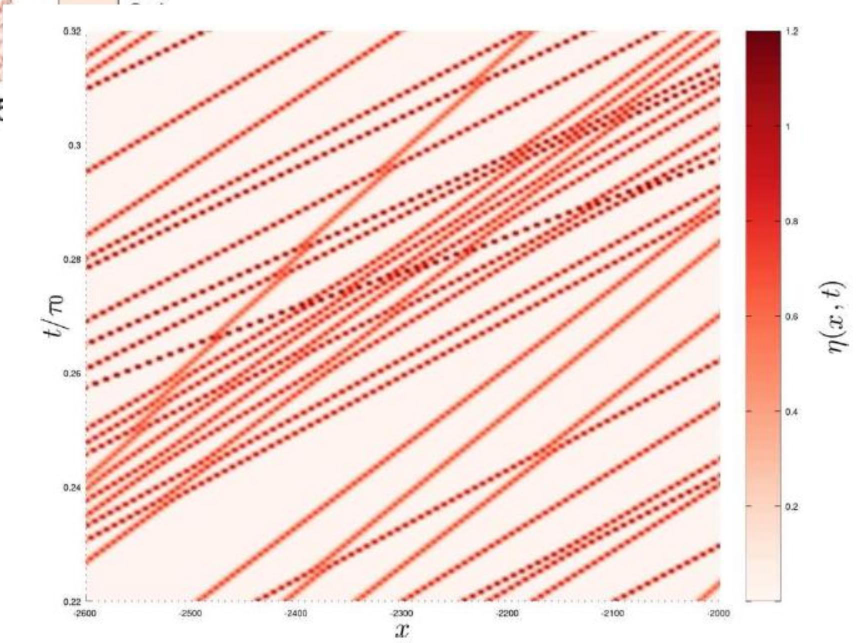


Invariants are conserved with accuracy of 10^{-11}

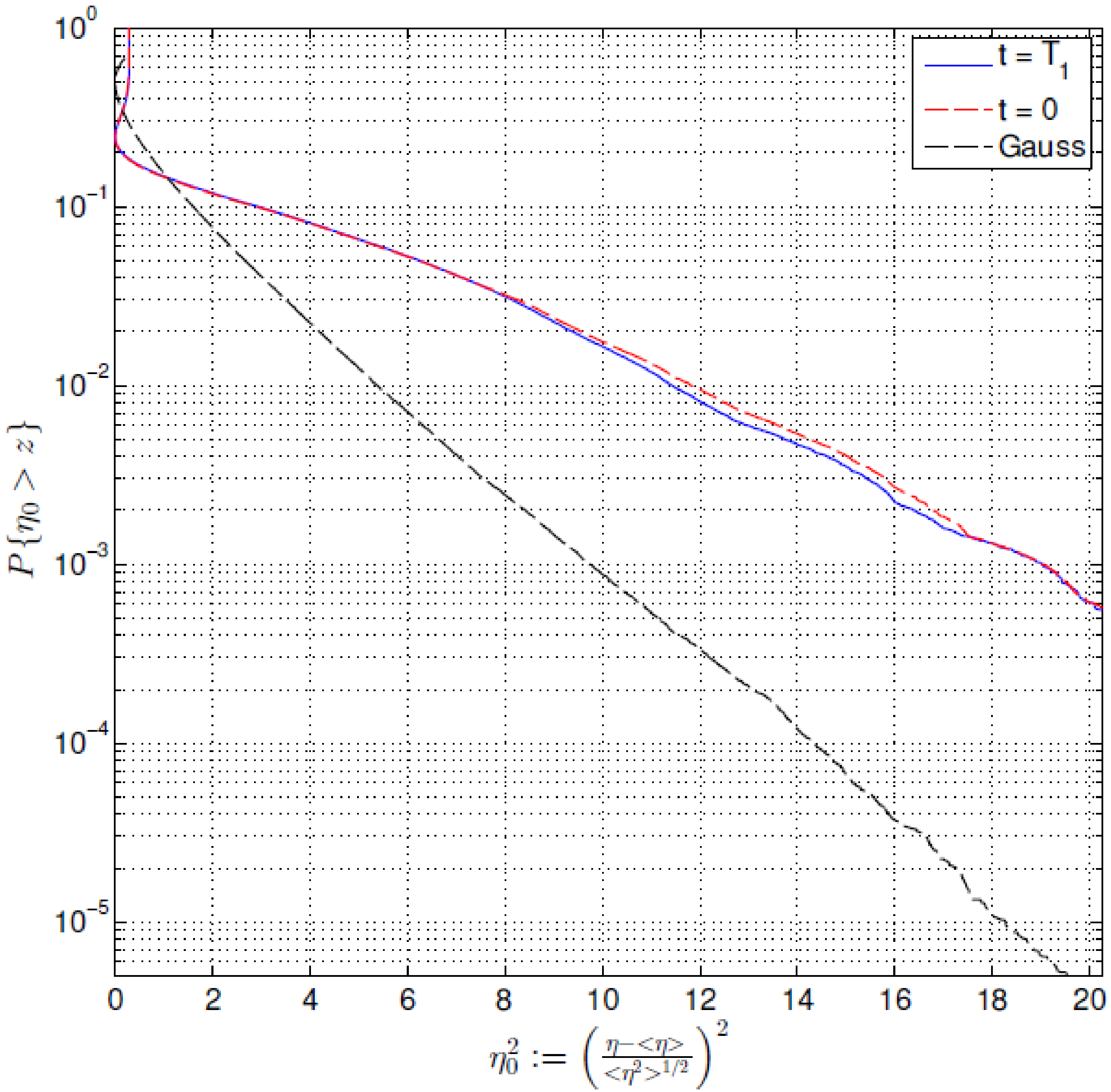


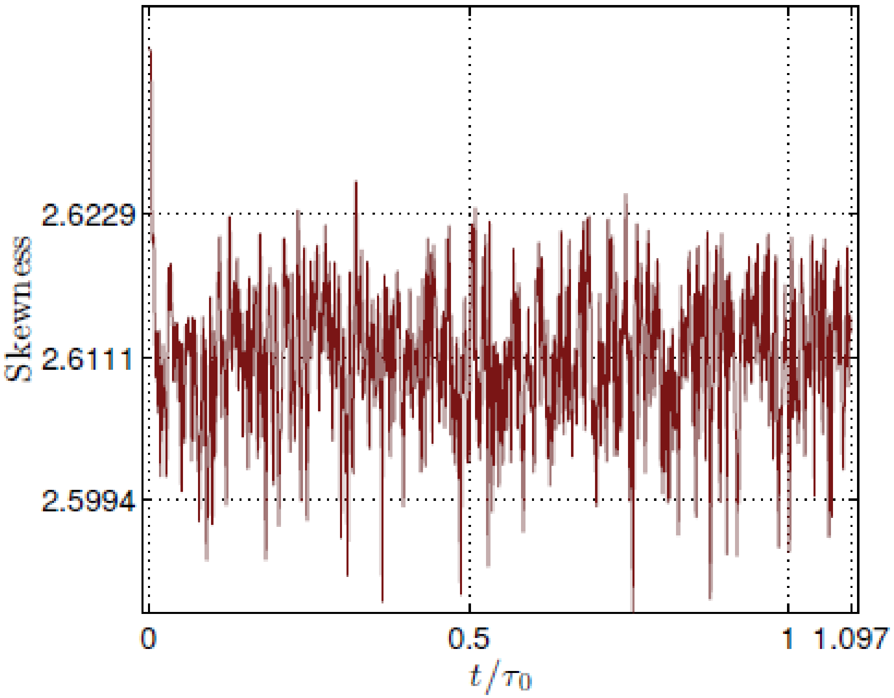
zoom

Trajectories

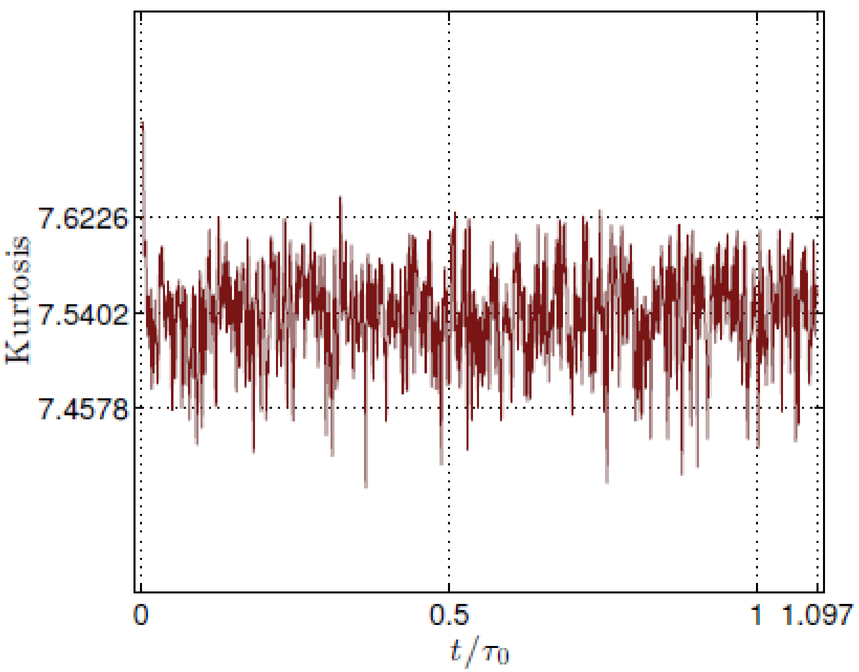


Exceedance Probability





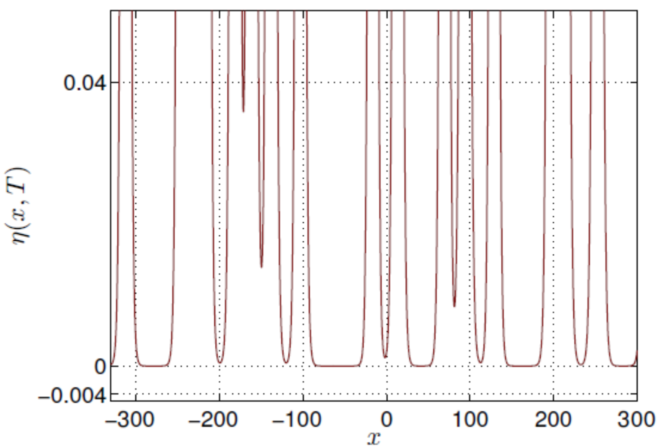
Interaction leads to decrease skewnes and kurtosis

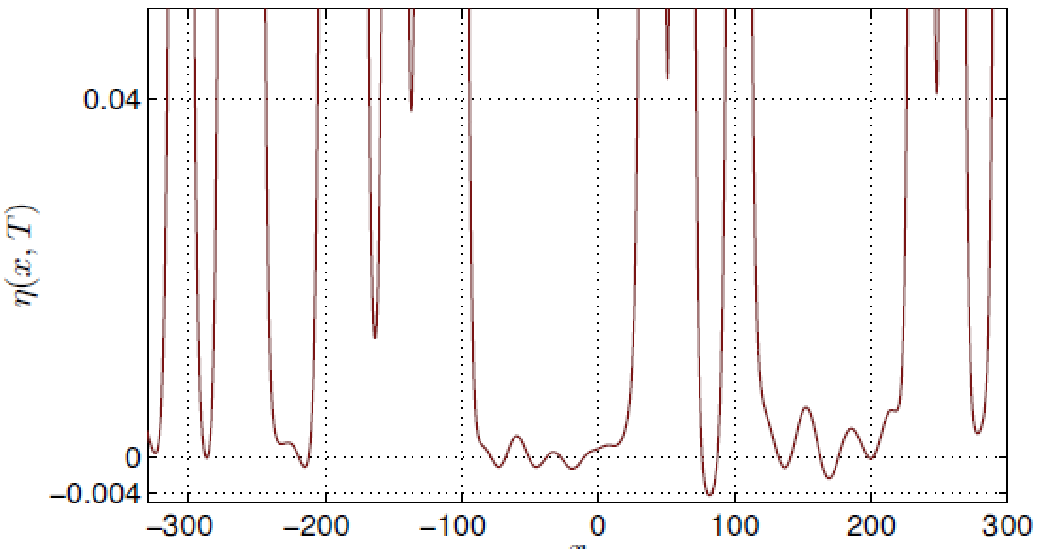


Not integrable BBM model

$$\eta_t + S\eta\eta_x + \eta_{xxx} - \delta\eta_{xxt} = 0$$

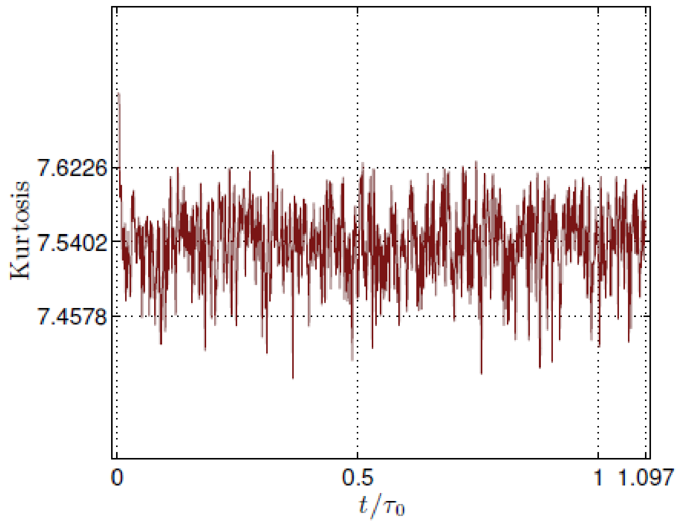
S=1
delta = 2



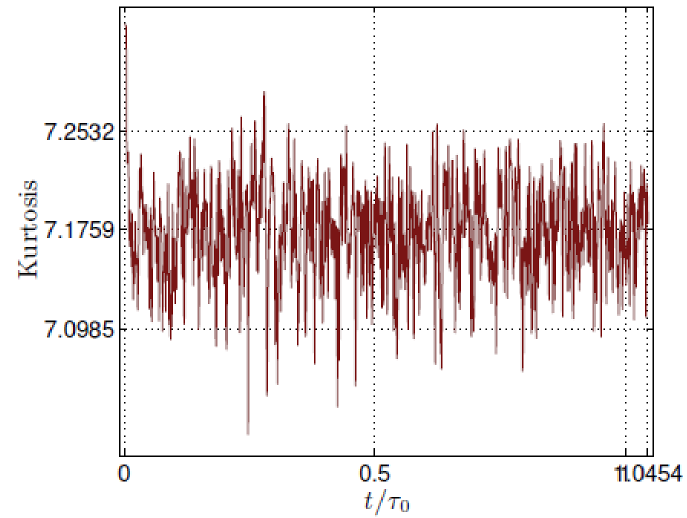


KdV
all values are positive

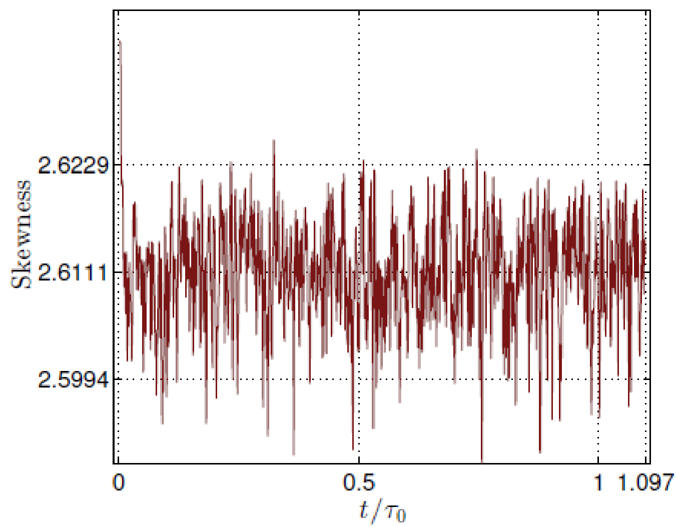
BBM
weak negative values



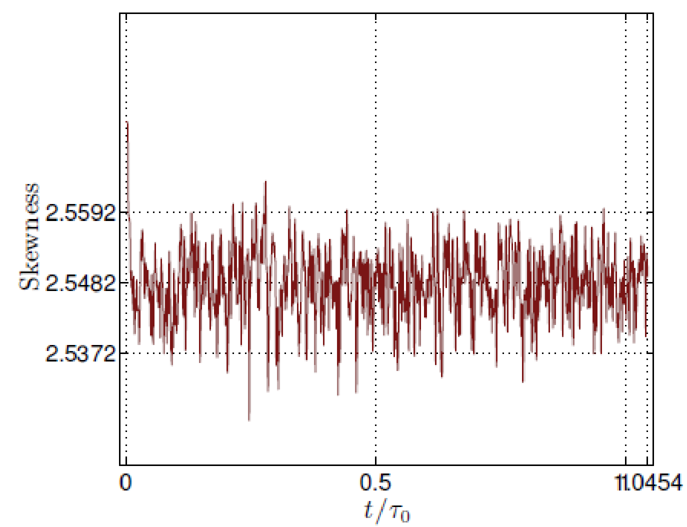
(a) Kurtosis — KdV



(b) Kurtosis — KdV–BBM



(c) Skewness — KdV



(d) Skewness — KdV–BBM

Weak difference in characteristics

Conclusions:

1. No rogue waves in unidirectional hyperbolic field
2. Rogue Waves in Interacted Riemann Waves
3. Positive skewness growing with Ur in KdV
4. Sign-Variable Kurtosis via Ur in KdV
5. Highest Probabilities for large Ur
6. Universal curves for 3d and 4th moments
7. Soliton interaction reduce moments
8. Soliton turbulence is not Gaussian process with small variations of moments

Recent References:

1. Didenkulova I., and Pelinovsky E. Rogue waves in nonlinear hyperbolic systems (shallow-water framework). *Nonlinearity*, 2011, **24**, R1-R18.
2. Slunyaev A., Didenkulova I., Pelinovsky E. Rogue waters. *Contemporary Physics*. 2011, **52**, No. 6, 571 – 590.
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4. Didenkulova, I., Pelinovsky, E., and Sergeeva, A. Statistical characteristics of long waves nearshore. *Coastal Engineering*, 2011, **58**, 94-102.
5. Denissenko P., Didenkulova I., Pelinovsky E., Pearson J. Influence of the nonlinearity on statistical characteristics of long wave runup. *Nonlinear Processes in Geophysics*, 2011, **18**, 967-975 (experiment).
6. Didenkulova I. Shapes of freak waves in the coastal zone of the Baltic Sea (Tallinn Bay). *Boreal Environment Research*, 2011, **16**, 138-149.
7. Nikolkina I., Didenkulova I. Rogue waves in 2006-2010. *Natural Hazards and Earth System Sciences*, 2011, **11**, 2913-2924.
8. Sergeeva, A., Pelinovsky, E., and Talipova, T. Nonlinear random wave field in shallow water: variable Korteweg – de Vries framework. *Natural Hazards and Earth System Science*, 2011, **11**, No. 1, 323-330.