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Processes of concentration of energy during the formation of rogue waves

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Rogue waves are sudden single surface waves with large heights (up to 30 m). Their suddenness and high amplitude pose serious danger to marine facilities.

These waves represent local concentration of energy.

The processes of concentration of energy and momentum are quantitatively estimated in the computer experiments in this talk. The capabilities of the numerical simulation of rogue waves make it possible “to look into” the processes responsible for their formation.

In our computer experiments, we considered waves traveling in one direction, which corresponds to ripples in the ocean. The experiments were based on the numerical solution of the Euler equation for an ideal fluid with a free surface and infinite depth in 2D-dimensional geometry:

$$-\infty < y < \eta(x,t)$$

$$0 < x < 2\pi$$

The boundary conditions in the variable x were 2π periodic. It was assumed that a flow is potential and the fluid is incompressible. Therefore,

$$v(x,y,t) = \nabla\Phi(x,y,t), \qquad div\, v = 0$$

Thus, the potential of the velocity field of the fluid Φ satisfies the Laplace equation

$$\Delta\Phi(x,y,t) = 0$$

For numerical calculations, we used the equations in conformal variables obtained Dr. A.I. Dyachenko.

We performed the conformal mapping of the region occupied by the fluid onto the lower half-plane, where the coordinates are specified as $w = u + iv$

This mapping is specified by the function

$$z = z(w,t), z = x + iy$$

The dynamic equations written in the Dyachenko variables

$$R = \frac{1}{z'}; V = i\frac{\partial \Phi}{\partial z}$$

have the form

$$\dot{R}(u,t) = i(UR' - UR')$$

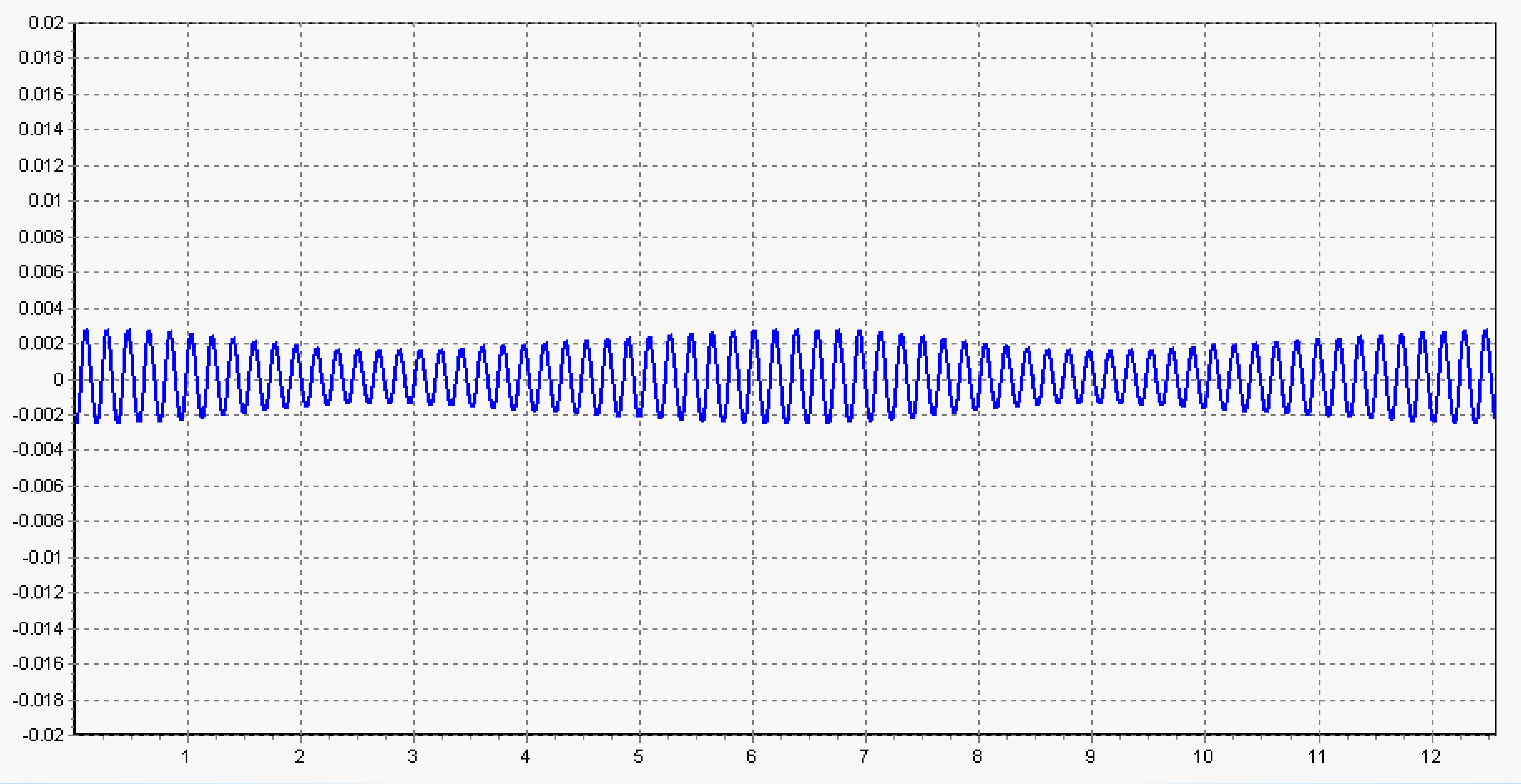
$$\dot{V}(u,t) = i(UV' - B'R) + g(R-1)$$

$$U = \frac{1}{2}(I + iH)\left[R\overline{V} + V\overline{R}\right], B = \frac{1}{2}(I + iH)\left[V\overline{V}\right]$$

Rogue waves are usually identified by the amplitude criterion

$$\nu = \frac{H_m(t)}{H_s(t)} \geq \nu^* = 2.1$$

Where H_m is the maximum height and H_s is the significant height of waves (average of one-third of the highest waves). The critical value v* = 2.1 was chosen empirically and was used in many studies of rogue waves.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In our computer experiments, we considered a train of waves periodic in the spatial variable. In this case, the profile of the free surface is specified by the function

$$y = y(x,t)$$

which is 2π-periodic in the variable x. The region between two local minima of the free surface is called an individual wave.

The energy E can be calculated for each wave by hydrodynamic formulas.

The quantitative measure of concentration of energy is calculated on the following formula:

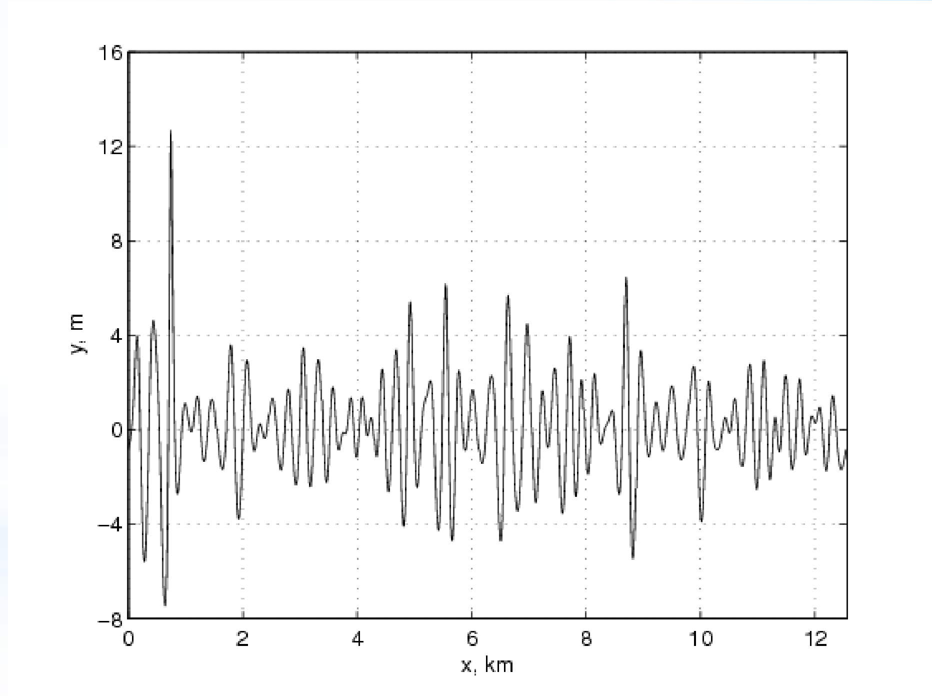
$$C_E = \frac{E_m}{\bar{E}},$$

where E_m is energy of maximal wave, $\bar{E}$ is average energy of waves.

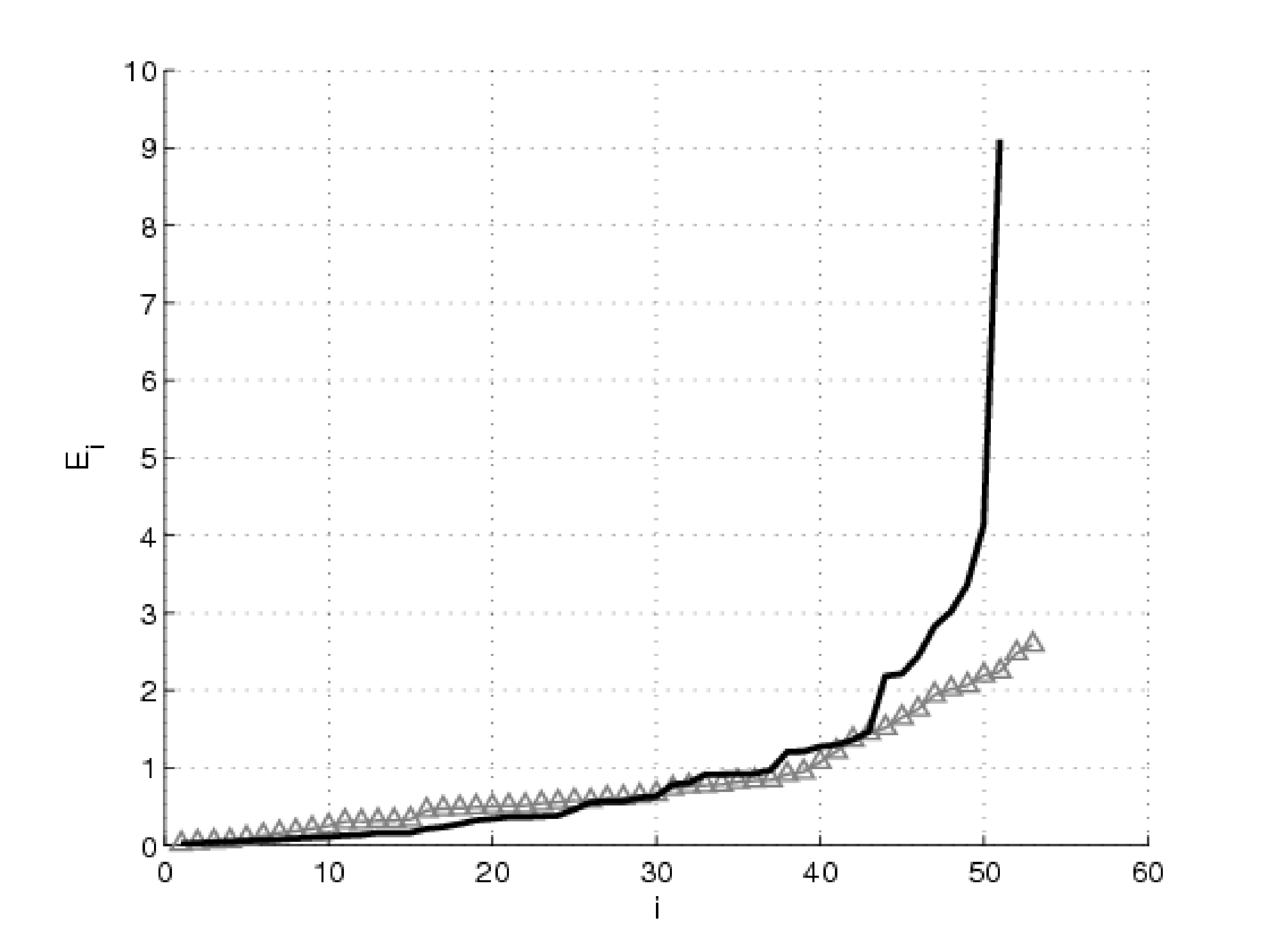
Now, we consider a typical experiment.

In this experiment, the initial field of waves consisted of a train of 53 waves traveling in one direction.

The square of average steepness was $\mu^2 = 3.72 \times 10^{-1}$. The duration of the experiment was approximately 3525 periods. A rogue wave appeared after 1412 periods.

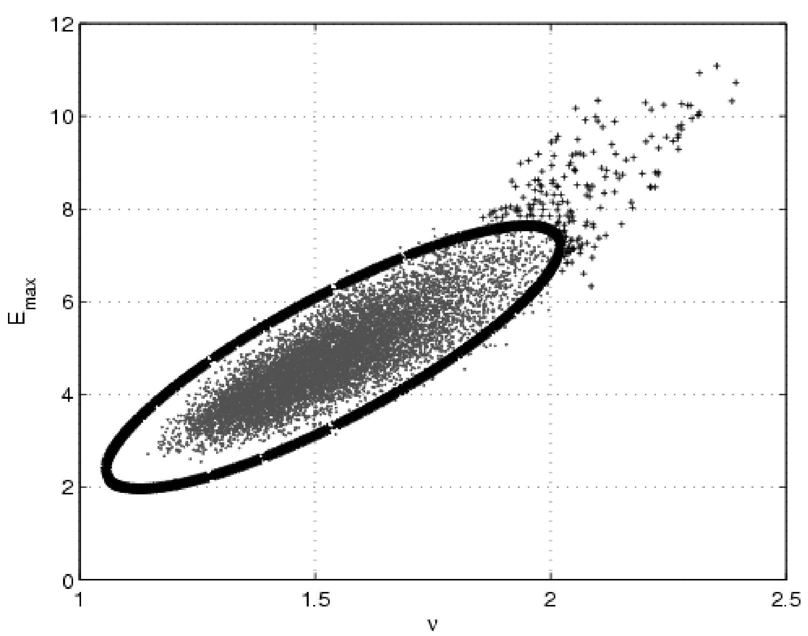


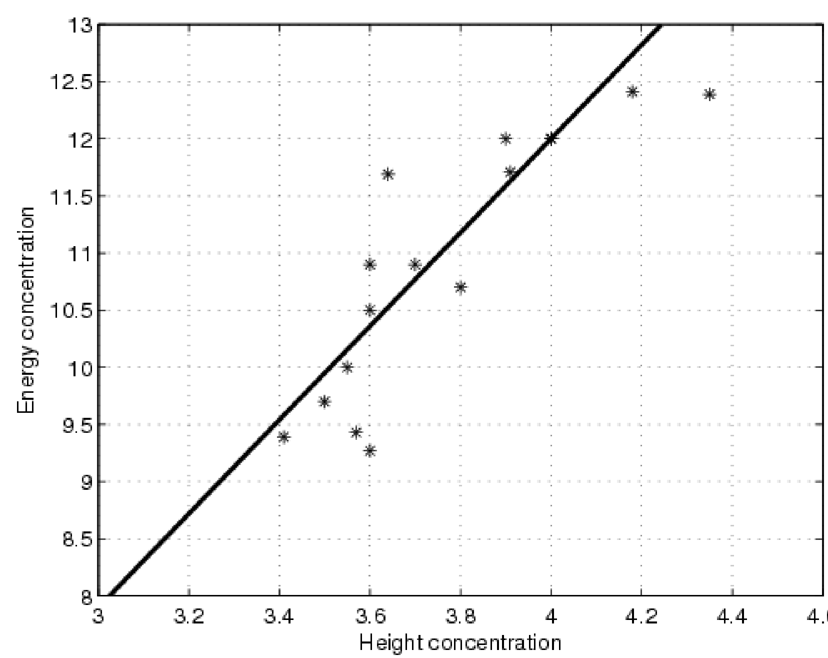
$\nu(t^*) = 2.26$



Energy concentration at the (thin line with triangles) initial time and (thick line) time of formation of the rogue wave.

We now consider the joint twodimensional distribution of the parameter v and the concentration of the energy C_E for the largest wave at each time of the computer experiment. About 98% of realizations, ν_m and C_E satisfy normal probability distributions





In summary, nonlinear processes of local concentration of the energy at the time of formation of extreme surface waves have been considered. Quantitative estimates of these processes have been obtained.

Significant correlation between the amplitude criterion of the rogue wave and energy concentration has been demonstrated. This allows a new insight into the definition of a rogue wave.

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