Self-similarity of wind input terms

and

magic numbers

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Outline

1. Group scientific motivation

2. Self-similarity analysis of HE to find new wind input term

3. Different physical framework

4. Different input term check for nonlinear effects, such as "magic numbers" and ZF spectrum $\varepsilon(\omega) = C \cdot \omega^{-4}$

5. Show identity of limited fetch numerical results and experimental observations for specific choice of wind input

Motivation

1. Try to get rid of tuning knobs in the model

2. Properly simulate “primitive”, but realizable in nature situations, like deep water limited fetch with permanent wind

3. We we suspecting that the truth might be hidden in complex nonlinear properties of HE

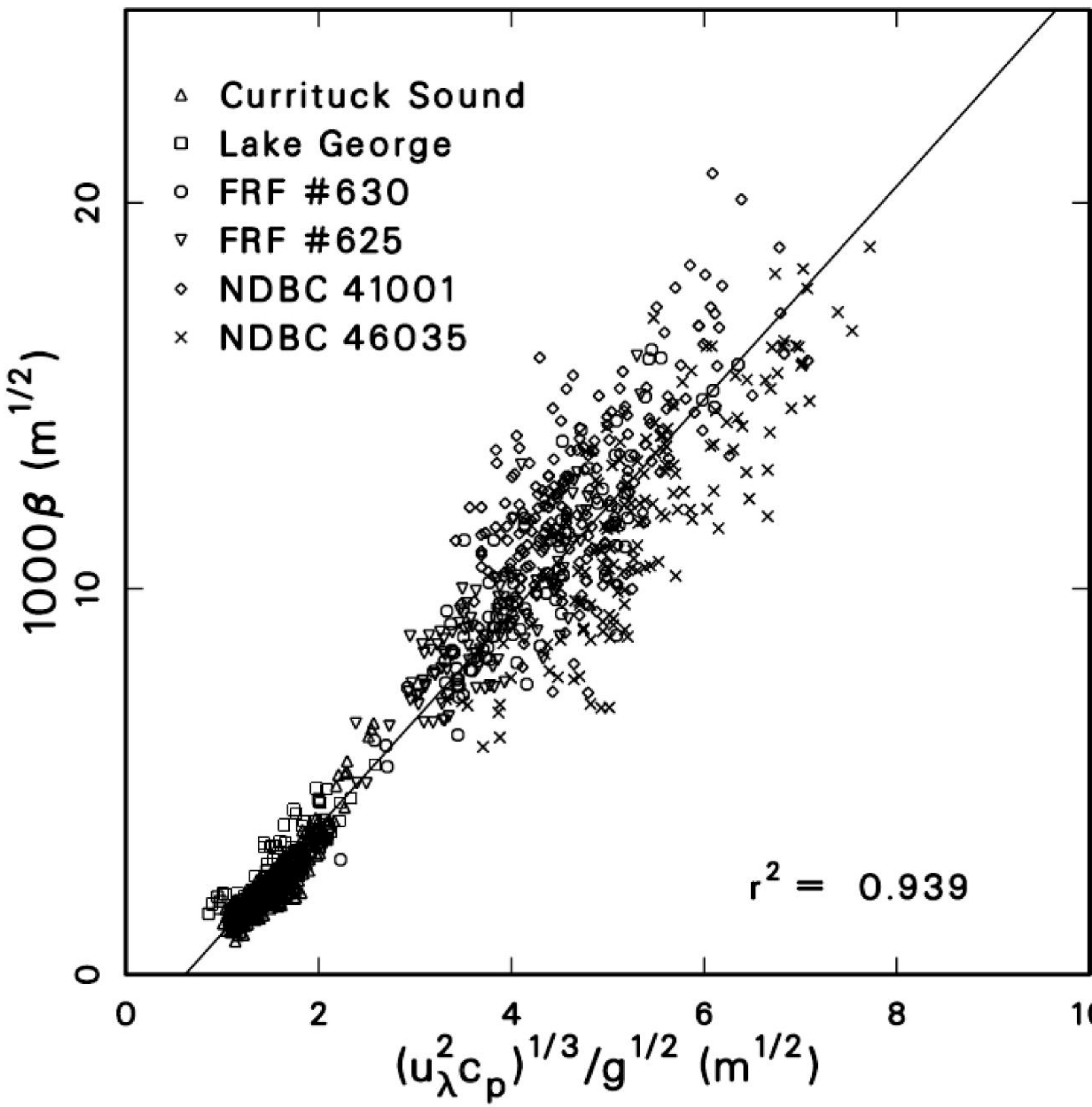
$$F(k)=\beta k^{-5/2}$$

$$\Updownarrow$$

$$\varepsilon(\omega)=C\cdot\omega^{-4}$$

Zakharov-Filonenko 1968

Resio and Long 2004-2007



Correlation of equilibrium range coefficient β *with* $(u_\lambda^2 c_p)^{1/3}/g^{1/2}$

Self-similarity analysis

Kinetic (Hasselmann) equation:

$$\frac{\partial \varepsilon}{\partial t} + \frac{\partial \omega_k}{\partial \vec{k}} \frac{\partial \varepsilon}{\partial \vec{r}} = S_{nl} + S_{in} + S_{diss}$$

Duration limited statement

$$\frac{\partial \varepsilon}{\partial t} = S_{nl} + S_{wind} + S_{diss}$$

Limited fetch statement

$$\frac{\partial \varepsilon}{\partial t} + \frac{g}{2\omega} \cos\theta \frac{\partial \varepsilon}{\partial x} = S_{nl} + S_{wind} + S_{diss}$$

$$\epsilon = \epsilon(\omega, \phi, t)$$

$$S_{nl} = \omega \left(\frac{\omega^5 \varepsilon}{g^2} \right)^2 \varepsilon$$

$$S_{wind} = \alpha \omega^{s+1} f(\phi) \epsilon$$

Duration limited self-similar solution

$$\varepsilon = t^{p+q} F(\omega t^{q}) \qquad\qquad 9q - 2p = 1$$

$$\text{If} \qquad S_{in} = \gamma \varepsilon \propto \omega^{s+1} \quad then$$

$$s = \frac{4}{3} \qquad \mathrm{q} = \frac{3}{7} \qquad p = \frac{10}{7}$$

published in

Zakharov, Resio, Pushkarev, arxiv, 2013

Fetch limited self-similar solution

$$\varepsilon = x^{p+q} F(\omega x^{q}) \qquad 10q - 2p = 1$$

$$\text{If} \quad S_{in} = \gamma \varepsilon \propto \omega^{s+1} \quad \text{then}$$

$$s = \frac{4}{3} \qquad \mathrm{q} = \frac{3}{10} \qquad p = 1$$

published in

Zakharov, Resio, Pushkarev, arxiv, 2013

ZRP model

$$\gamma = 0.05 \frac{\rho_{air}}{\rho_{water}} \omega \left(\frac{\omega}{\omega_0} \right)^{4/3} f(\theta)$$

$$f(\theta) = \begin{cases} \cos^2\theta \ \text{ for } -\pi/2 \le \theta \le \pi/2 \\ 0 \ \text{ otherwise} \end{cases}$$

$$\omega_0 = \frac{g}{u_{10}}, \qquad \frac{\rho_{air}}{\rho_{water}} = 1.3 \cdot 10^{-3}$$

Similar to ***Resio, Perrie, 1989***

$$\varepsilon = \varepsilon_0 \chi^{p}$$

$$\omega = \omega_0 \chi^{-q}$$

Different physical framework

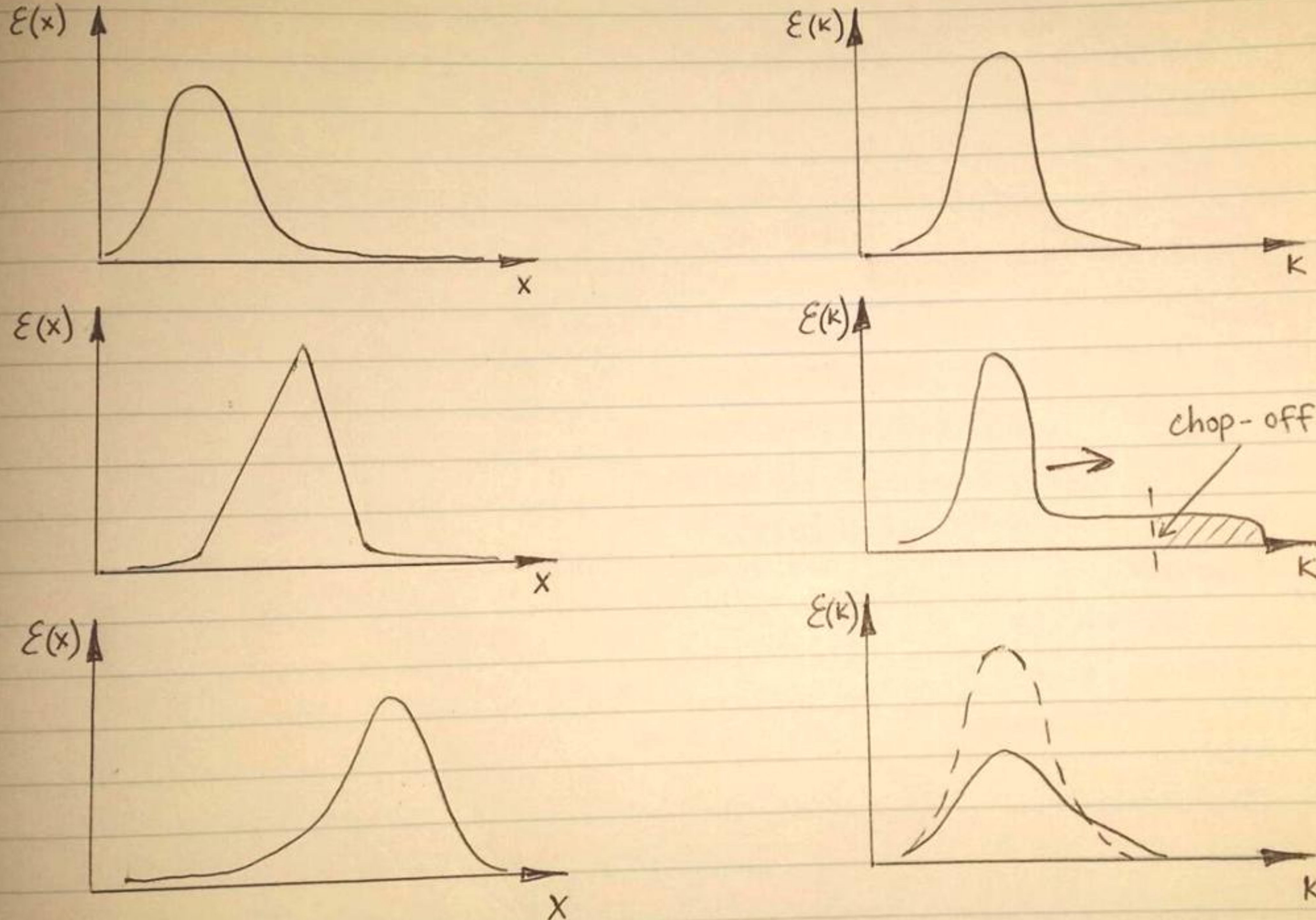
1. *No need for spectral maximum peak dissipation!*

2. *Only high-frequency* *implicit* $\sim f^{-5}$ *dissipation starting at* $f = 1.1\,\mathrm{Hz}$ *(Resio, Long 2007)*

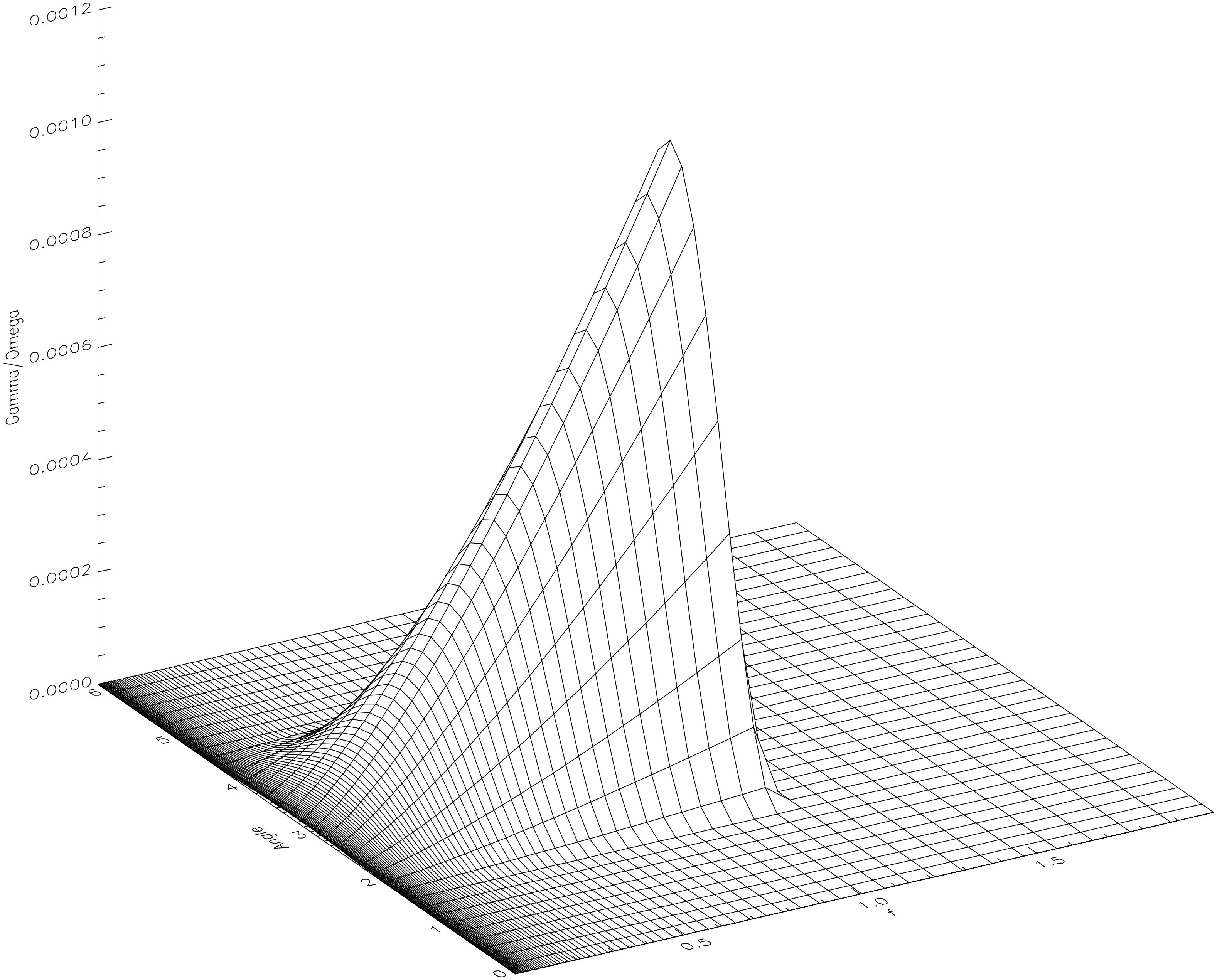
Why we don’t need spectral maximum dissipation?

Because HF dissipation works as a cigar cutter !

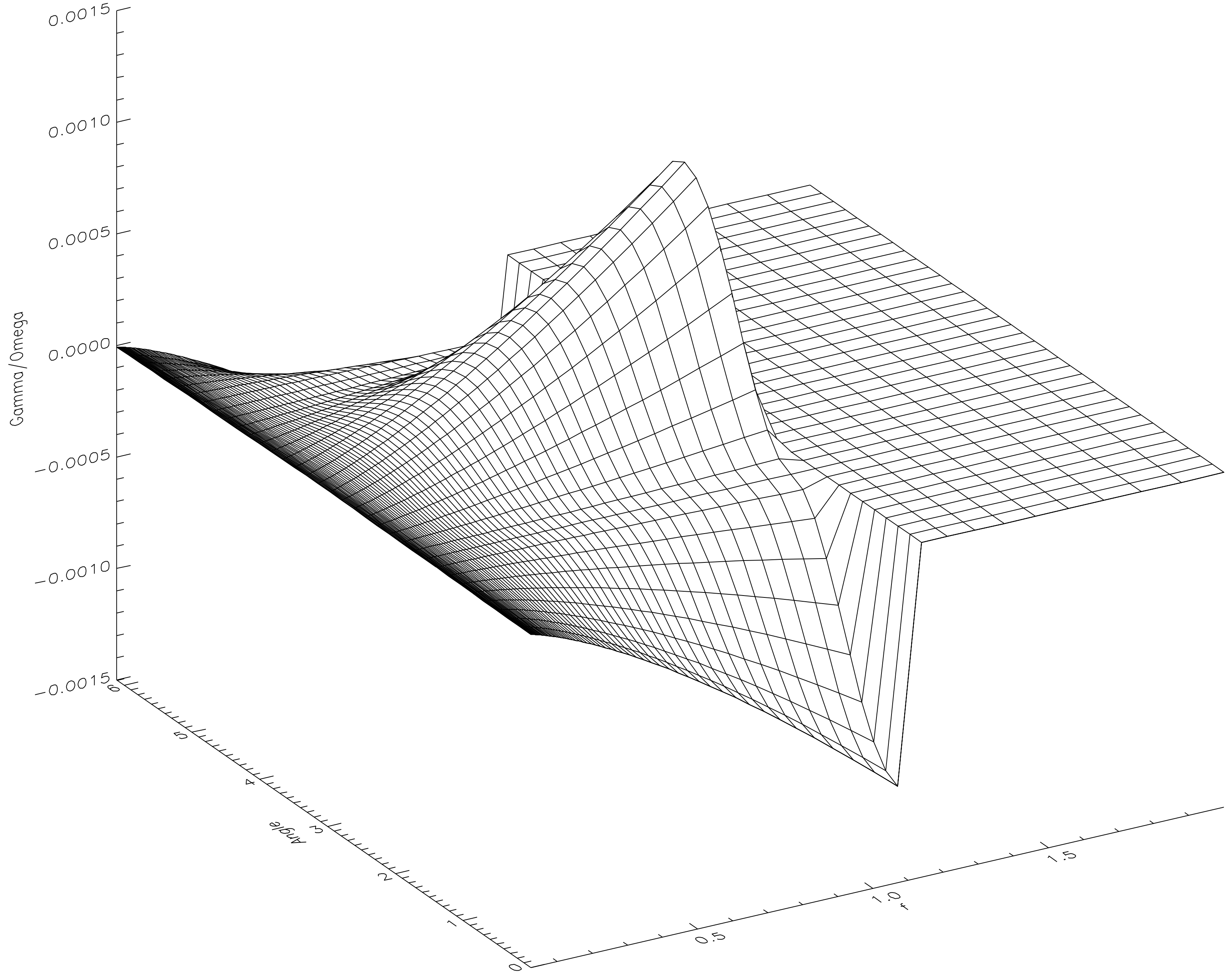
Real space Fourier space



- Radar "troubled" by breaking waves (sea spikes)
  - Review (of one: UMass FOPAIR)
  - NRL coherent radar analysis (tower)
  - NRL InSAR analysis (airborne)
- Key results
  - Breaking velocity: 2.7±0.7 m/s (U_{10}<~15m/s, c_p similar range)
  - Swell influence
    - Decreases/broadens breaking lengthscale
    - Decreases breaking threshold
- Really, really short scale waves: breaking and roughness spectrum



Normalized wind input function as a function of frequency and angle in polar coordinates.

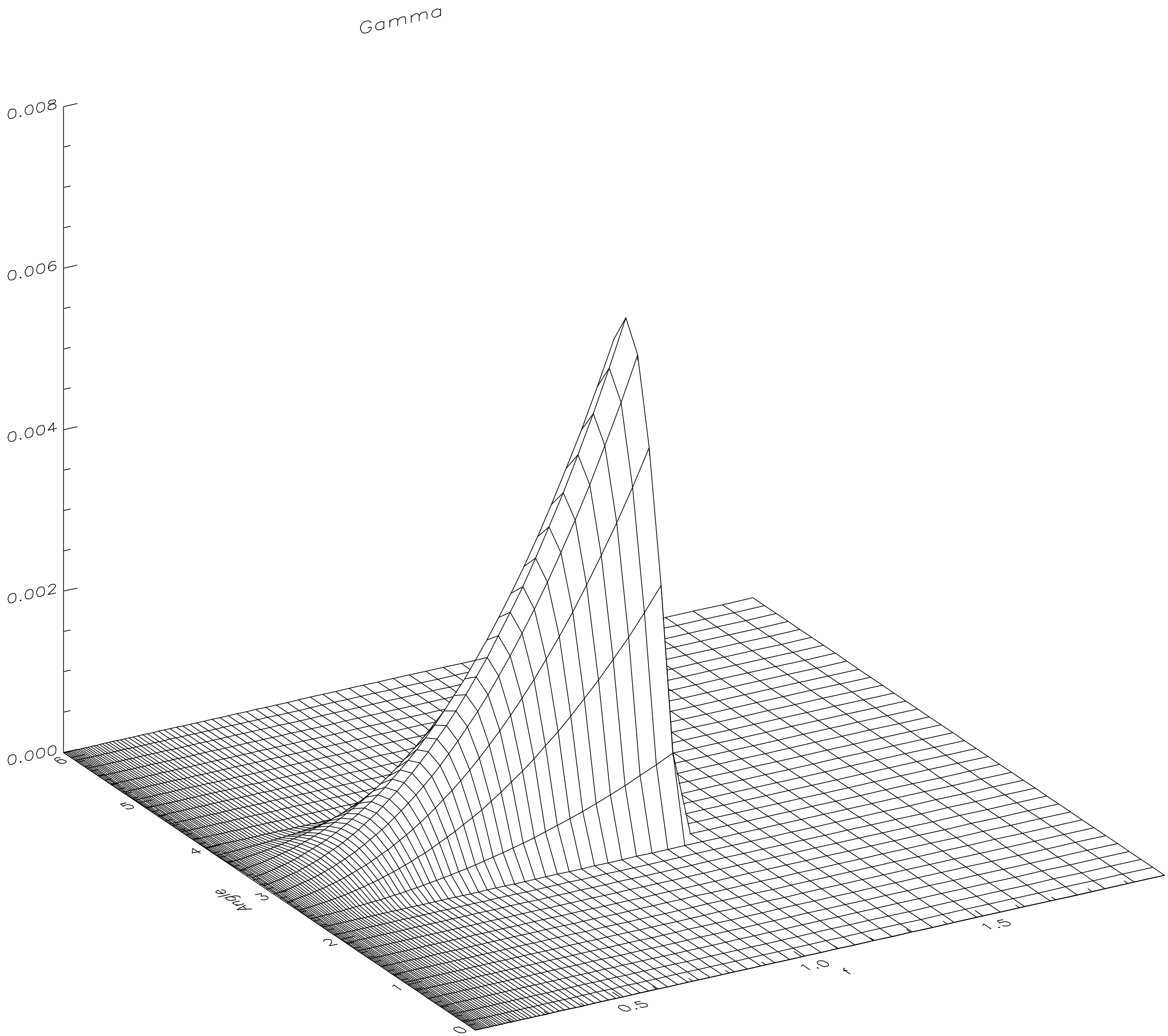


Alternative normalized wind input function similar to Tolman and Chalikov, 1996

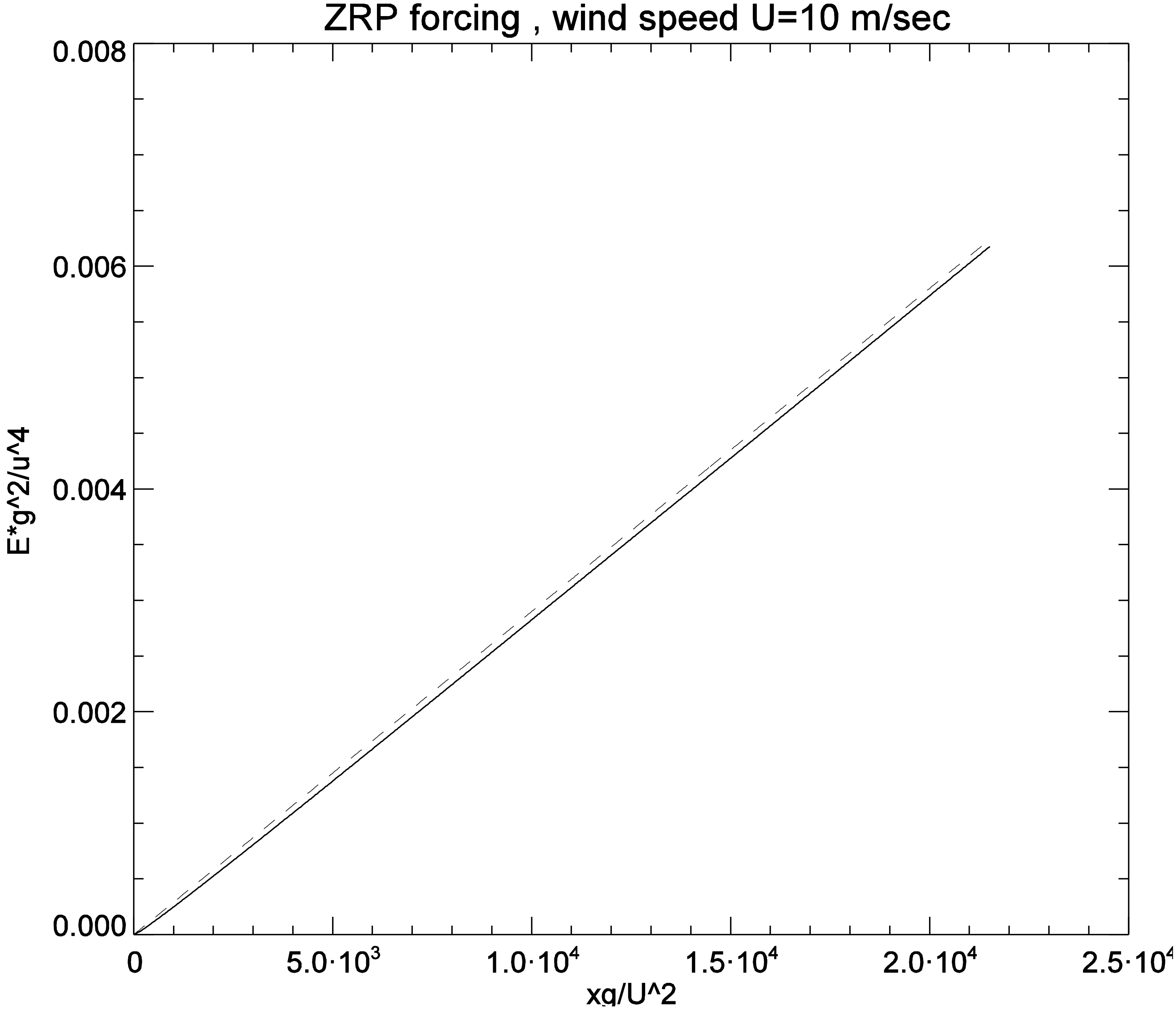
Different wind input terms check against nonlinear evidence

ZRP model

Dissipation

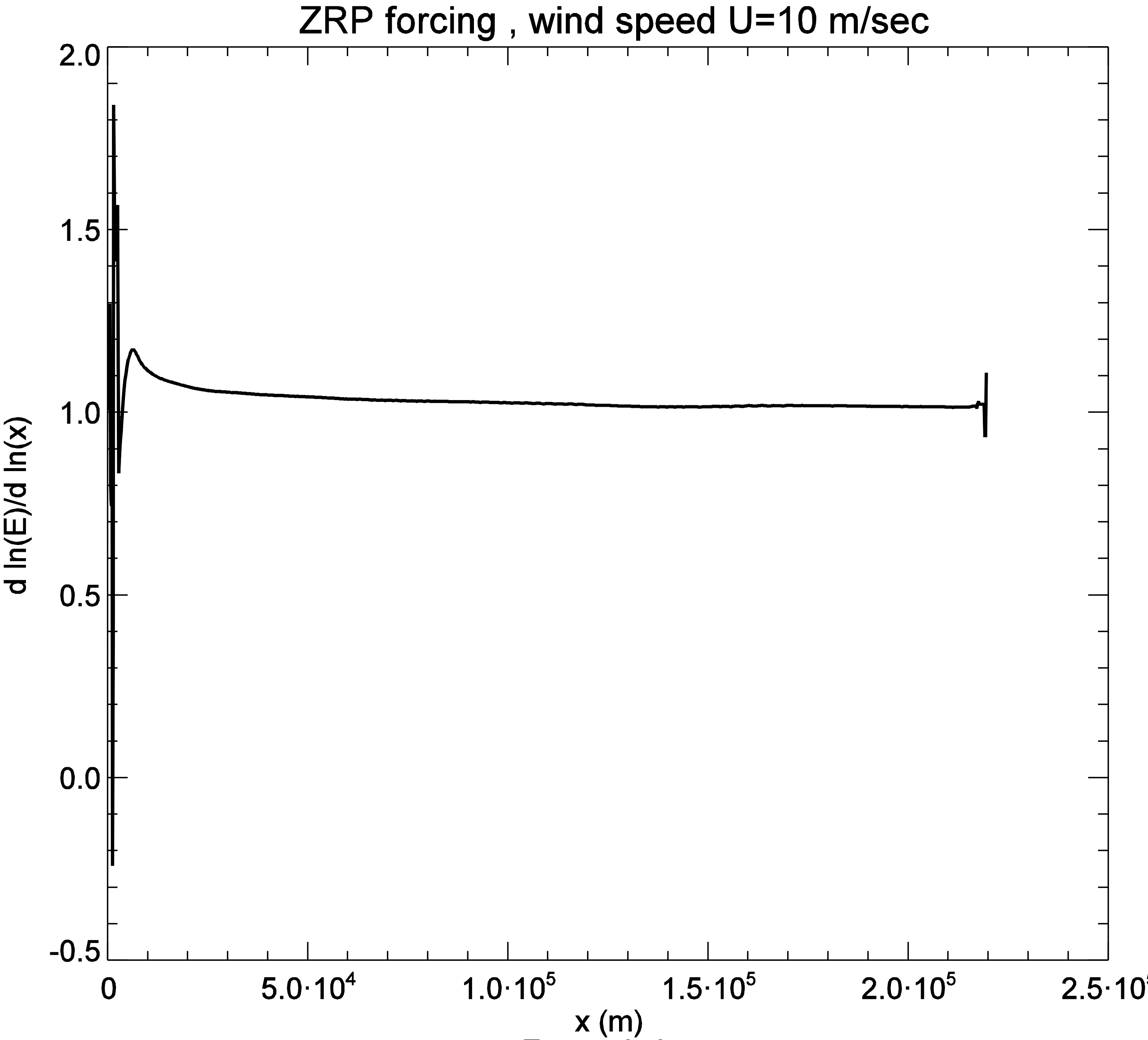


Dimensionless energy



Solid line - numerical experiment; dashed line - fit by $2.9 \cdot 10^{-7} \cdot \dfrac{xg}{U^2}$

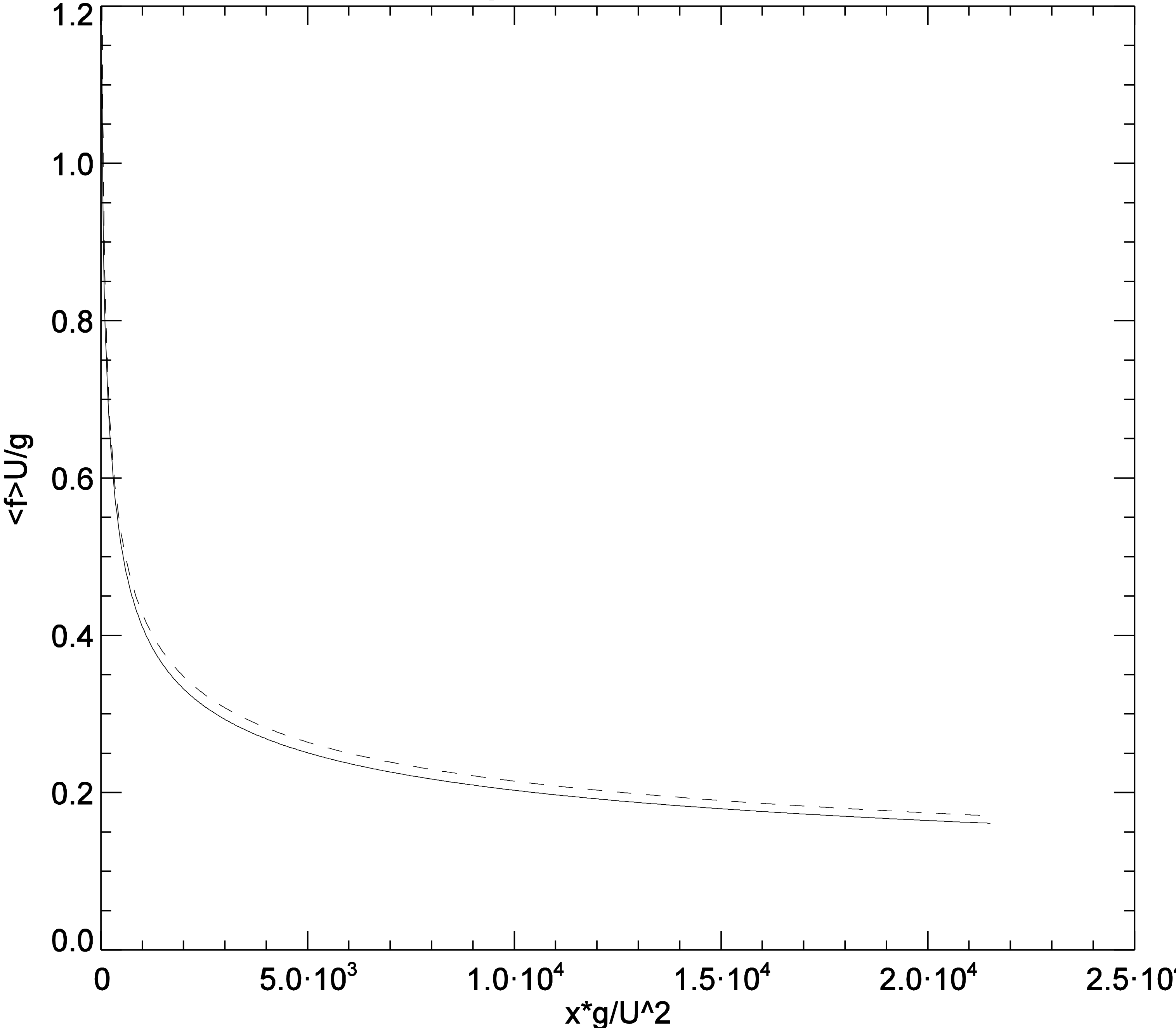
Energy index



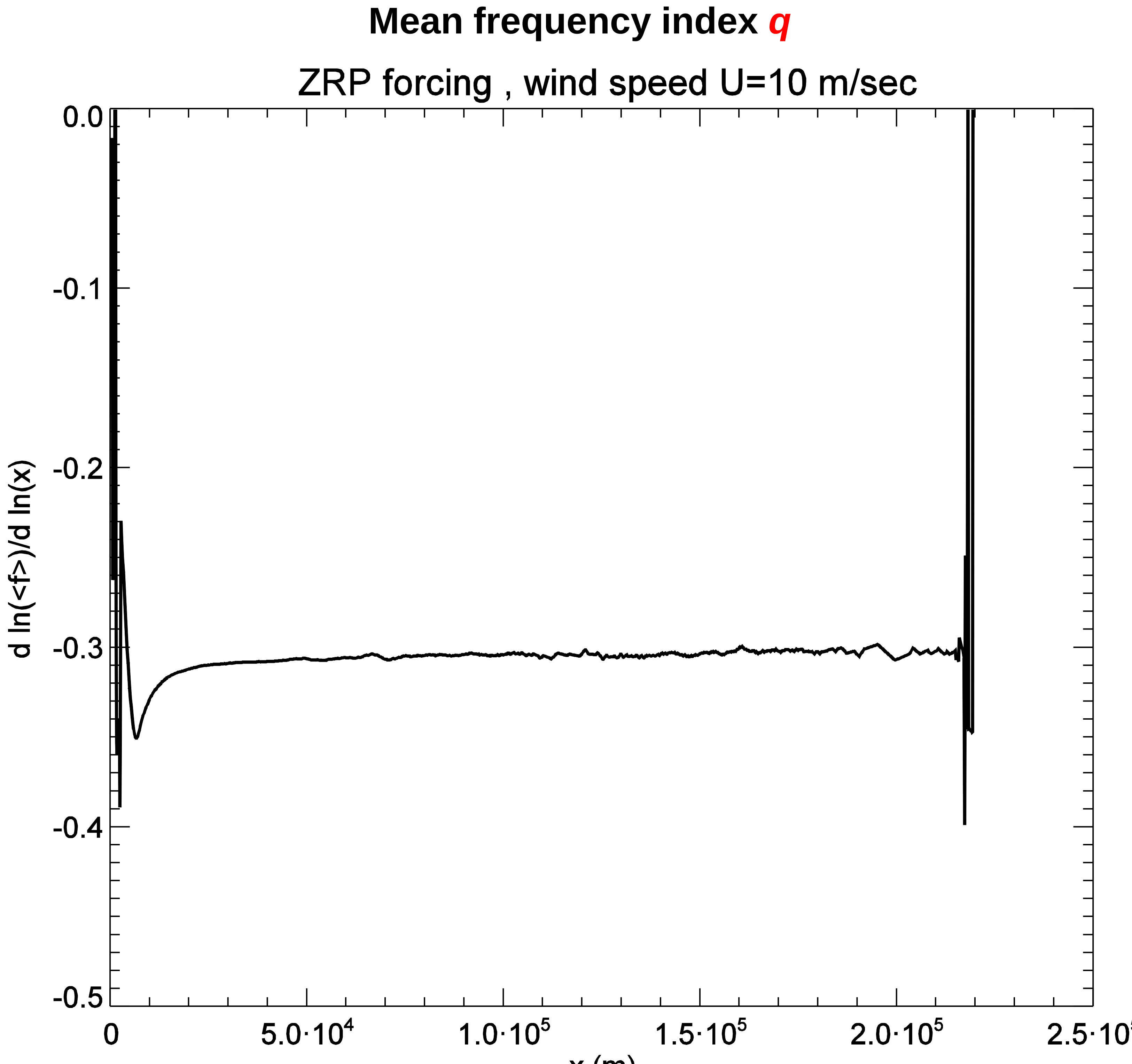
Energy index

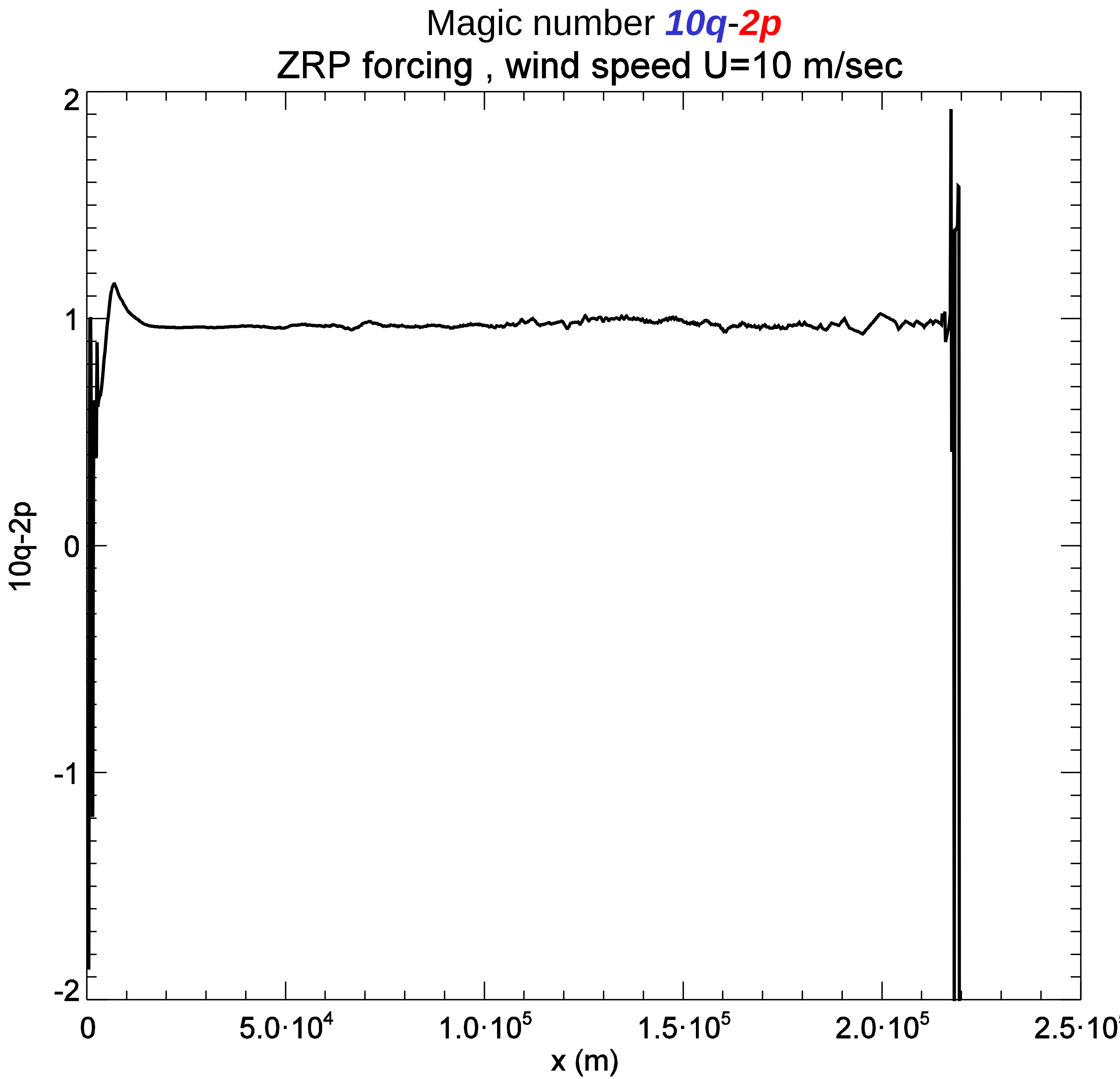
Dimensionless frequency

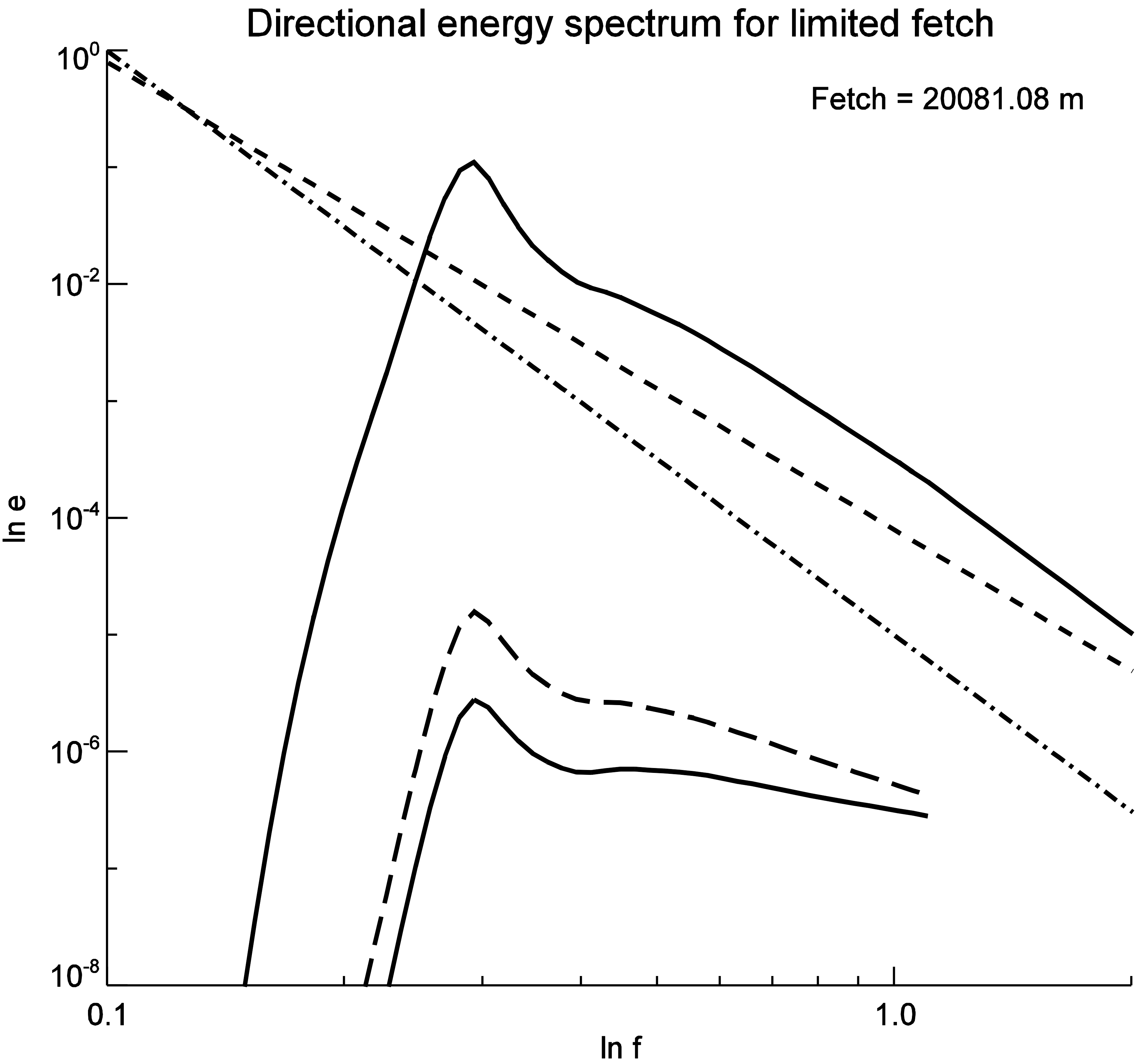
ZRP forcing , wind speed U=10 m/sec



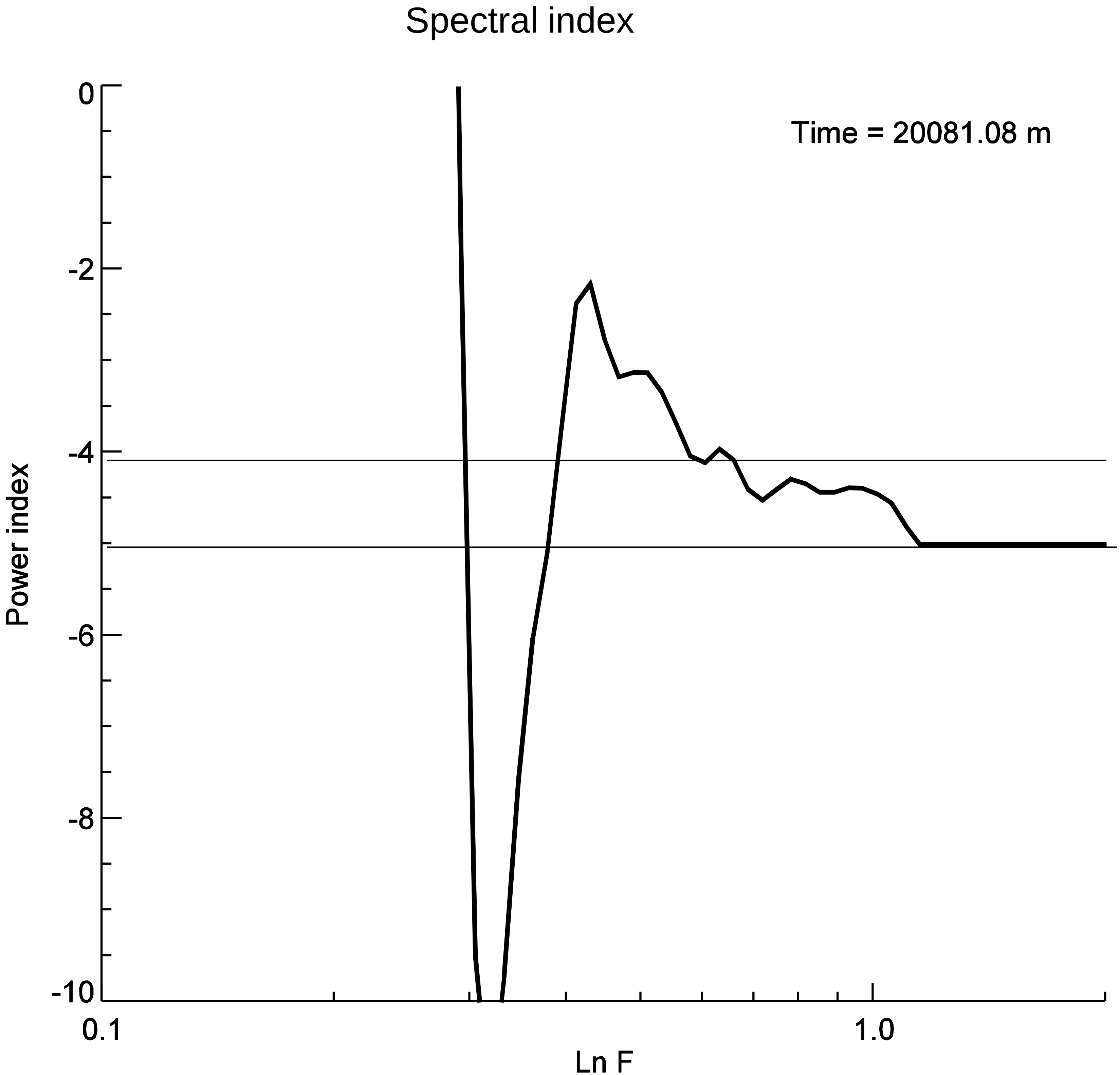
Solid line - numerical experiment; dashed line - fit by $21.36\cdot\dfrac{xg}{U^2}$


Mean frequency index q
ZRP forcing , wind speed U=10 m/sec
d ln(<f>)/d ln(x)
0.0
-0.1
-0.2
-0.3
-0.4
-0.5
0
5.0·10^4
1.0·10^5
1.5·10^5
2.0·10^5
2.5·10^5
x (m)


Magic number 10q-2p
ZRP forcing , wind speed U=10 m/sec
2
1
0
-1
-2
10q-2p
0
5.0·10^4
1.0·10^5
1.5·10^5
2.0·10^5
2.5·10^5
x (m)



Directional energy spectrum for limited fetch
Fetch = 20081.08 m
10^0
10^-2
10^-4
10^-6
10^-8
ln e
0.1
1.0
ln f



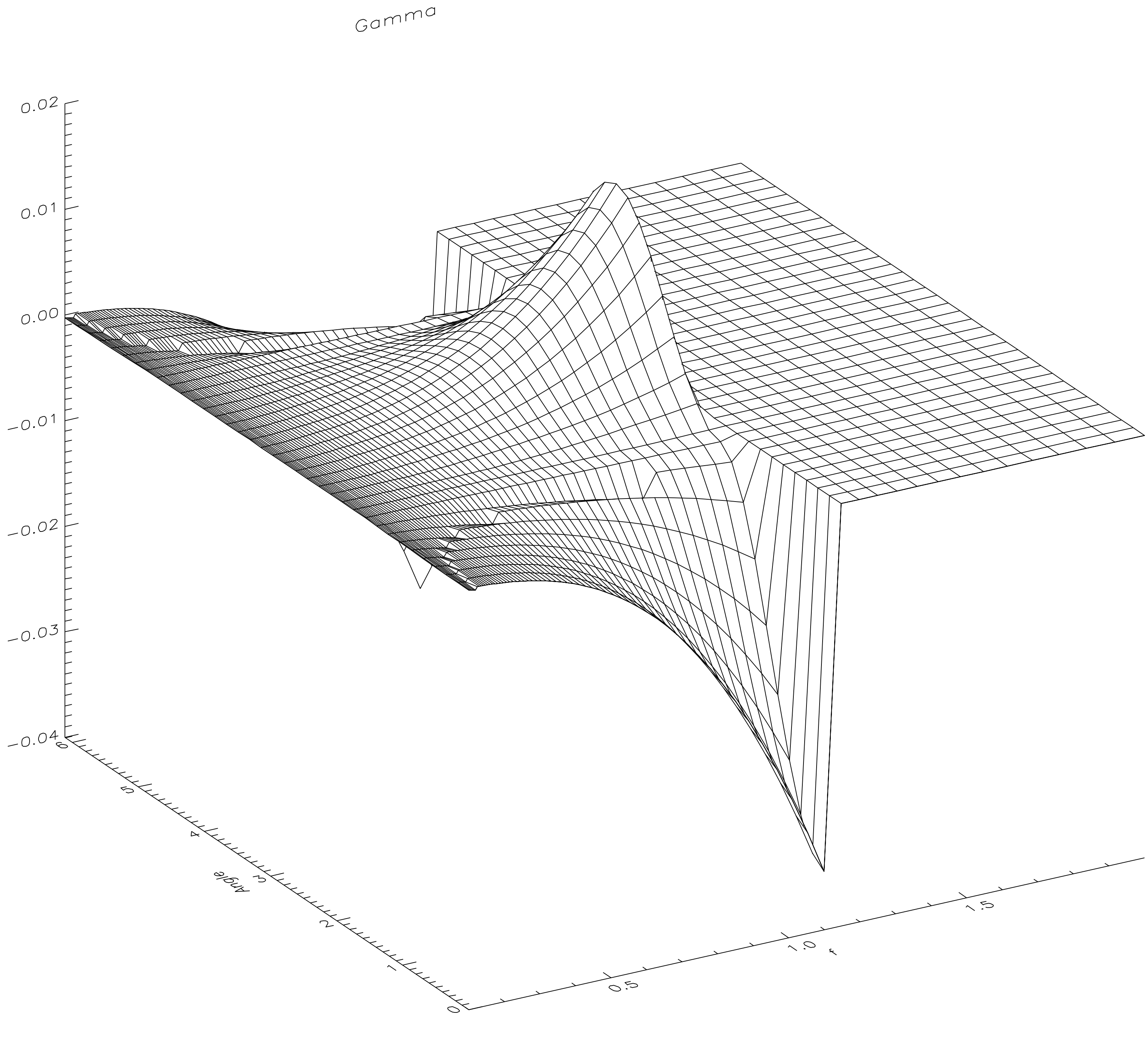
Spectral index
Time = 20081.08 m
Power index
0
-2
-4
-6
-8
-10
0.1
1.0
Ln F

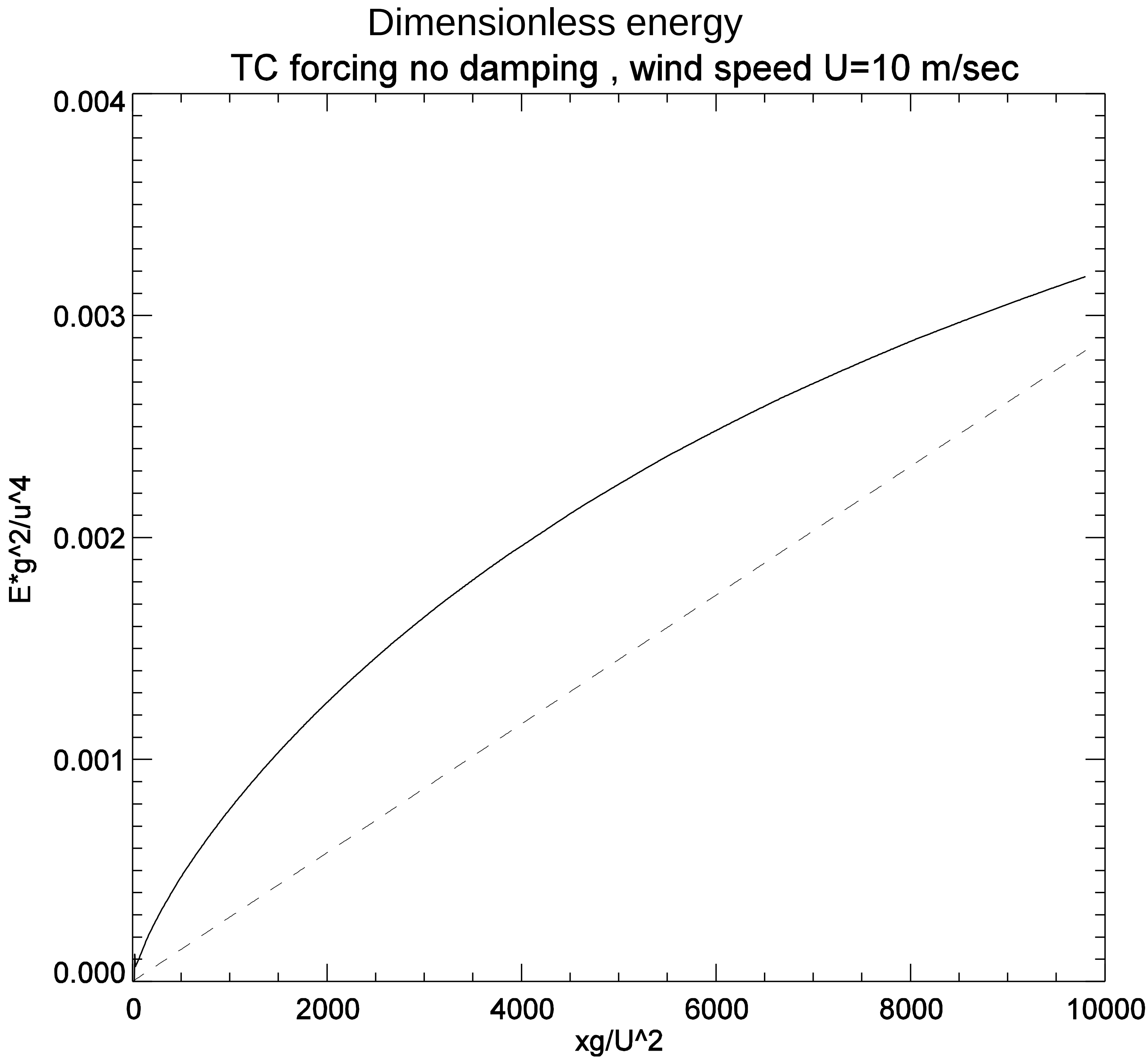
Experiment	$\tilde{\varepsilon}_0 \cdot 10^7$	p	$\tilde{\omega}_0$	q
Black Sea (Babanin & Soloviev 1998b)	4.41	0.89	15.14	0.275
Walsh et al. (1989) US coast	1.86	1.0	14.45	0.29
Kahma & Calkoen (1992) unstable	5.4	0.94	14.2	0.28
Kahma & Calkoen (1992) stable	9.3	0.76	12.0	0.24
Kahma & Pettersson (1994)	5.3	0.93	12.66	0.28
JONSWAP by Davidan (1980)	4.363	1.0	16.02	0.28
JONSWAP by Phillips (1977)	2.6	1.0	11.18	0.25
Kahma & Calkoen (1992) composite	5.2	0.9	13.7	0.27
Kahma (1981, 1986) rapid growth	3.6	1.0	20	0.33
Kahma (1986) average growth	2.0	1.0	22	0.33
Donelan *et al.* (1992) St Claire	1.7	1.0	22.62	0.33
Ross (1978), Atlantic, stable	1.2	1.1	11.94	0.27
Liu & Ross (1980), Michigan, unstable	0.68	1.1	12.88	0.27
JONSWAP (Hasselmann *et al.* 1973)	1.6	1.0	21.99	0.33
Mitsuyasu *et al.* (1971)	2.89	1.008	19.72	0.33
ZRP numerics	**2.9**	**1.0**	**21.36**	**0.3**

Exponents and pre-exponents of wind-wave growth in fetch-limited experiments.

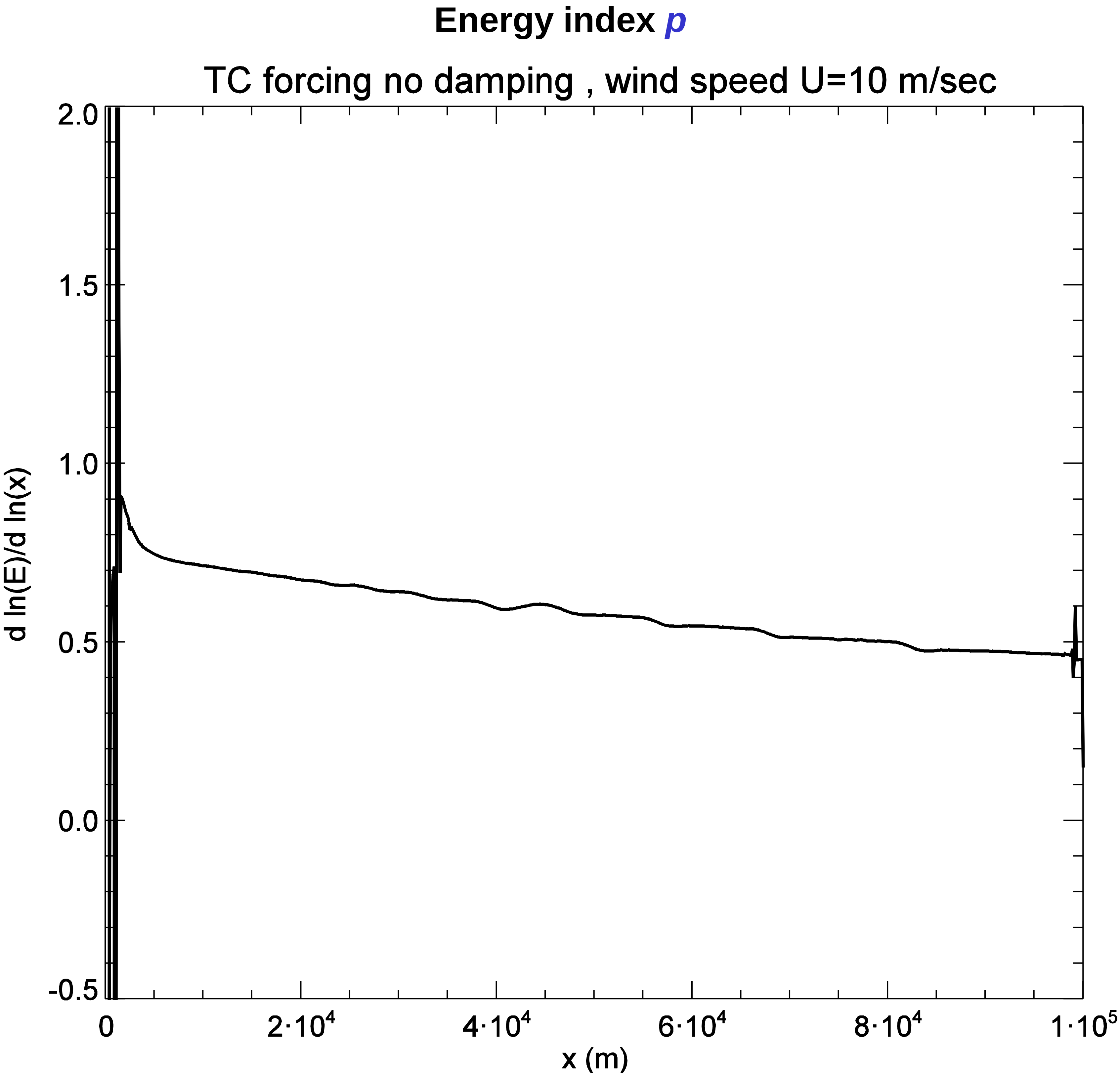
Adopted from ***Badulin, Babanin, Zakharov*, *Resio* 2007**

Chalikov-Tolman model (no damping)

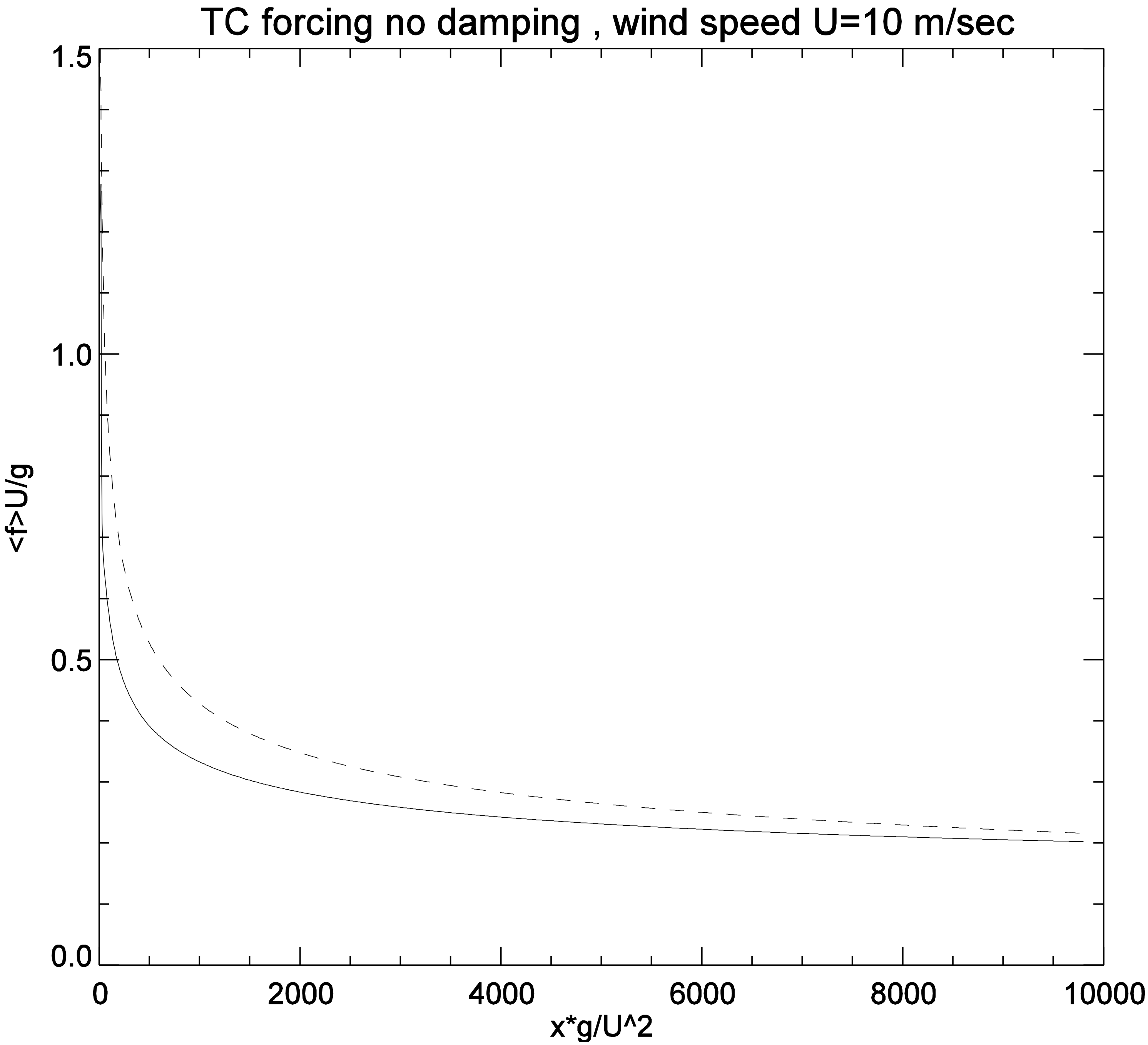




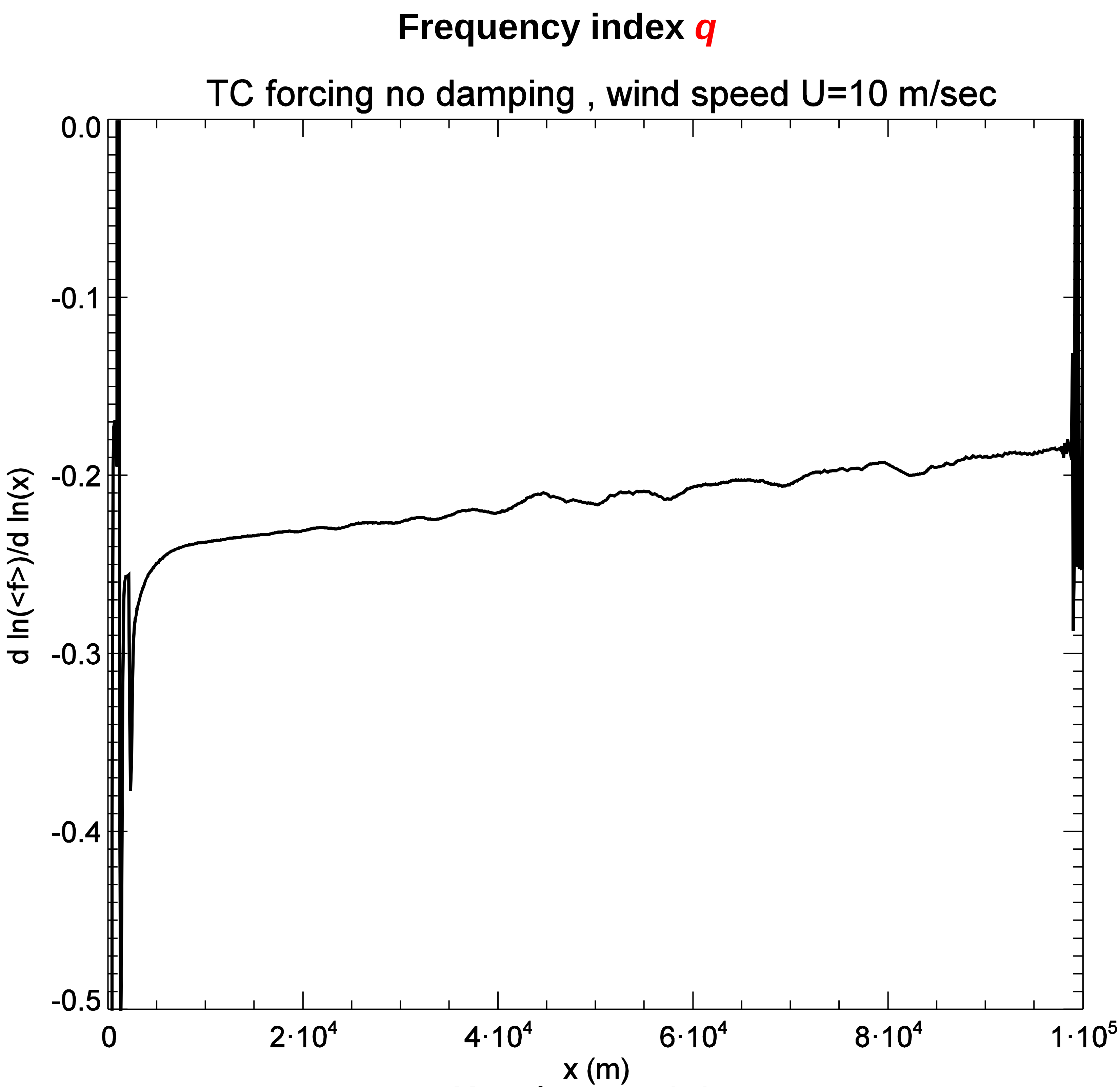
Solid line - numerical experiment; dashed line - fit by $2.9 \cdot 10^{-7} \cdot \dfrac{xg}{U^2}$


Energy index p
TC forcing no damping , wind speed U=10 m/sec
2.0
1.5
1.0
0.5
0.0
-0.5
d ln(E)/d ln(x)
0
2·10^4
4·10^4
6·10^4
8·10^4
1·10^5
x (m)

Dimensionless frequency

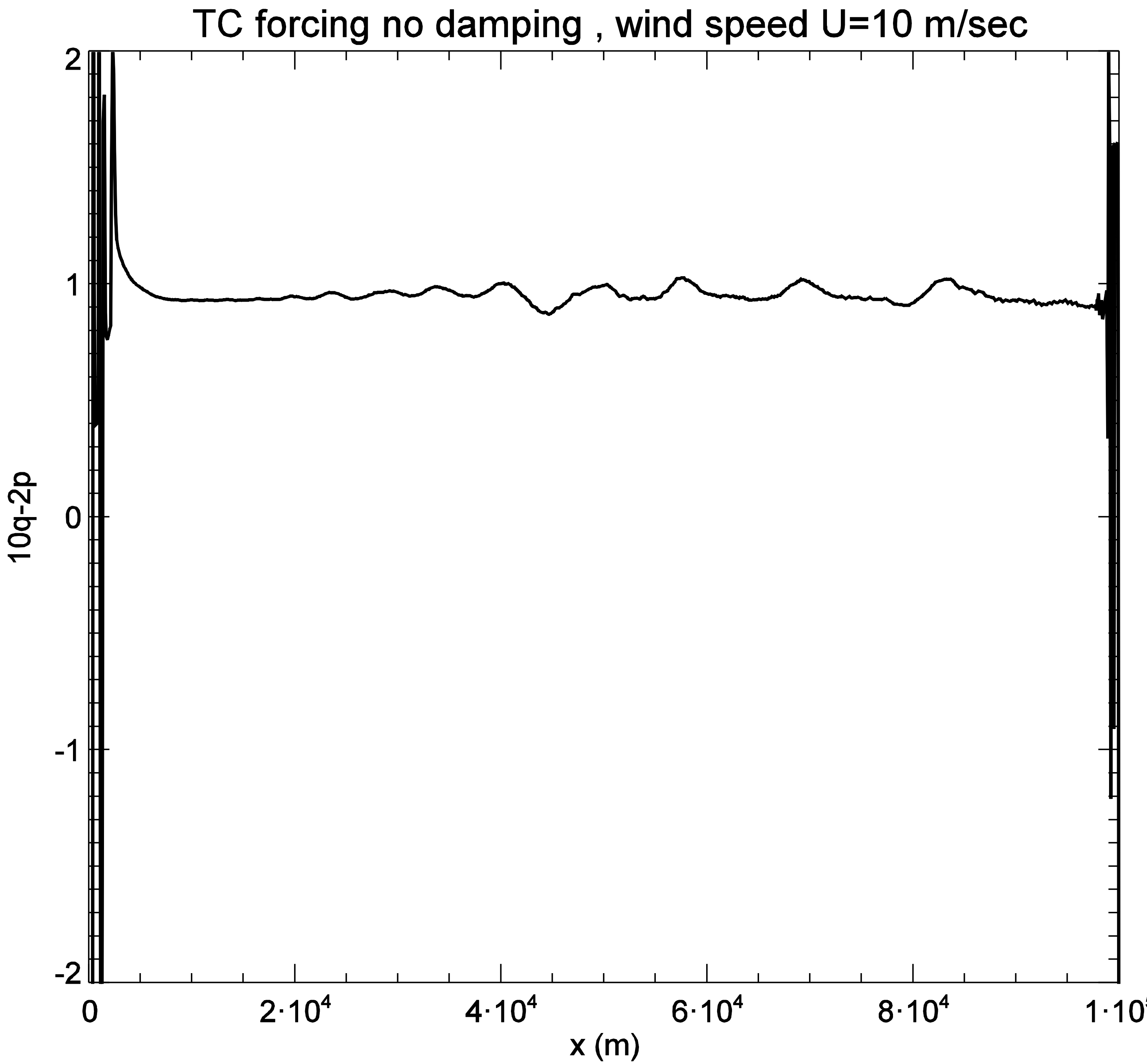


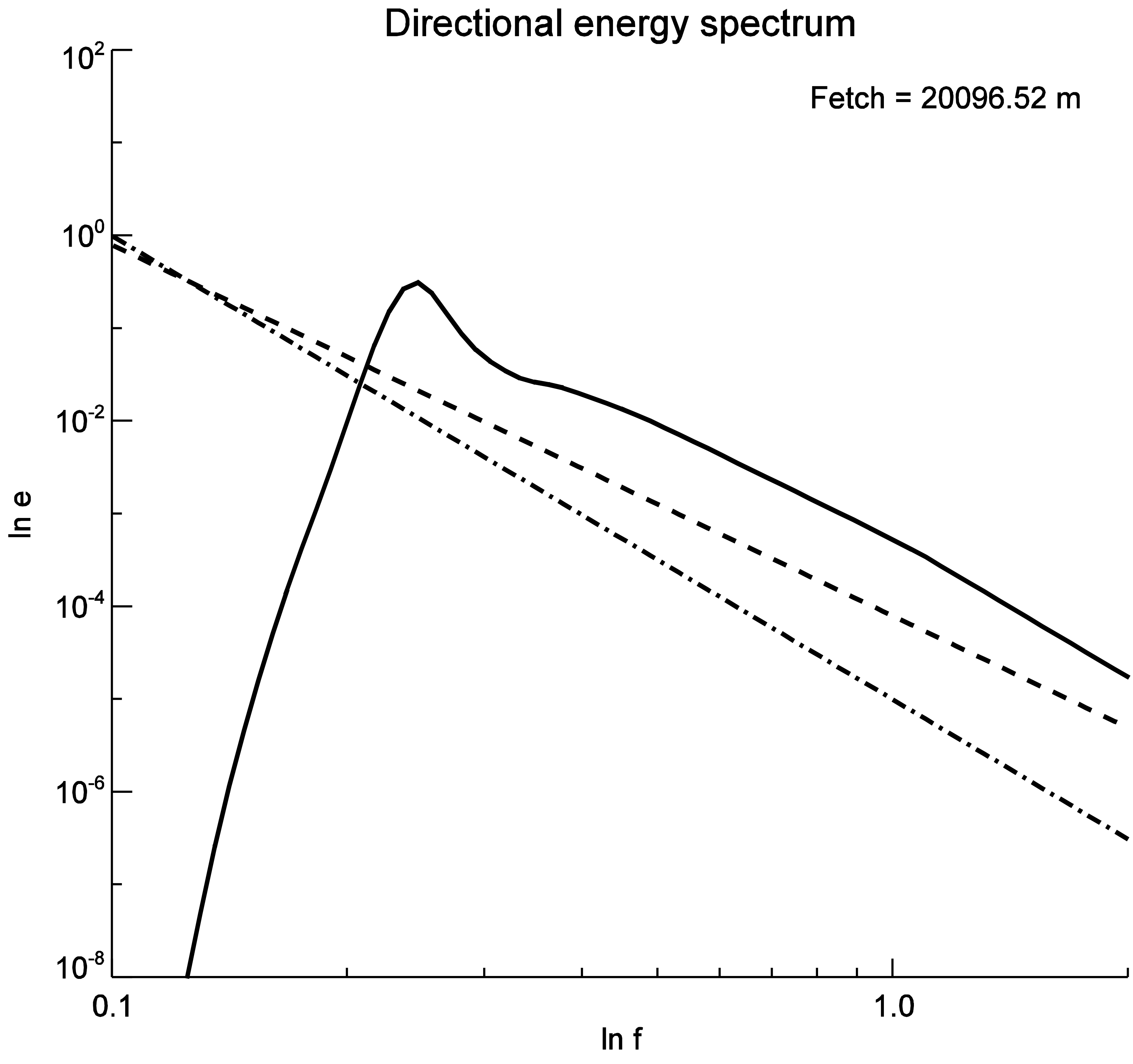
Solid line - numerical experiment; dashed line - fit by $21.36 \cdot \left(\frac{xg}{U^2}\right)^{-0.3}$



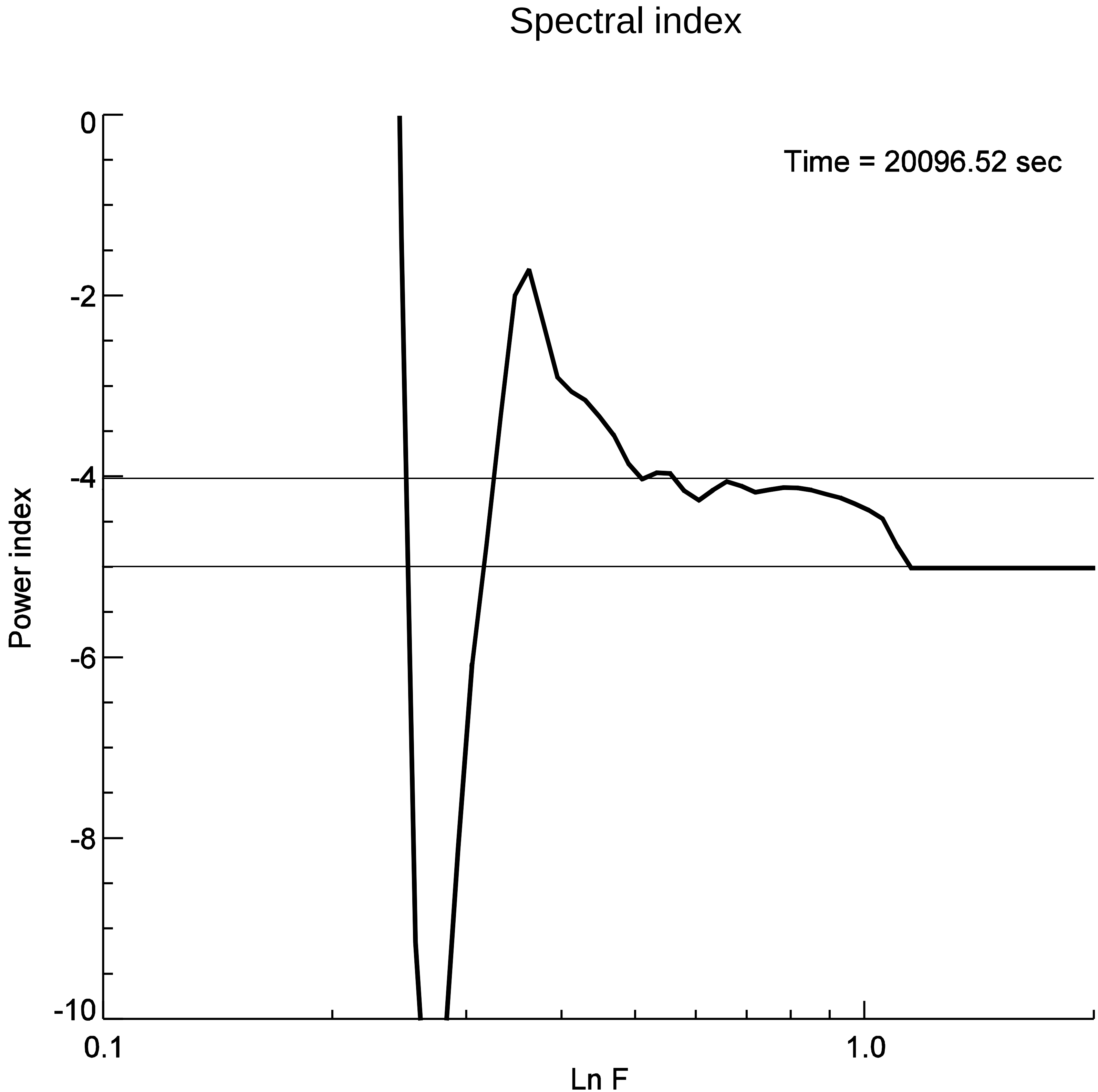
Mean frequency index

Magic number *10q*-*2p*

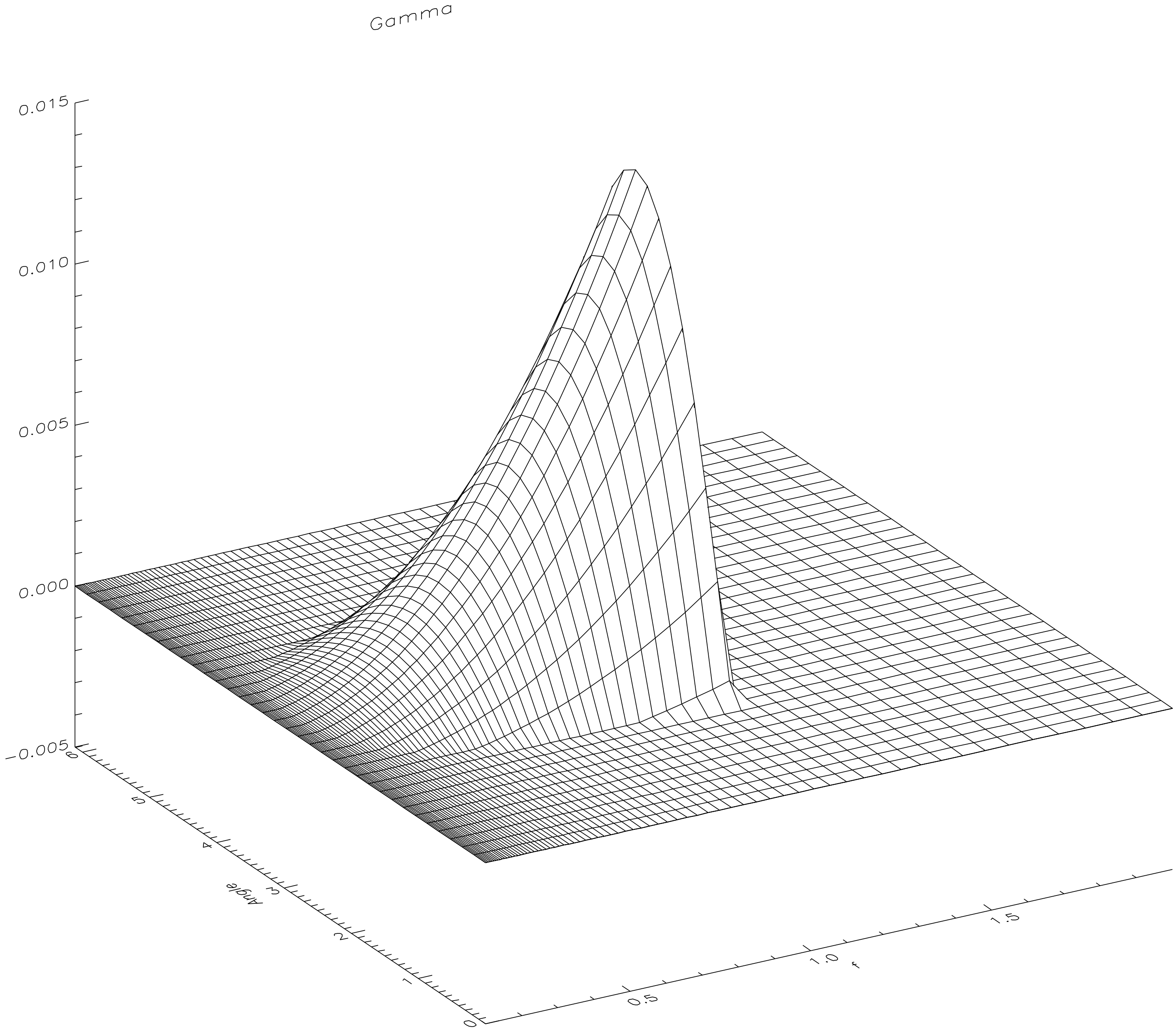


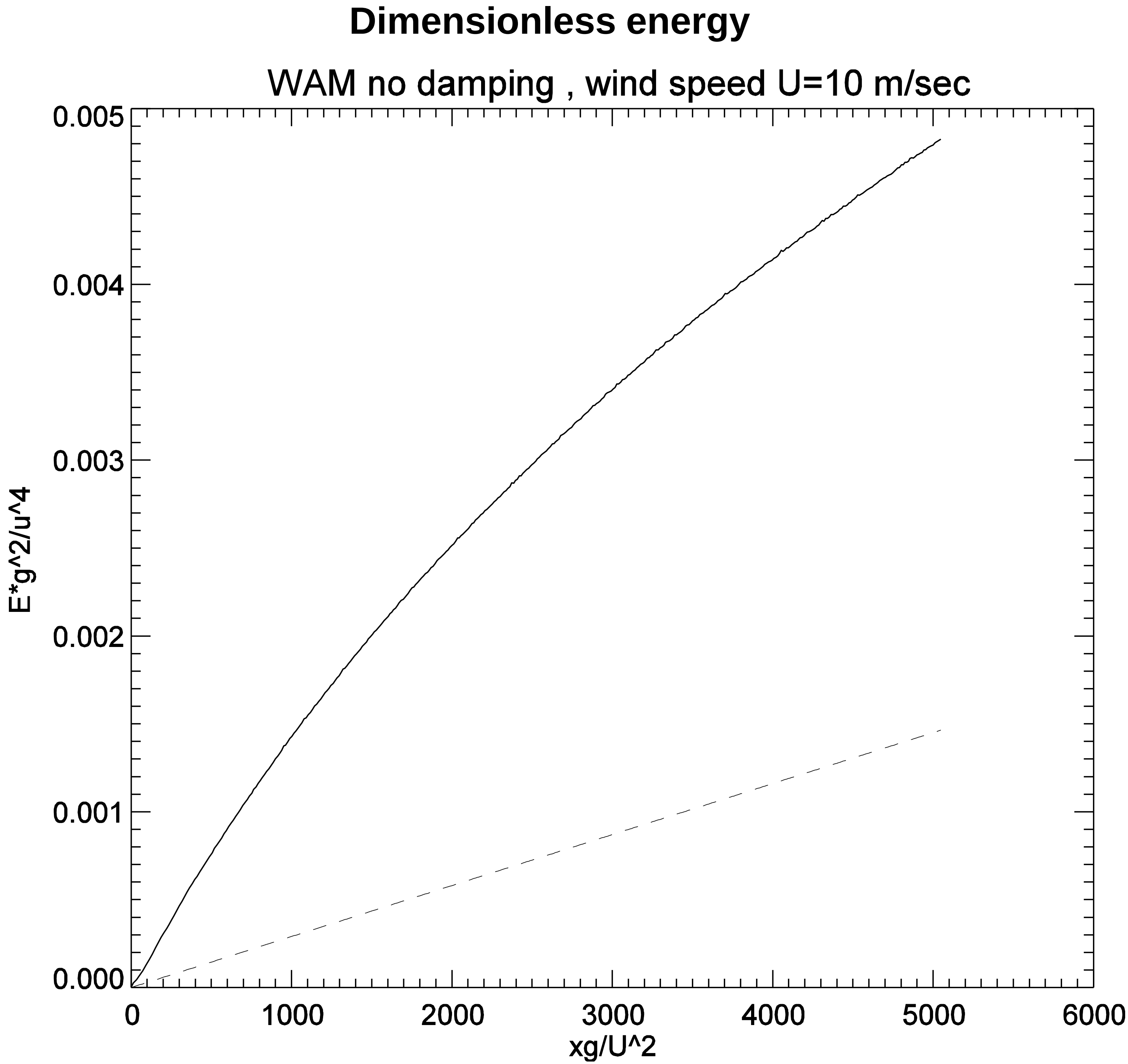


Directional energy spectrum
Fetch = 20096.52 m
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
ln e
0.1
1.0
ln f

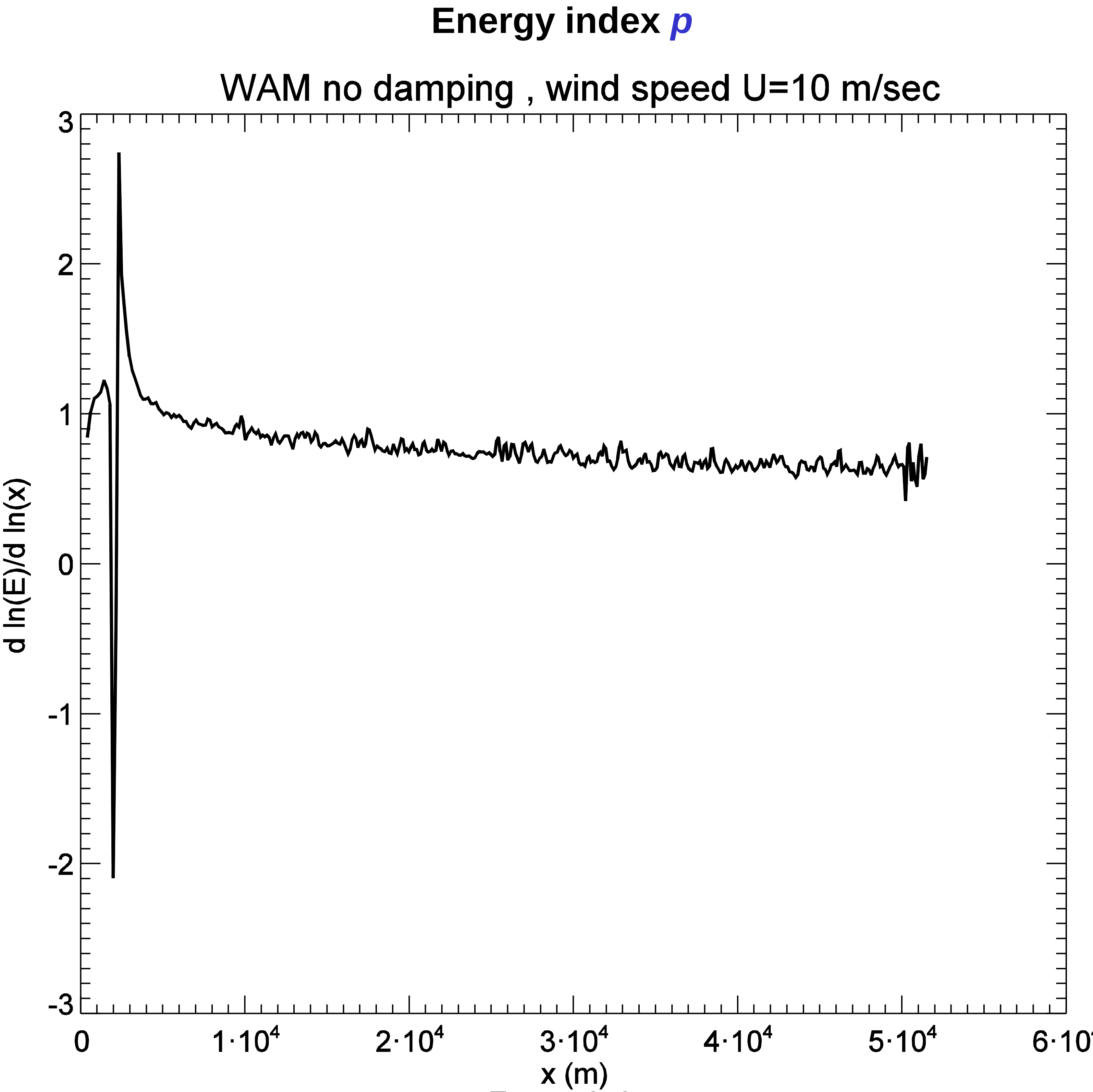

Spectral index
Time = 20096.52 sec
0
-2
-4
-6
-8
-10
Power index
0.1
1.0
Ln F

WAM no damping (Snyder)



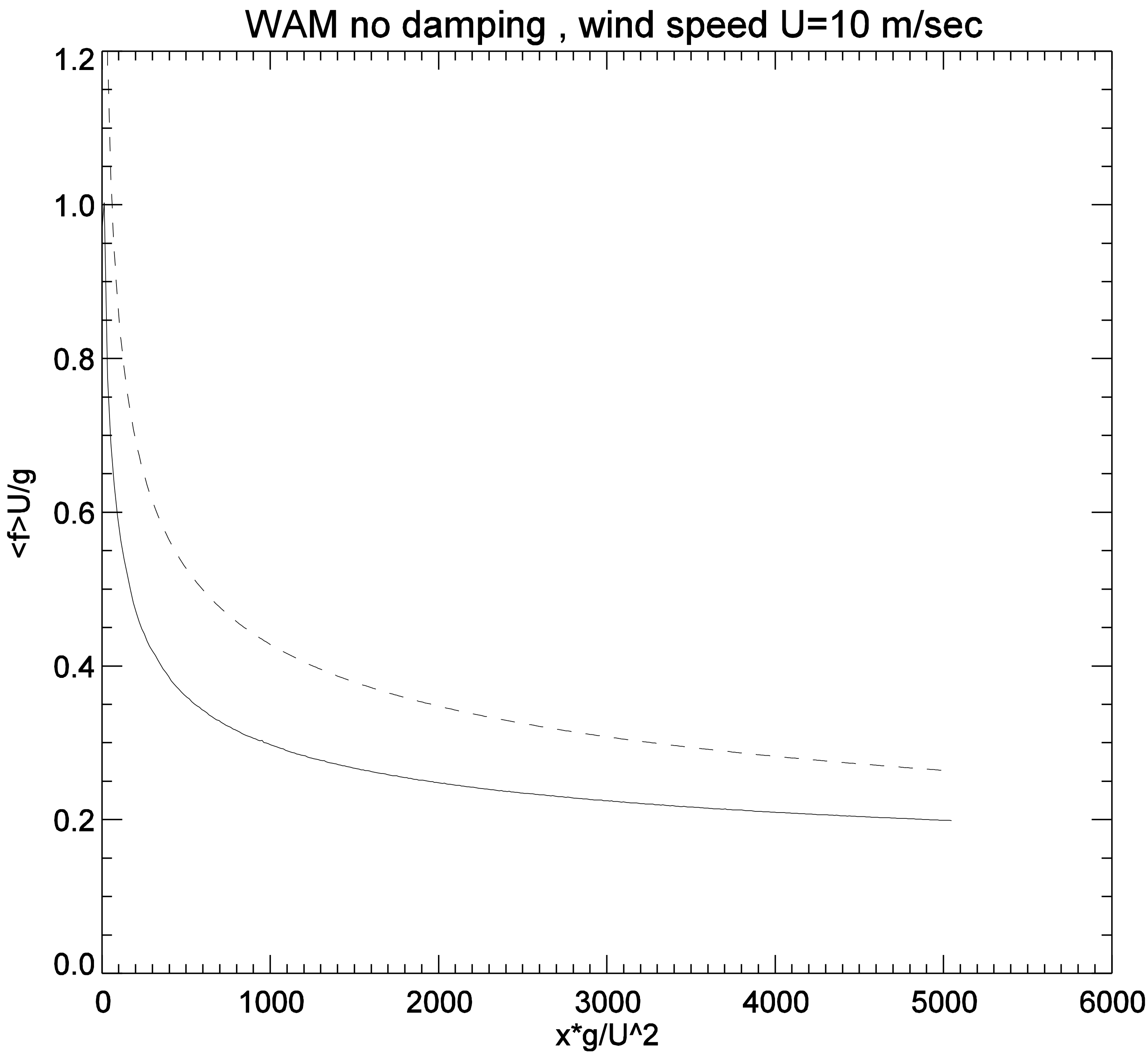


Solid line - numerical experiment; dashed line - fit by $2.9 \cdot 10^{-7} \cdot \dfrac{xg}{U^2}$



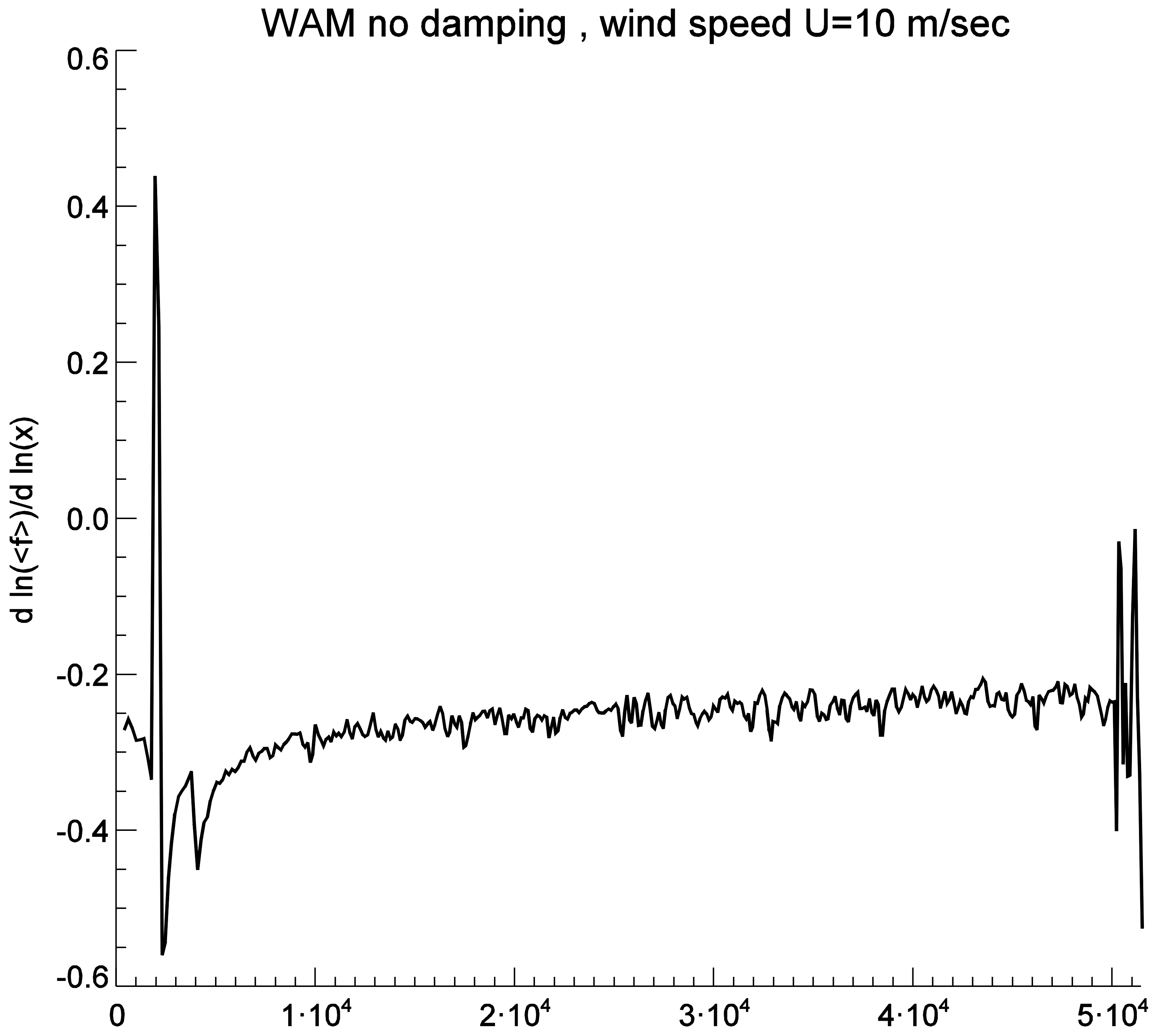
Energy index

Dimensionless frequency

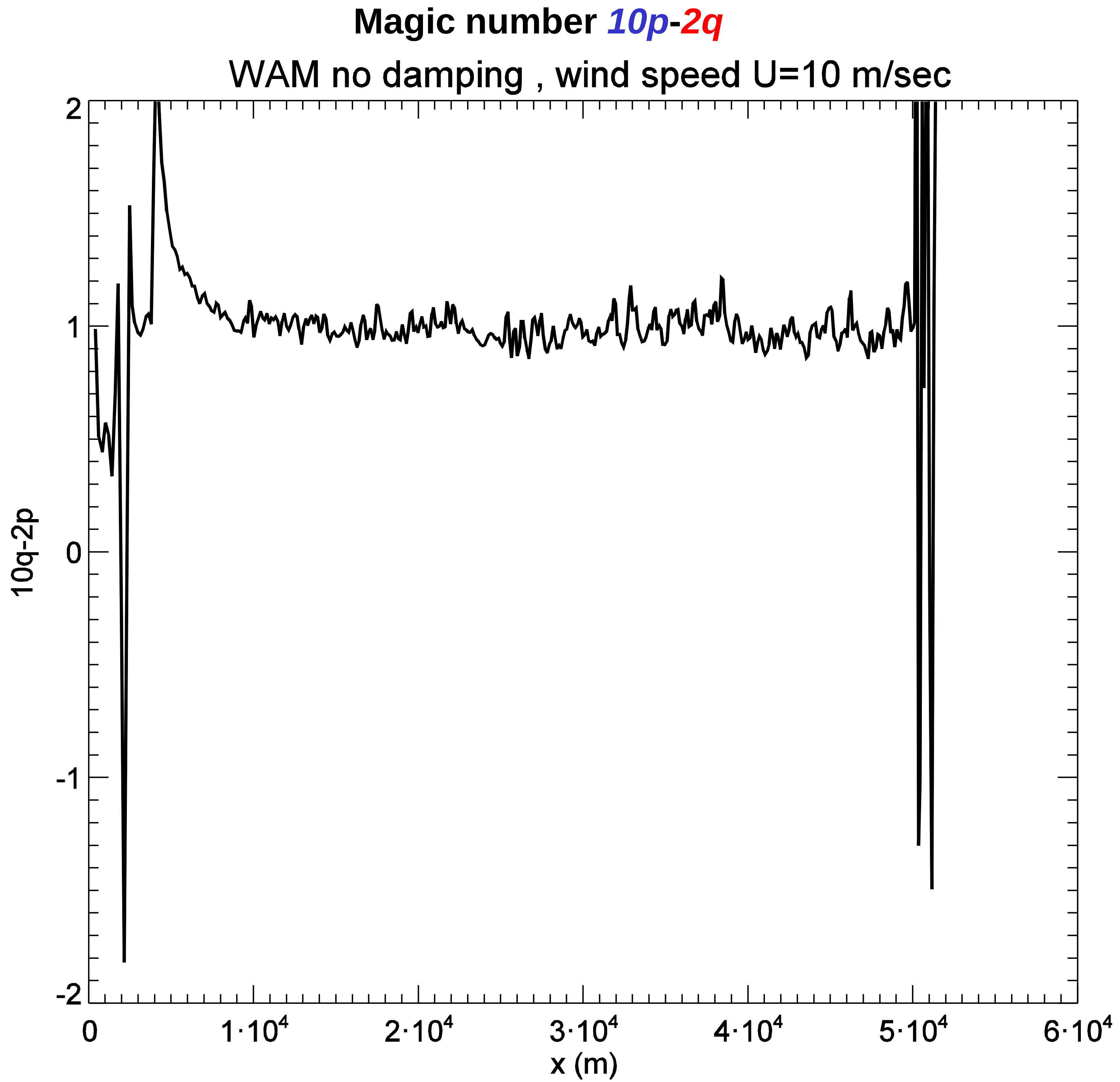


Solid line - numerical experiment; dashed line - fit by $21.36 \cdot \left(\frac{xg}{U^2} \right)^{-0.3}$

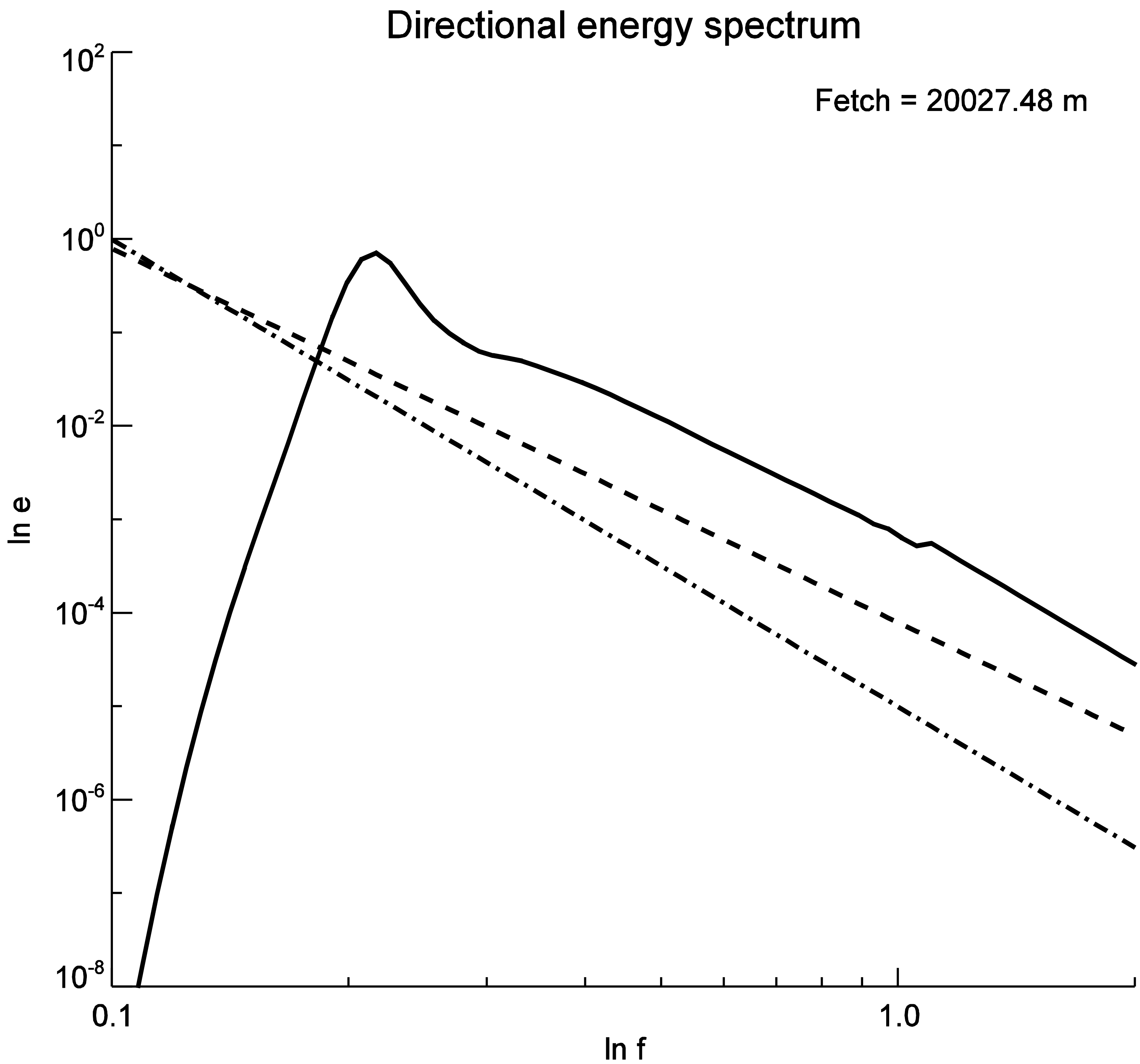
Frequency index *q*



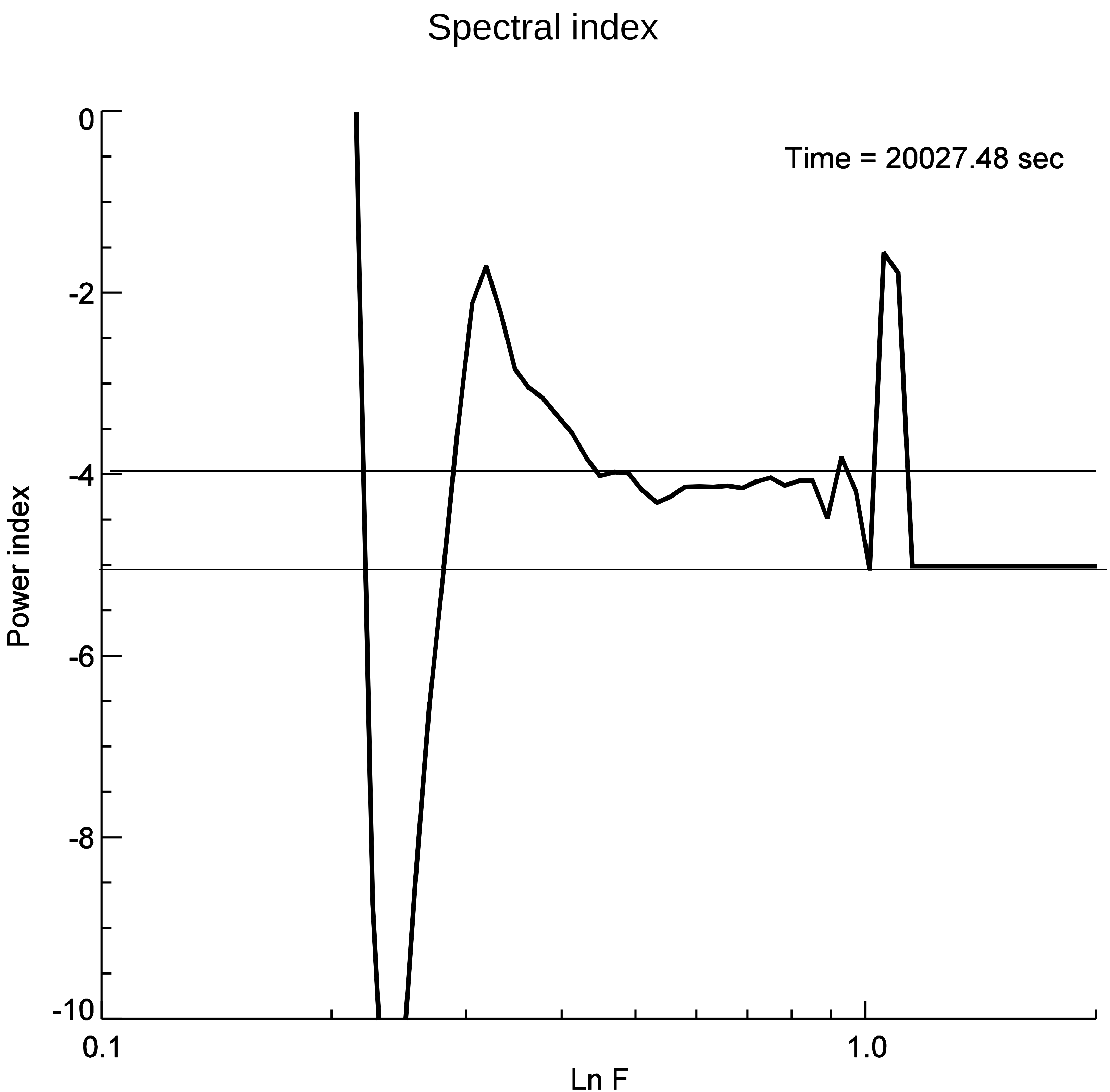
Mean frequency index



Magic number 10p-2q
WAM no damping , wind speed U=10 m/sec
10q-2p
x (m)
2
1
0
-1
-2
0
1·10^4
2·10^4
3·10^4
4·10^4
5·10^4
6·10^4

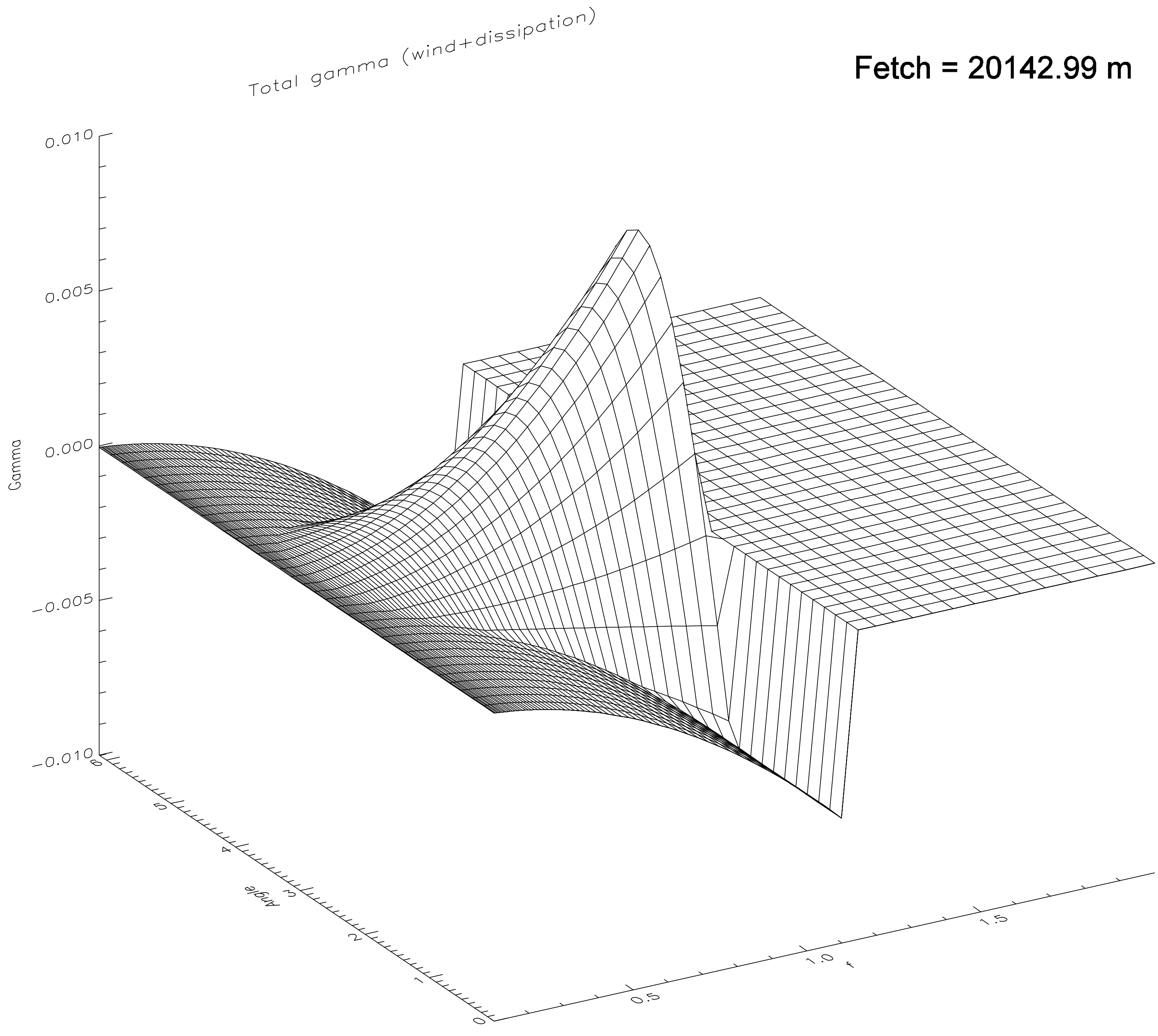


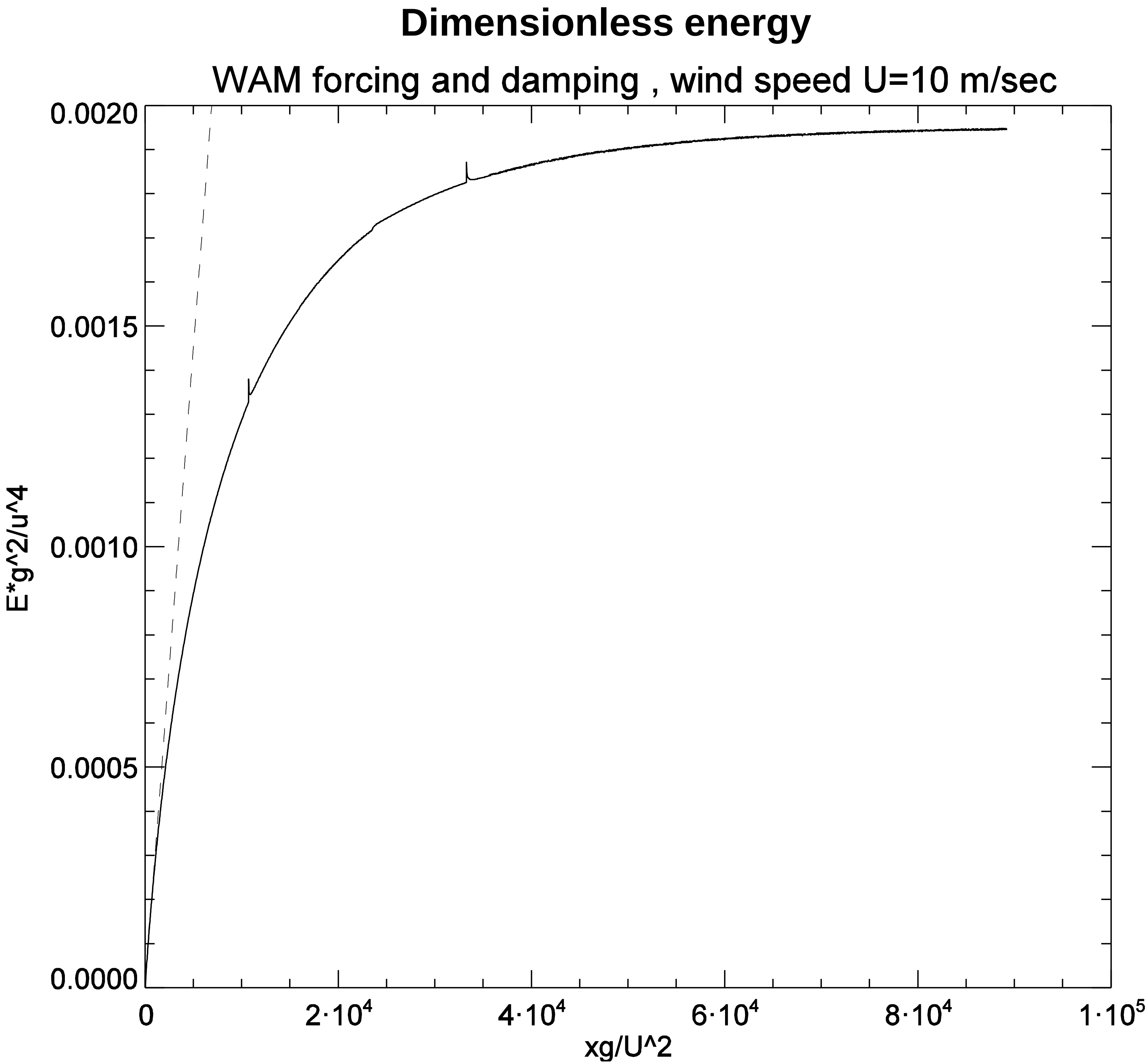
Directional energy spectrum
Fetch = 20027.48 m
10^2
10^0
10^-2
10^-4
10^-6
10^-8
ln e
0.1
1.0
ln f



Spectral index
Time = 20027.48 sec
Power index
0
-2
-4
-6
-8
-10
0.1
1.0
Ln F

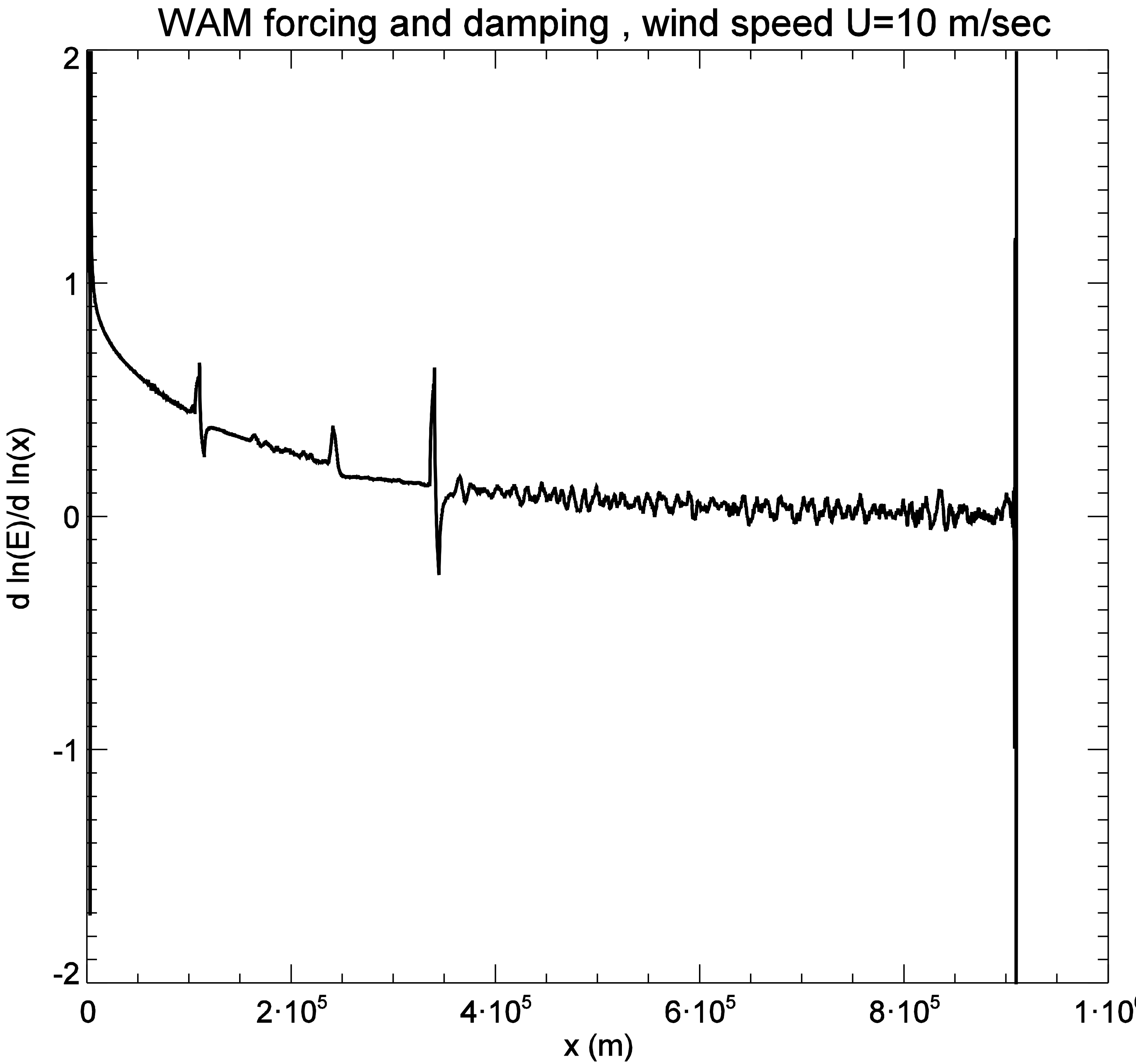
WAM with damping



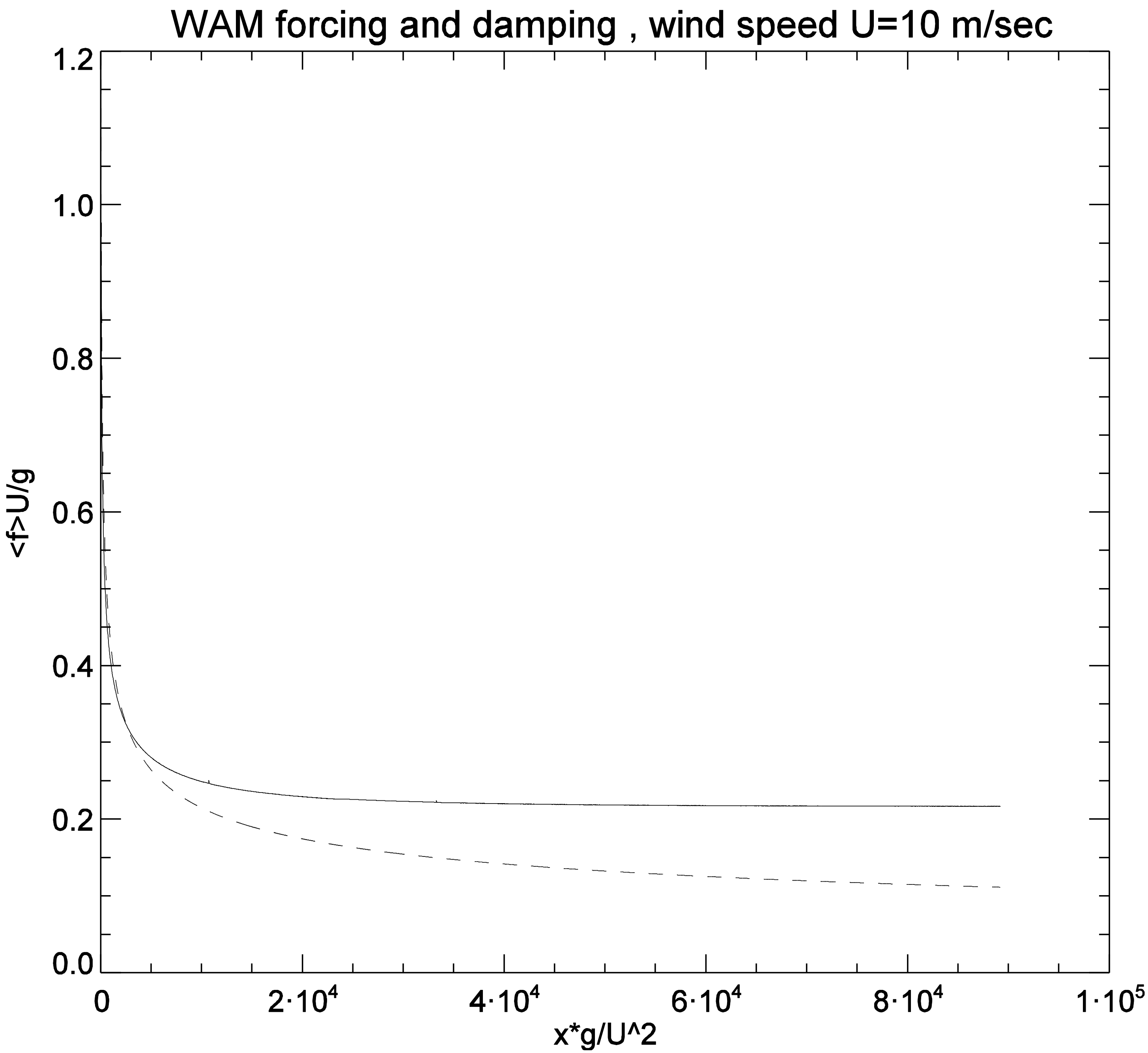


Solid line - numerical experiment; dashed line - fit by $2.9 \cdot 10^{-7} \cdot \dfrac{xg}{U^2}$

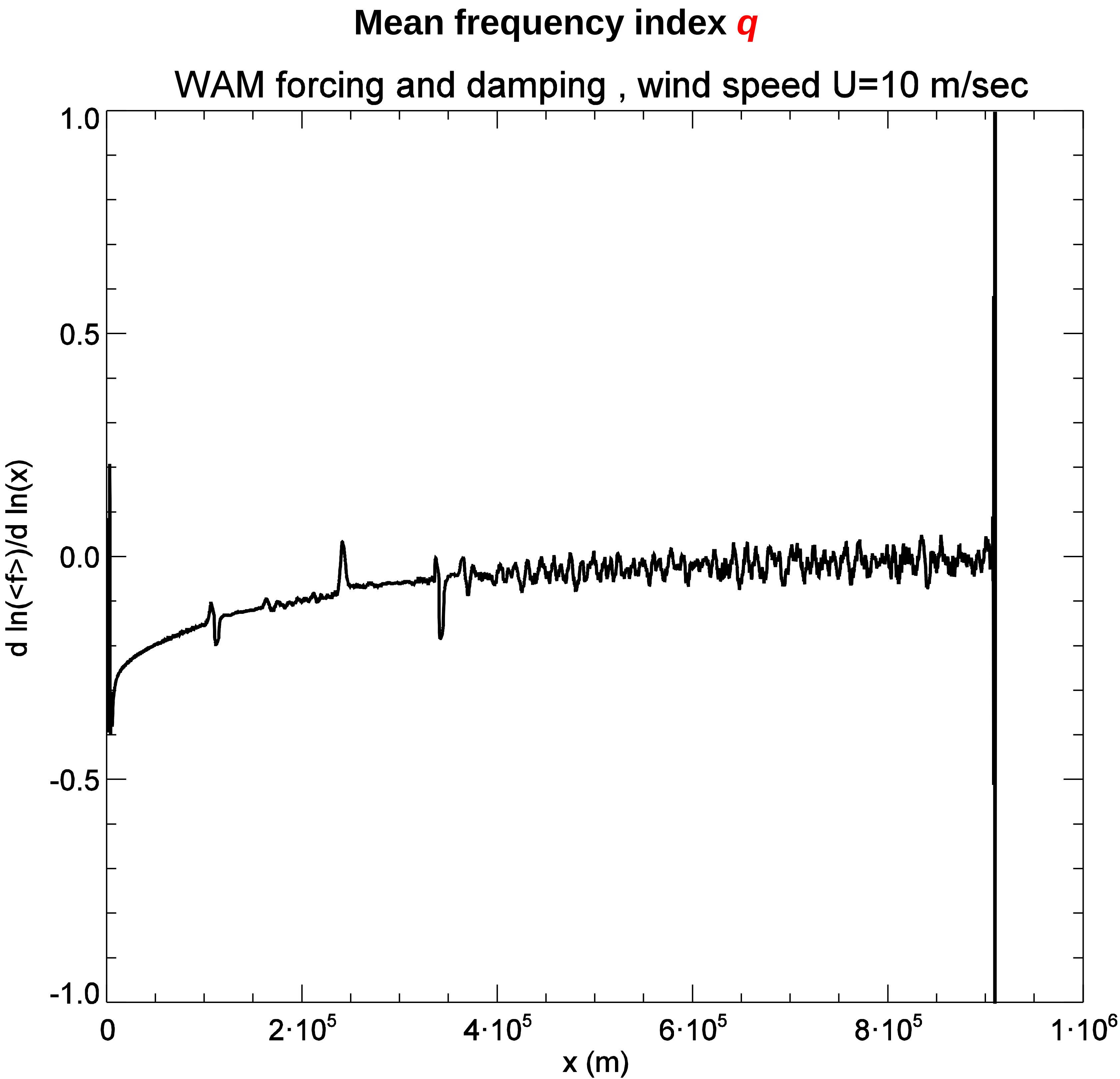
Energy index *p*

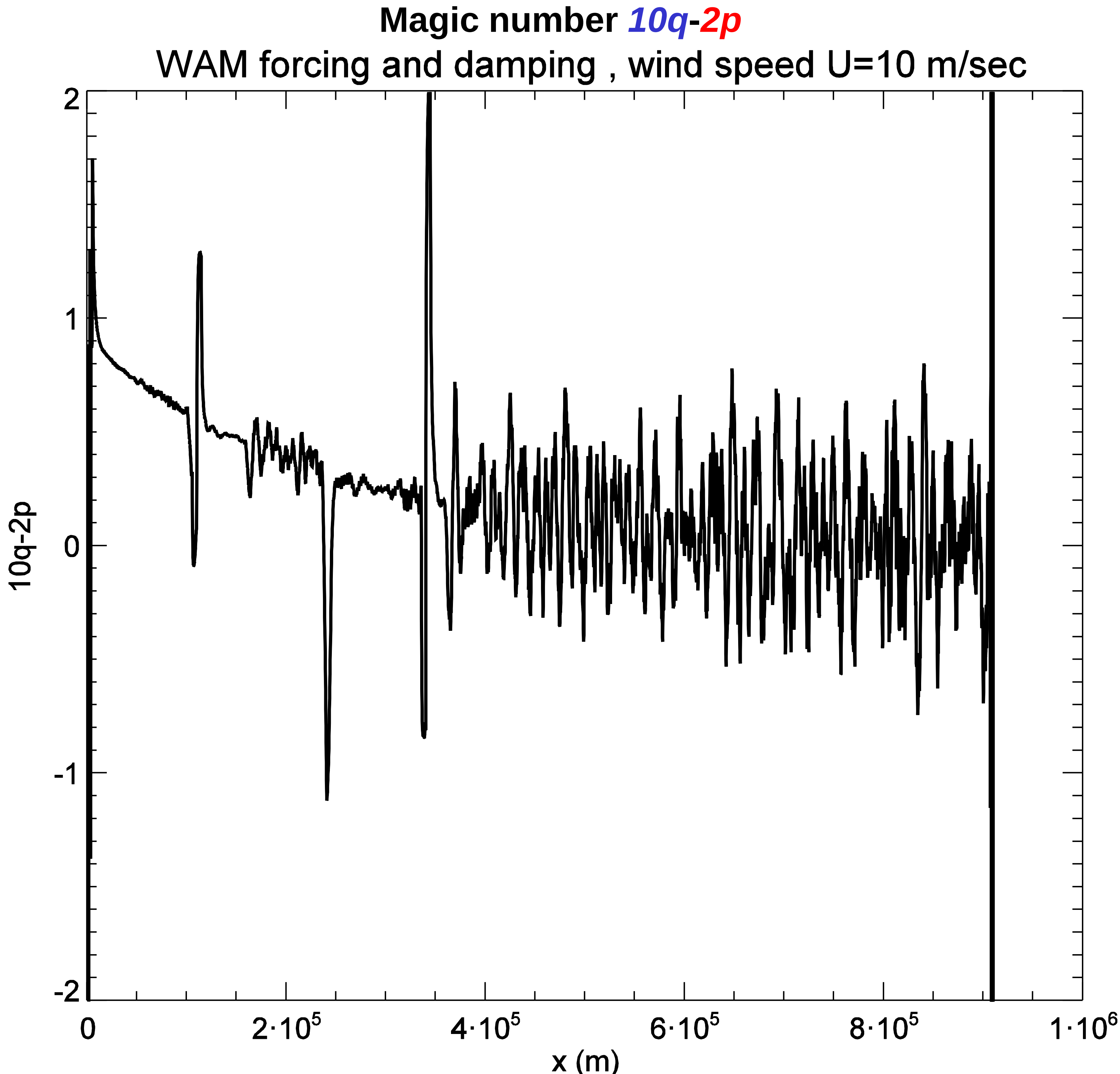


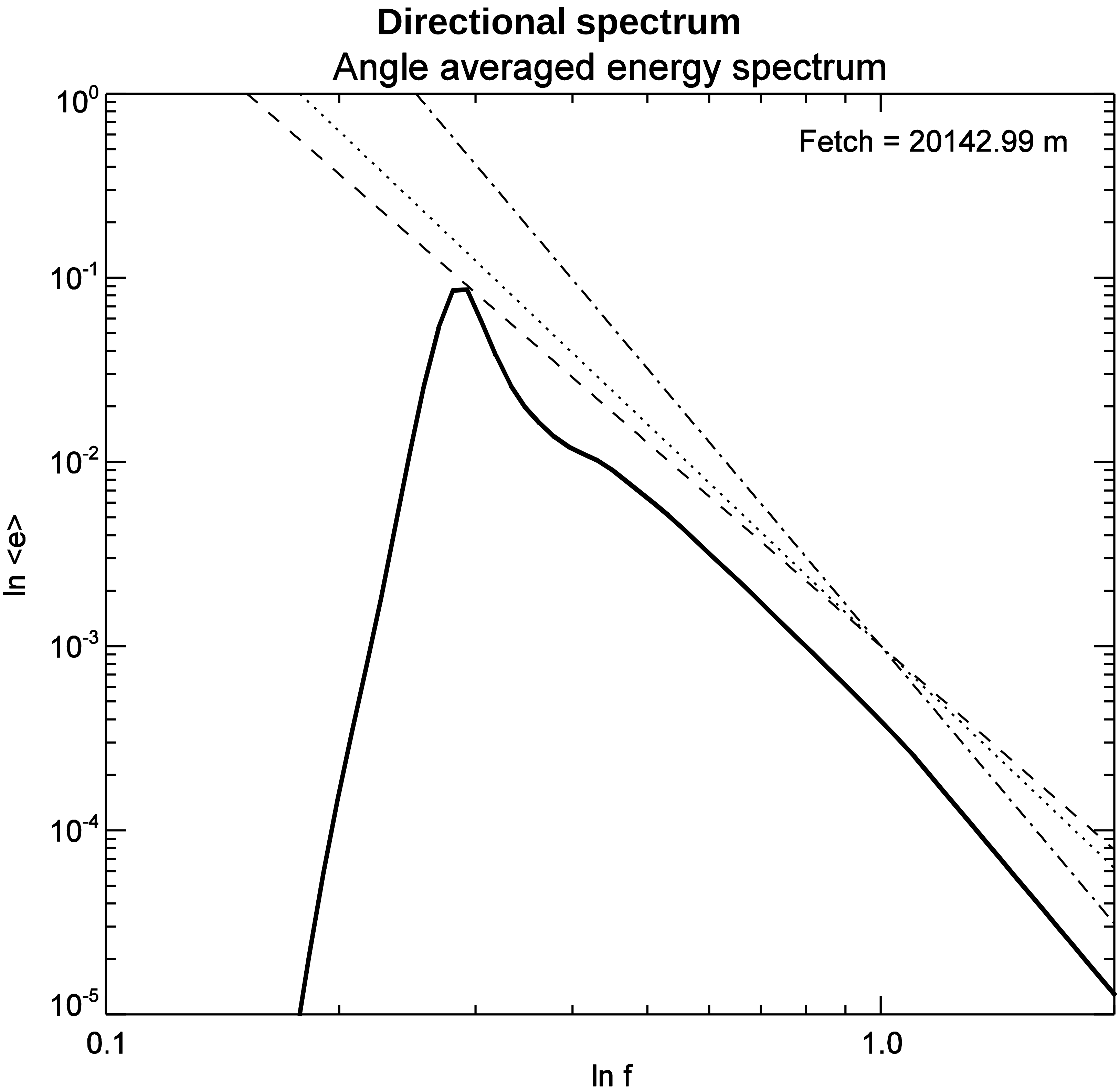
Dimensionless frequency



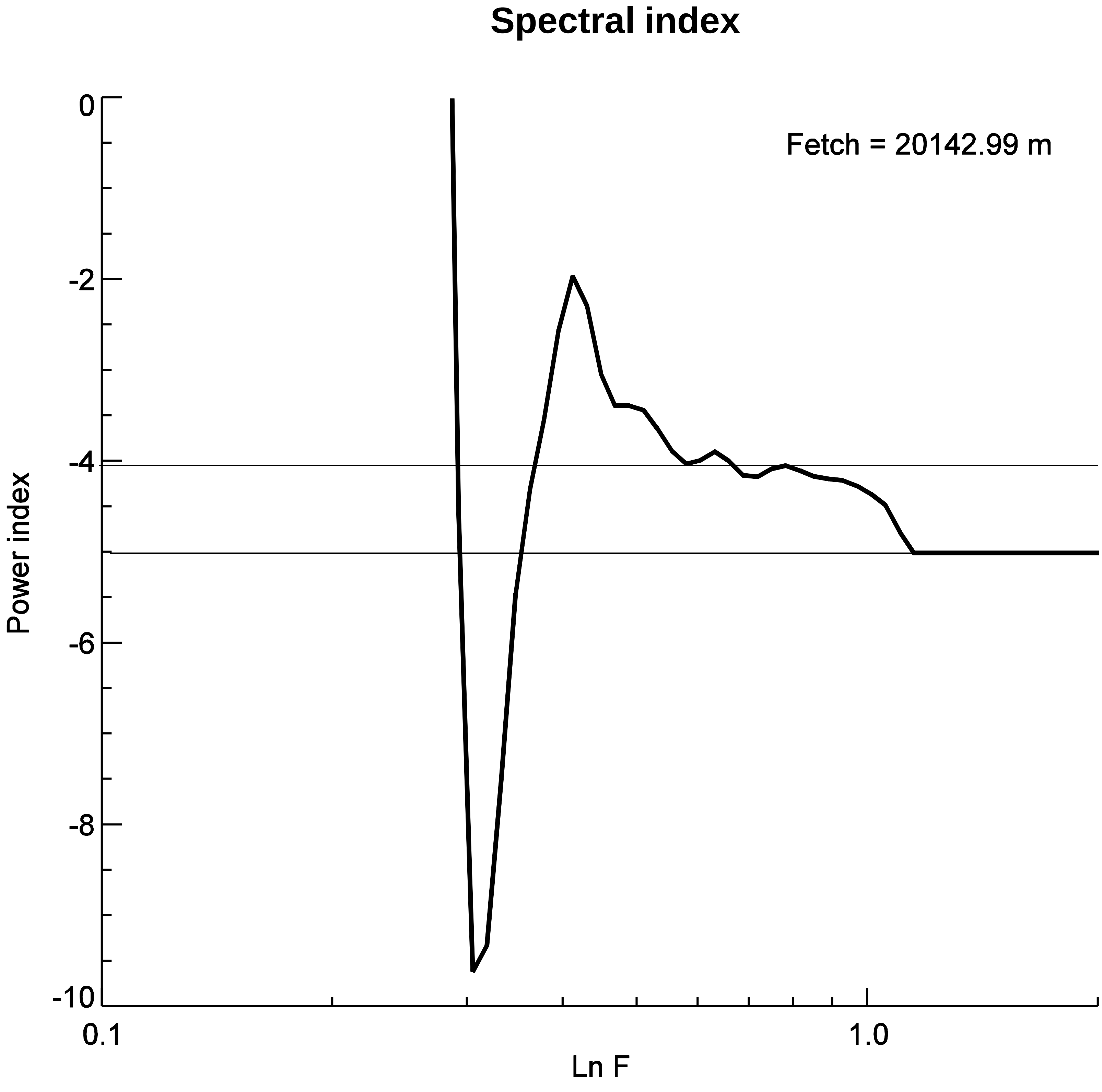
Solid line - numerical experiment; dashed line - fit by $21.36 \cdot \left(\frac{xg}{U^2} \right)^{-0.3}$


Mean frequency index q
WAM forcing and damping , wind speed U=10 m/sec
1.0
0.5
0.0
-0.5
-1.0
d ln(<f>)/d ln(x)
0
2·10^5
4·10^5
6·10^5
8·10^5
1·10^6
x (m)


Magic number 10q-2p
WAM forcing and damping , wind speed U=10 m/sec
10q-2p
x (m)
2
1
0
-1
-2
0
$2\cdot10^5$
$4\cdot10^5$
$6\cdot10^5$
$8\cdot10^5$
$1\cdot10^6$



dash-dotted line: power -5; dotted line: power -4; dashed line: power -11/3



Spectral index
Fetch = 20142.99 m
Power index
Ln F
0
-2
-4
-6
-8
-10
0.1
1.0

CONCLUSION

- ***ZRP*** forcing term is in the agreement with at least 15 fetch-limited experimental observations
- Many source terms and experimental observations exhibit similar nonlinear effects in the form of Zakharov-Filonenko spectra and "magic numbers" -- evidence of (quazi) self-similarity
- There is *no need* to use spectral maximum damping – **Occam's razor** principle
- It is necessary to use correct wind input – exact solution of HE

TO-DO'S:

- Check as a much as possible existing wind input terms against ZF spectra, magic numbers, correspondence to experiments.
- Use self-similarity characteristics as selection tools for wind input terms
- Try to explain the difference in self-similarity parameters for stable and unstable atmosphere cases