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SOLITONS, COLLAPSES AND TURBULENCE
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Self-organization and generation of large scale flows in quasi 2D turbulence

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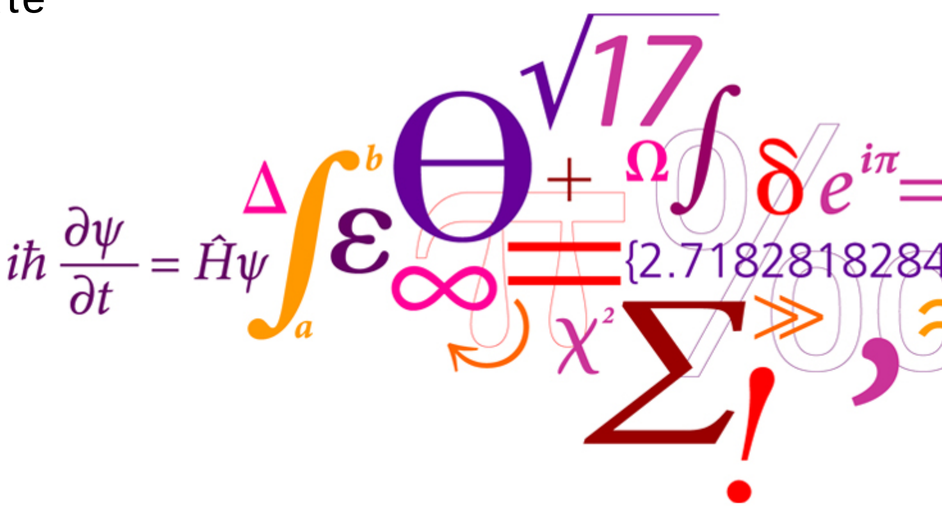


DTU Physics
Department of Physics

Motivation and outline I

- Generation of large scale flows –zonal flows - by the rectification of small scale turbulent fluctuations in quasi-2D turbulent flows

- Great importance both in geophysical flows and in magnetically confined plasmas.

- The flows regulate the turbulence by suppressing the small scale structures and set up effective transport barriers.

- The morphology of zonal flows, the basic mechanisms for their generation and their influence on turbulence and the associated transport in magnetized plasmas and rotating fluids.

Motivation and outline II

- Zonal flow generation in a fluid experiment in a rotating tank with radial symmetric bottom topography, by exploiting the Lagrangian invariance of the potential vorticity, PV.

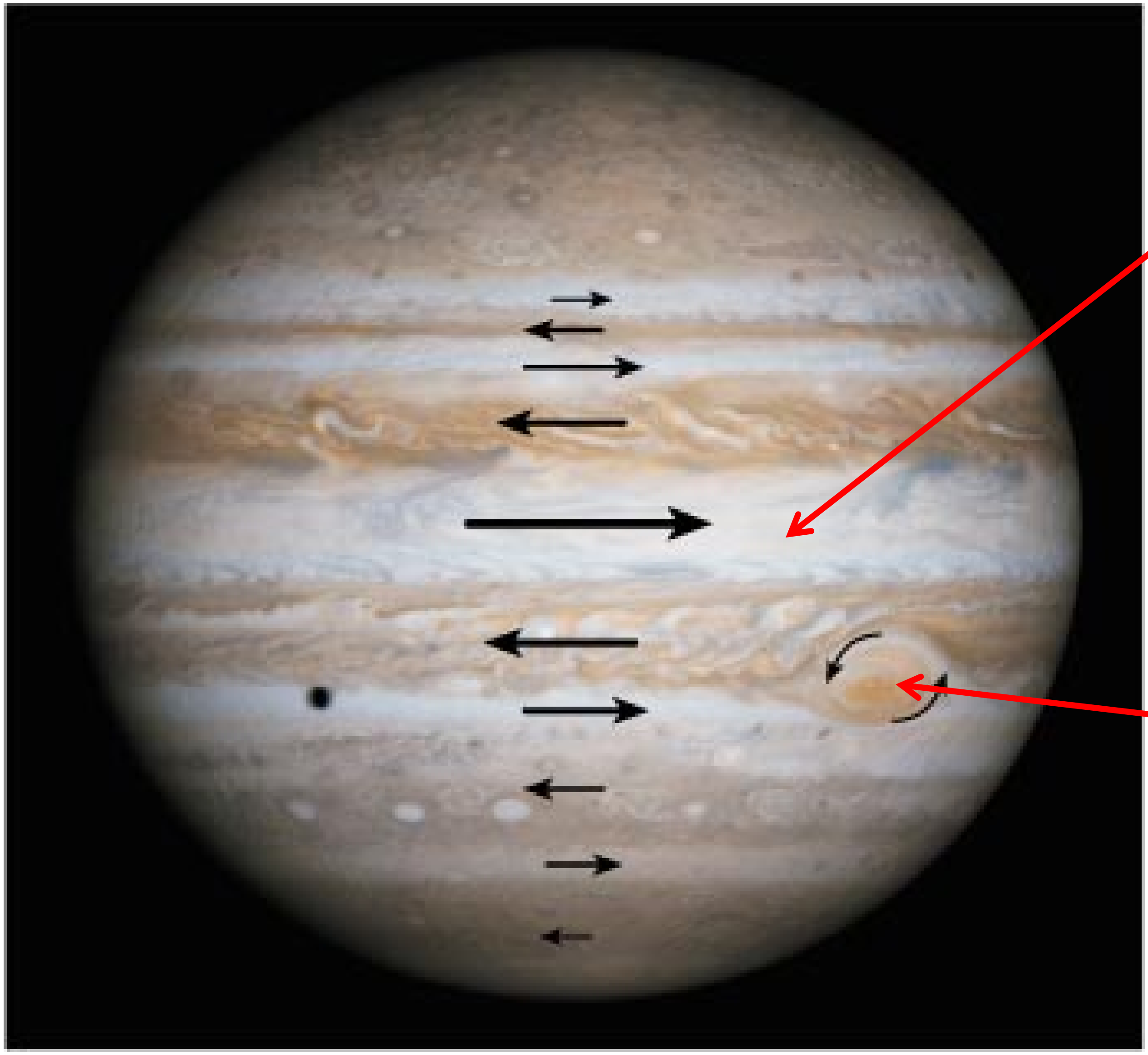
- This mechanism is widely applied in quasi-2D geostropic turbulent flows for explaining zonal flow bands on planets.

- In magnetically confined plasmas sheared poloidal zonal flows reduce the radial turbulent transport and are instrumental in the transition to an enhanced confinement state (the H-mode), with suppressed turbulent transport.
-- Turbulent transport is the dominating transport channel in magnetically confined plasma.

- The H-mode is envisaged for the ITER experiment for fulfilling the goal of demonstrating beak-even.

- And it is the H-mode that future fusion reactors will rely on.

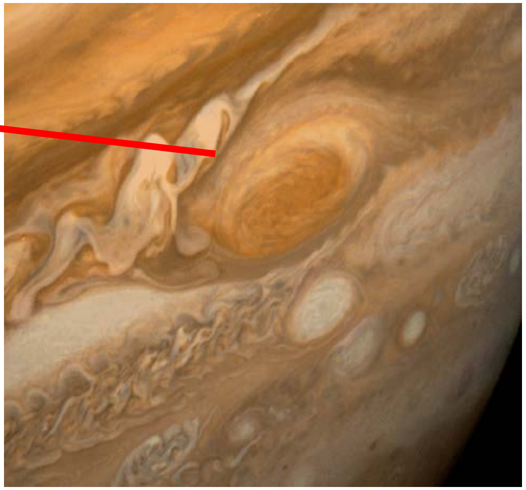
Zonal flows - Jupiter



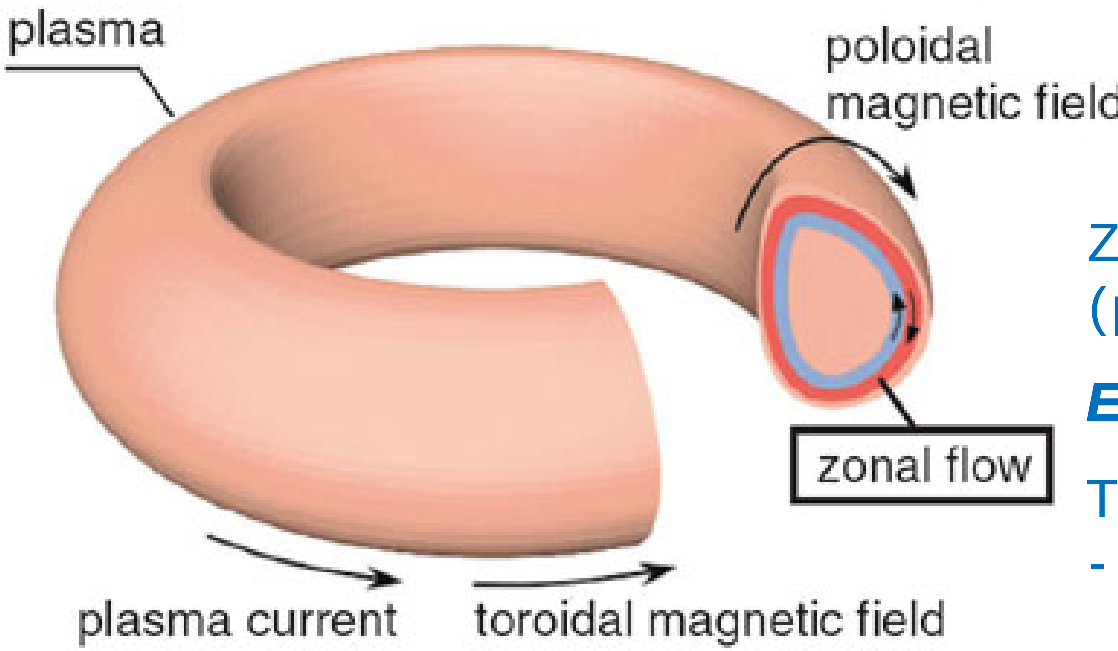
Zonal flow on major planets

Typical width related to stability

Great red spot – not for this talk



Zonal flow in magnetically confined plasma



Zonal flow : $\boldsymbol{v}_{ZF}$ = $\boldsymbol{E_r}$ **x** $\boldsymbol{B}$ (poloidal)

$\boldsymbol{E_r}$ – radial electric field

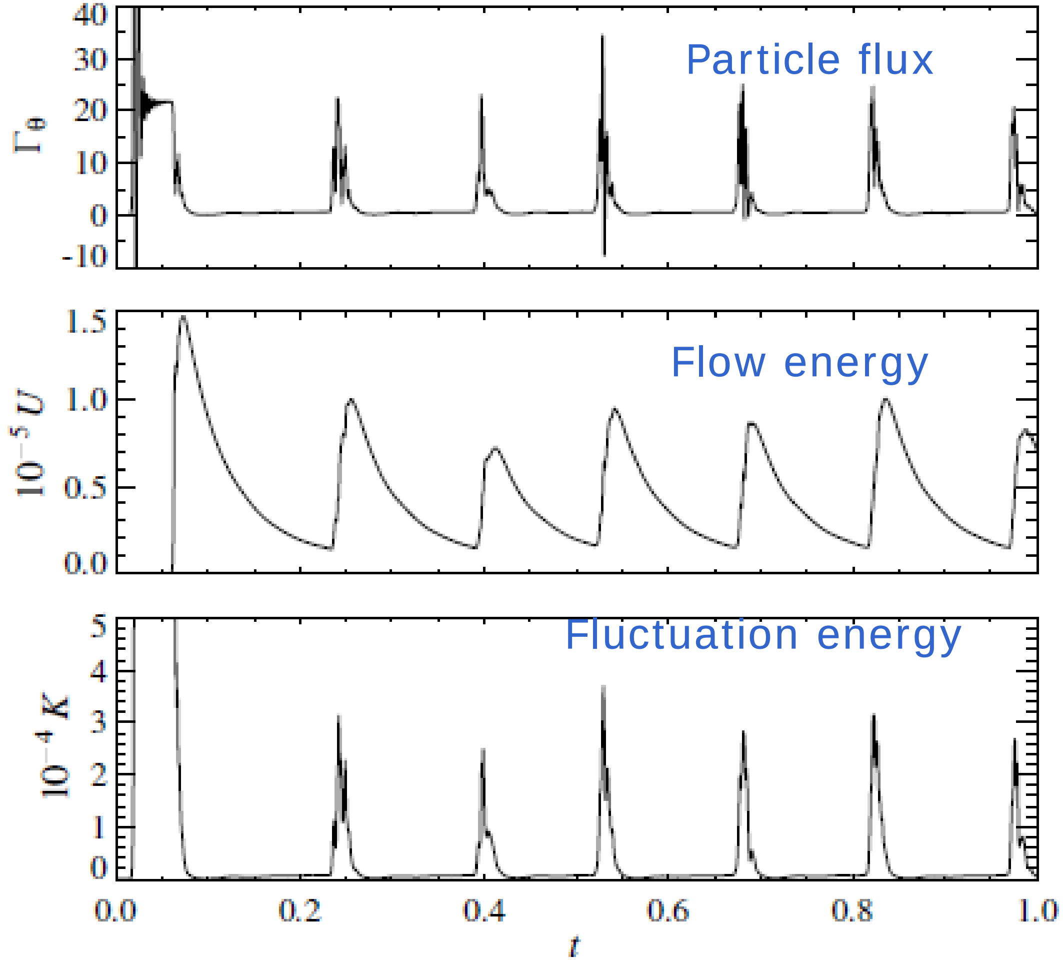
Turbulent velocity $\boldsymbol{v}$ = $\boldsymbol{E}$ **x** $\boldsymbol{B}$
- $\boldsymbol{v}$ = (u,v) in poloidal plane

Terminologi :

Zonal Flows : small scale flows driven by rectified turbulent fluctuations - local transport barrier Diamond et al. PPCF **47,** R35 (2005)

Mean Flows – global poloidal flows : large scale flows in the plasma edge – driven by radial force balance and neoclassical effects – ETB: edge transport barrier

Turbulence-Flow-Flux



Γ_θ flux; U, K energy in the flow, fluctuations

Garcia and Bian PRE **68**, 047301 (2003)

Simulation of convection model, plasma in an inhomogeneous magnetic field.

The turbulent intensity and the radial particle flux across the magnetic field is strongly modulated by the zonal flow generation.

Typical behaviour.

Zonal flow generation by Reynolds stress

$$\frac{\partial \omega}{\partial t} + \{\phi, \omega\} + \mathcal{K}(n+T) = \mu \nabla^2 \omega \,.$$

Reynolds decomposition

$\omega = \Omega + \tilde{\omega},\ \phi = \Phi + \tilde{\phi},\ \mathbf{v} = \mathbf{V} + \tilde{\mathbf{v}}$

$$\Omega = \langle \omega \rangle \equiv \frac{1}{L_y}\int_0^{L_y} \omega dy; \quad \langle \tilde{\omega} \rangle \equiv 0$$

Zonal velocity $V = \langle v \rangle$; $\langle u \rangle = 0$

Re-stress flux of zonal momentum

$$\frac{\partial V}{\partial t} = -\frac{\partial}{\partial x}\langle uv \rangle + \mu \frac{\partial^2}{\partial x^2} V \qquad (\langle \mathcal{K}(n+T) \rangle = 0)$$

Quasilinear approximation: Contribution from the k'te wave-component:

$$\partial_x \langle uv \rangle = -2k \partial_x (|\psi_k|^2 \partial_x \theta_k)$$

θ_k is the phase of ψ_k.

Flow generation for $\partial_x \theta_k \neq 0$ Radial propagation

Diamond and Kim, Phys. Fluids B 3, 1626 (1991)

Flow generation takes energy from the turbulence and limits the turbulent transport – the flow do not contribute to transport

Zonal flow generation and potential vorticity

Homogenization of potential vorticity (PV) in quasi 2-D flows (geophysical flows)

P. Rhines *The Sea (1977); Ann. Rev. Fluid Mech.* ***11,*** *401 (1979)*

Dritschel and McIntyre, *J. Atmos. Sci.* ***65***, *855 (2008)* – PV staircase

$$\frac{D\Pi}{Dt} = \frac{D}{Dt}\left(\frac{\omega + f}{H(r)}\right) = 0$$

Ertel, 1942 – (G.K. Vallis, *Atmospheric and oceanic fluid dynamics. 2006)*

barotropic flows - $\nabla P \times \nabla \rho = 0$

$D/Dt \equiv \partial/\partial t + \mathbf{v} \cdot \nabla \mathbf{v}$, ω is the relative vorticity of a fluid element, f is background vorticity, $H(r)$ is the depth of the fluid layer.

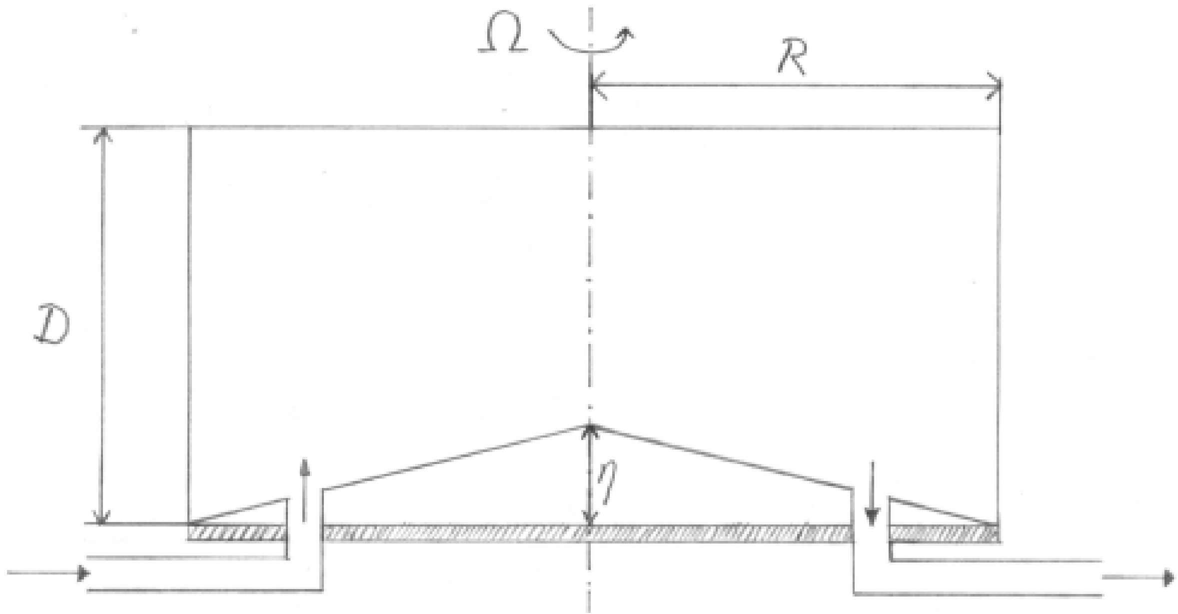
Movement towards deeper regions stretch the vortices and enhance ω; towards shallower regions compress the vortices and decrease ω. Mixing of $\Pi \rightarrow$ low relative vorticity over shallow regions and higher relative vorticity over deeper regions.

Plasma case: Ion vorticity equation (cold ions):

$$\frac{D\Pi_i}{Dt} = \frac{D}{Dt}\left(\frac{\omega + \omega_{ci}}{n(r)}\right) = 0$$

"barotropic flows" - $\nabla P \times \nabla n = 0$

Experiment – PV homogenization



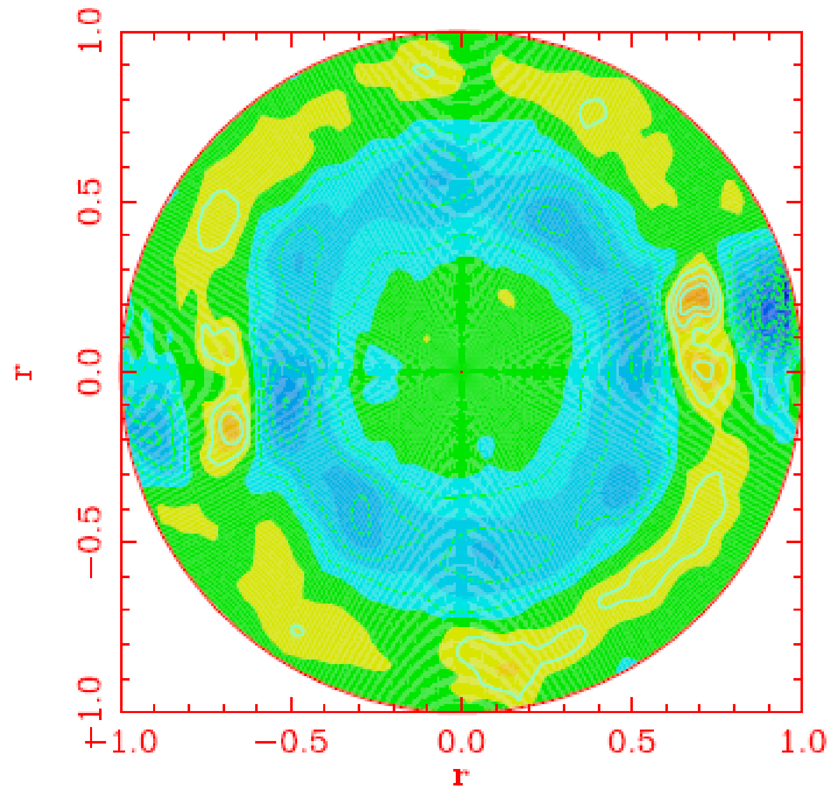
Experimental setup, rotating tank with a rigid lid. R = 19.4 cm, D = 20 cm, $\eta = 5$ cm, rotation rate 12 rpm.

$\Pi = \omega + \beta r$ (expansion $H(r) = 1 - \beta r$)

Mixing: periodically pumping water in and out of two holes (diameter $2\,cm$). Forcing period: T_F ($T_F = 6.6\,s$) Diagnostics: particle tracking: instantaneous velocity field

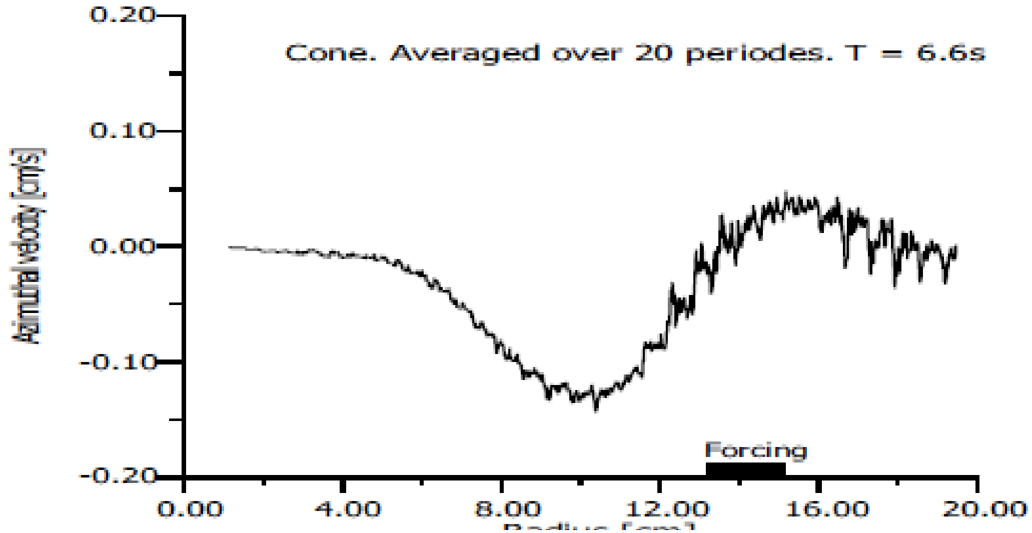
Rasmussen et al Physica Scripta **T122,** 44 (2006)

Azimuthal velocity



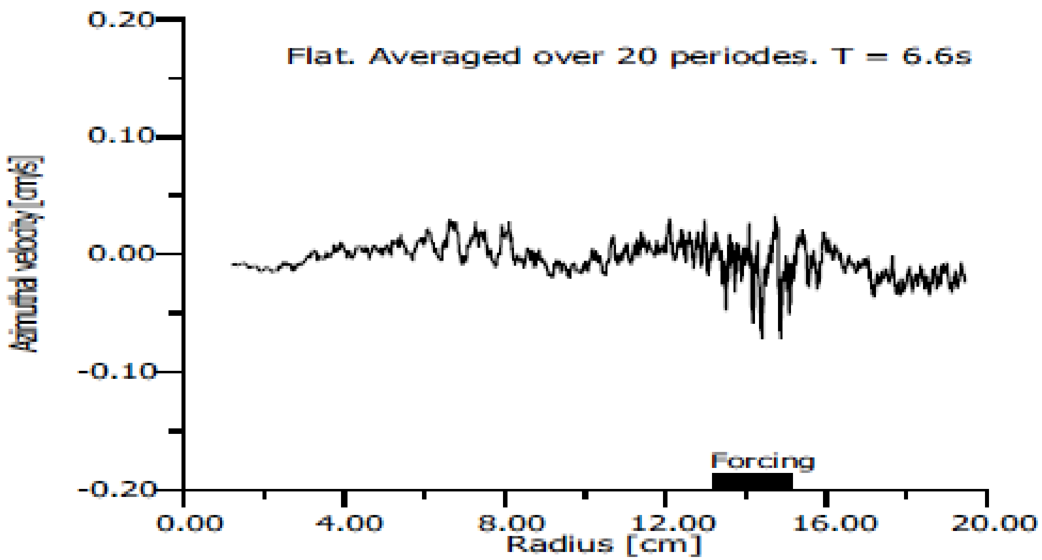
The azimuthal velocity component averaged over 20 forcing periods. Blue designates negative velocity, i.e. anti-cyclonic motion and red designates positive velocity

Azimuthal velocity profile



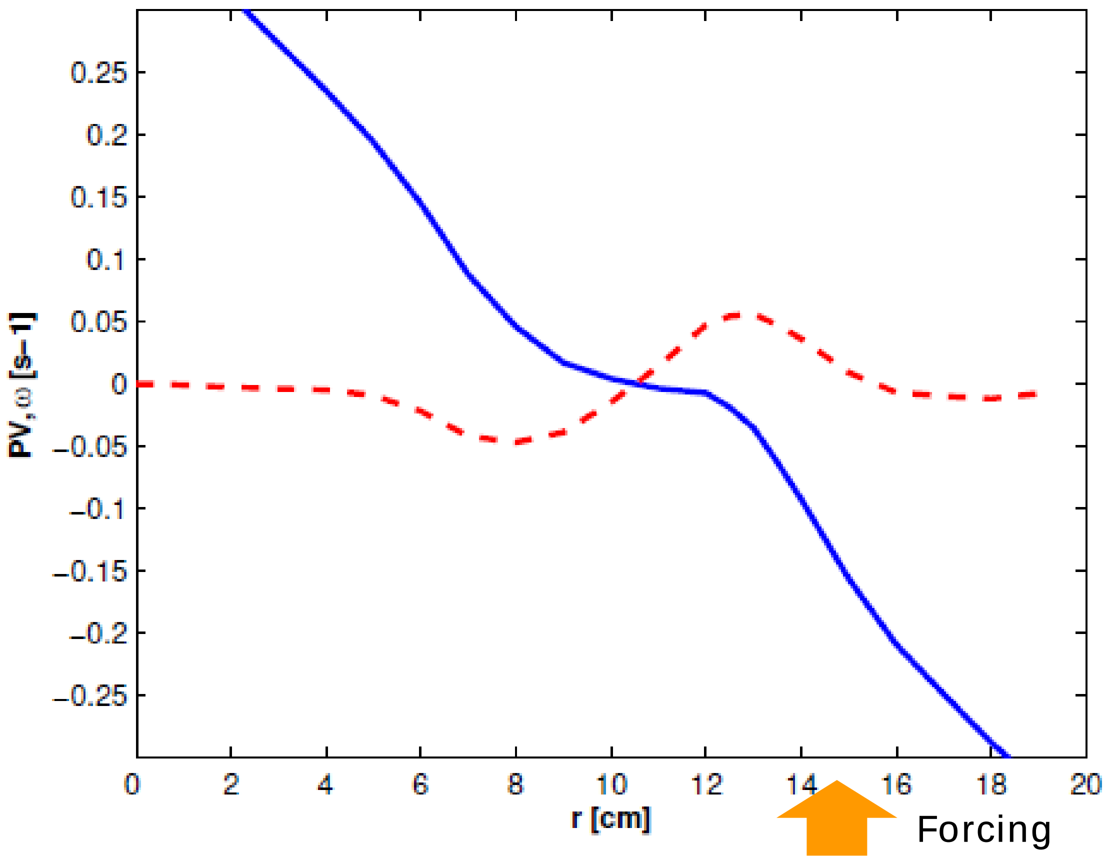
Azimuthally averaged velocity profile –

cone bottom



flat bottom – no flow!

Potential vorticity profile



Azimuthally averaged potential vorticity and fluid vorticity

Maximum velocity set by total homogenization of PV

Numerical modelling

The forced quasi-geostrophic vorticity equation on a disk with no-slip boundary conditions at the walls.

$$\frac{\partial \omega}{\partial t} + \frac{1}{r}[\phi, \omega] - \frac{\beta}{r}\frac{\partial \phi}{\partial \theta} = -\nu\omega + \frac{1}{Re}\nabla^2\omega + F \ ,$$

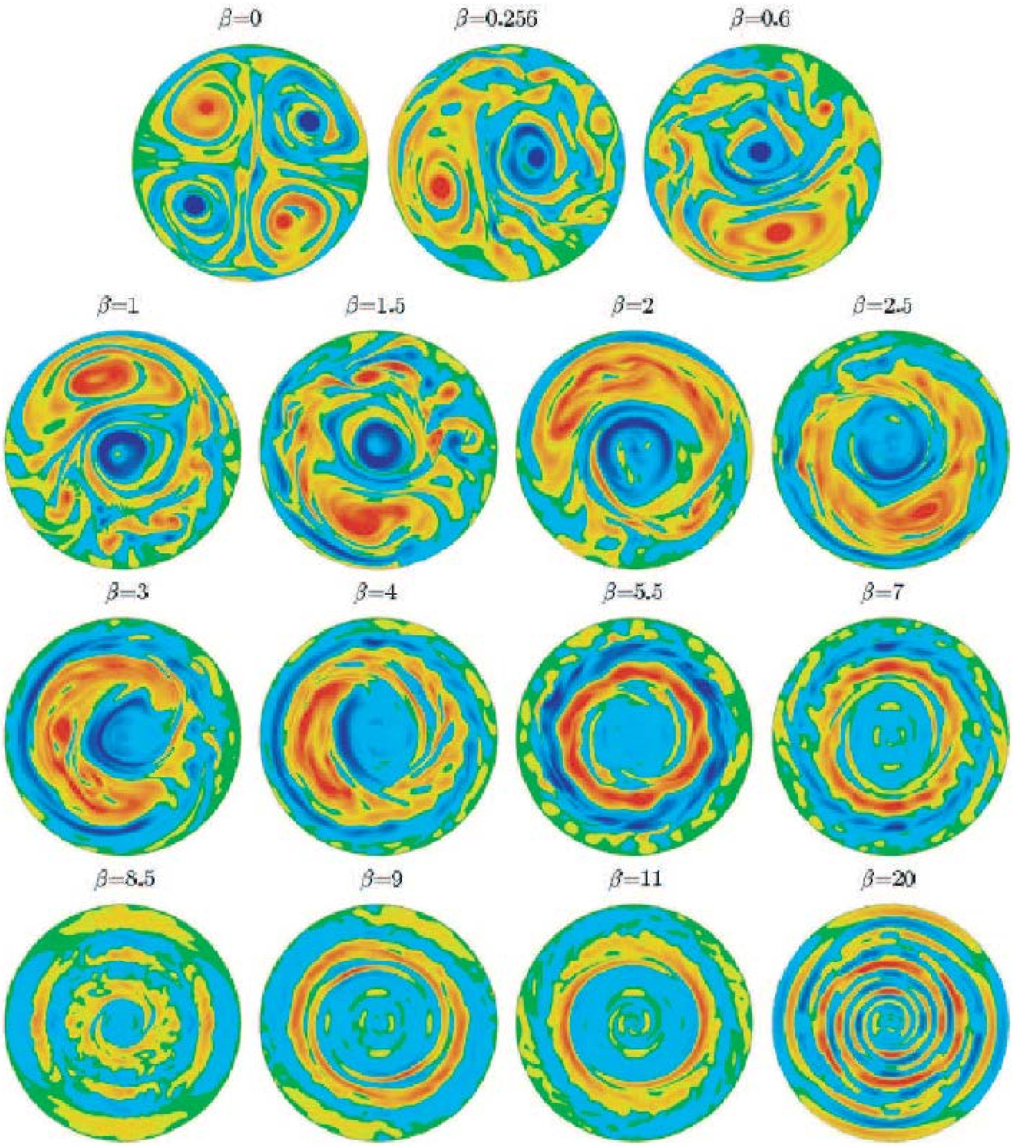
Length is scaled as R, time as f^{-1}, and β by f/R. $\nu = \sqrt{E}$, Ekman number $E = \mu/D^2\Omega$ with a spin down time $\tau_E \approx 90\, s$.

The forcing is modeled by localized vorticity sources with alternating positive and negative vorticity:

$F = A_0[G(x, y; r_1)\sin(\sigma_F t) + G(x, y; r_2)\sin(\sigma_F t + \pi)]$, $G(x, y, r_{1,2})$ localized at the positions of the two holes.

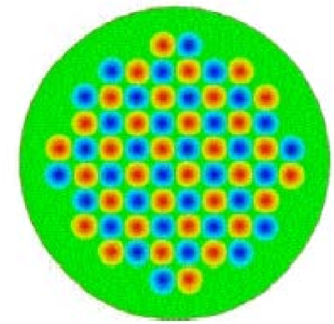
For the experimental condition the scaled values of $\beta = 0.256$ and $E = 4.55 \times 10^{-4}$. While $Re \approx 80.000$ and volume viscosity is negligible.

Vorticity - simulations



Vorticity for different values of β.

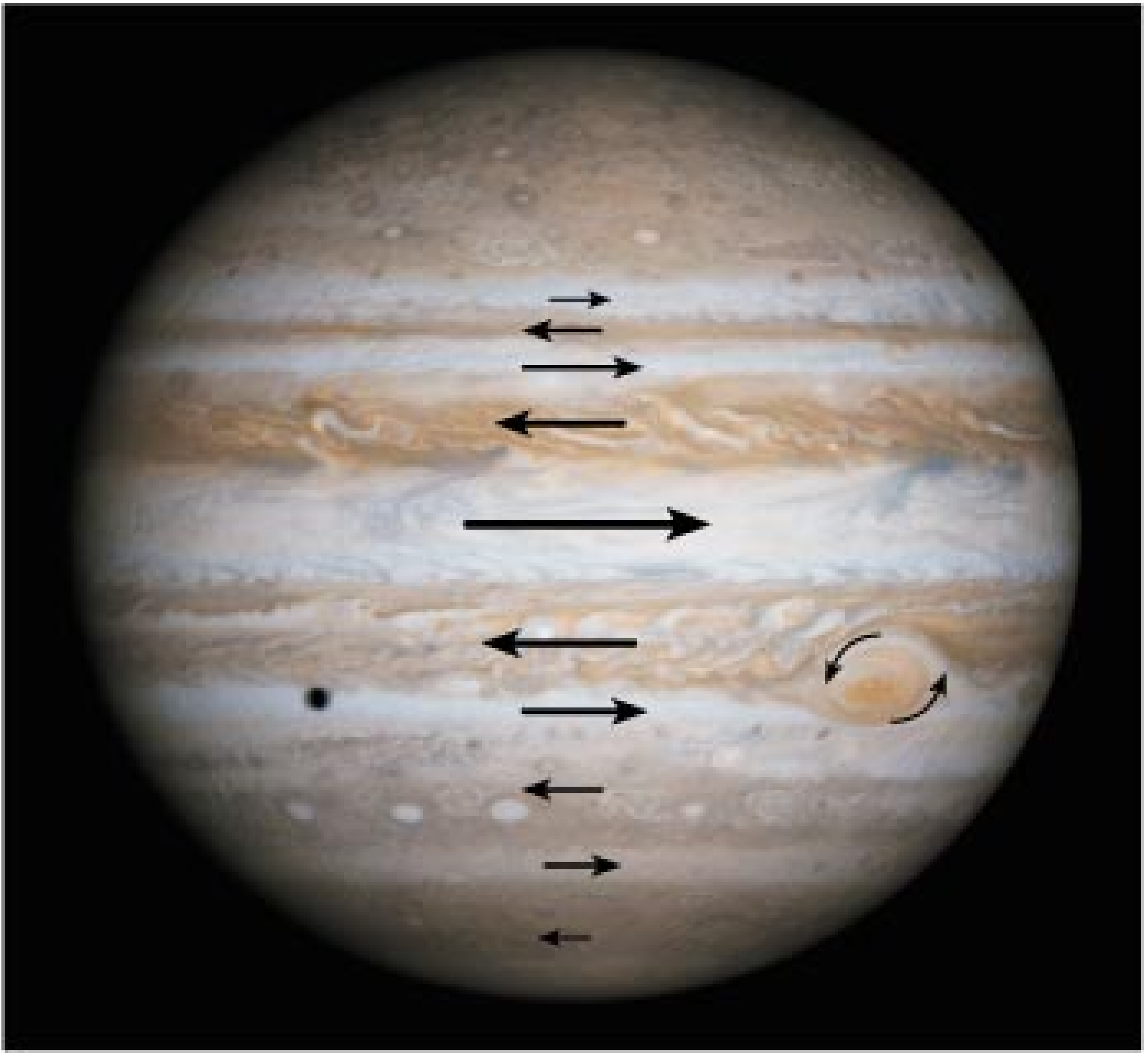
Forced turbulence by array of vortices with oscillating vorticity:



Number of zonal bands increas with β – width realated to the Rhines scale

$$L_\beta = (2<u>/\beta)^{1/2}$$

Jupiter zonal flow bands



Modeled by PV homogenization – almost – GRS anomaly

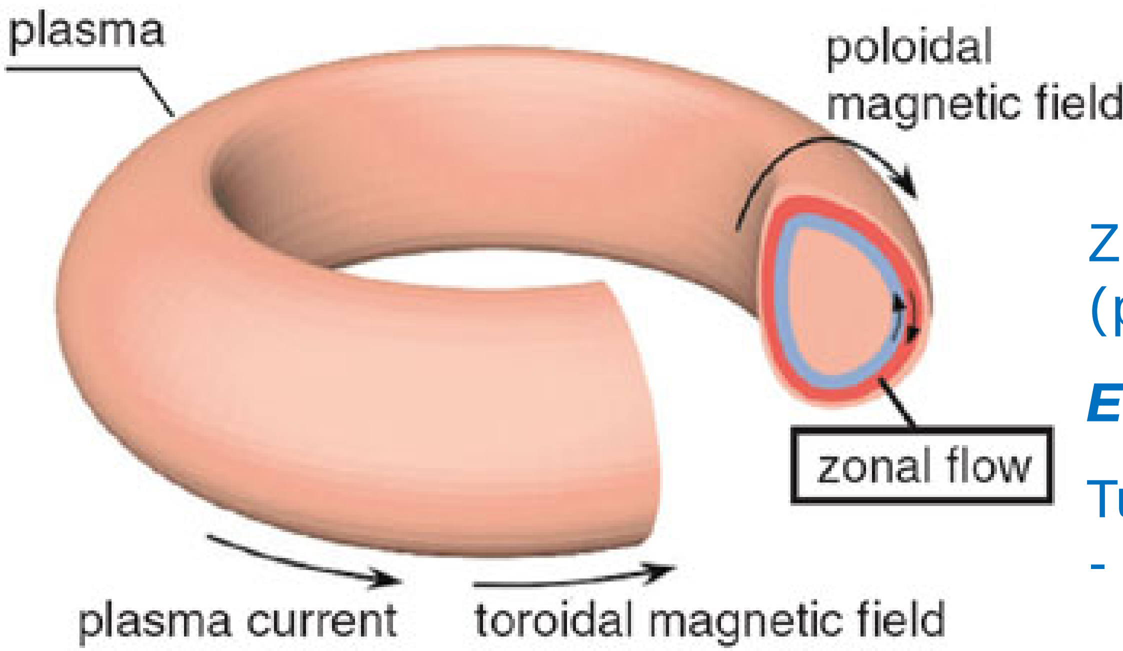
PV-staircase – piecewise constant PV

Marcus and Shetty *Phil. Trans. R. Soc. A* ***369,*** *771 (2011)*

Flow generation in magnetically confined plasmas –

L-H transition

Zonal flow in magnetically confined plasma



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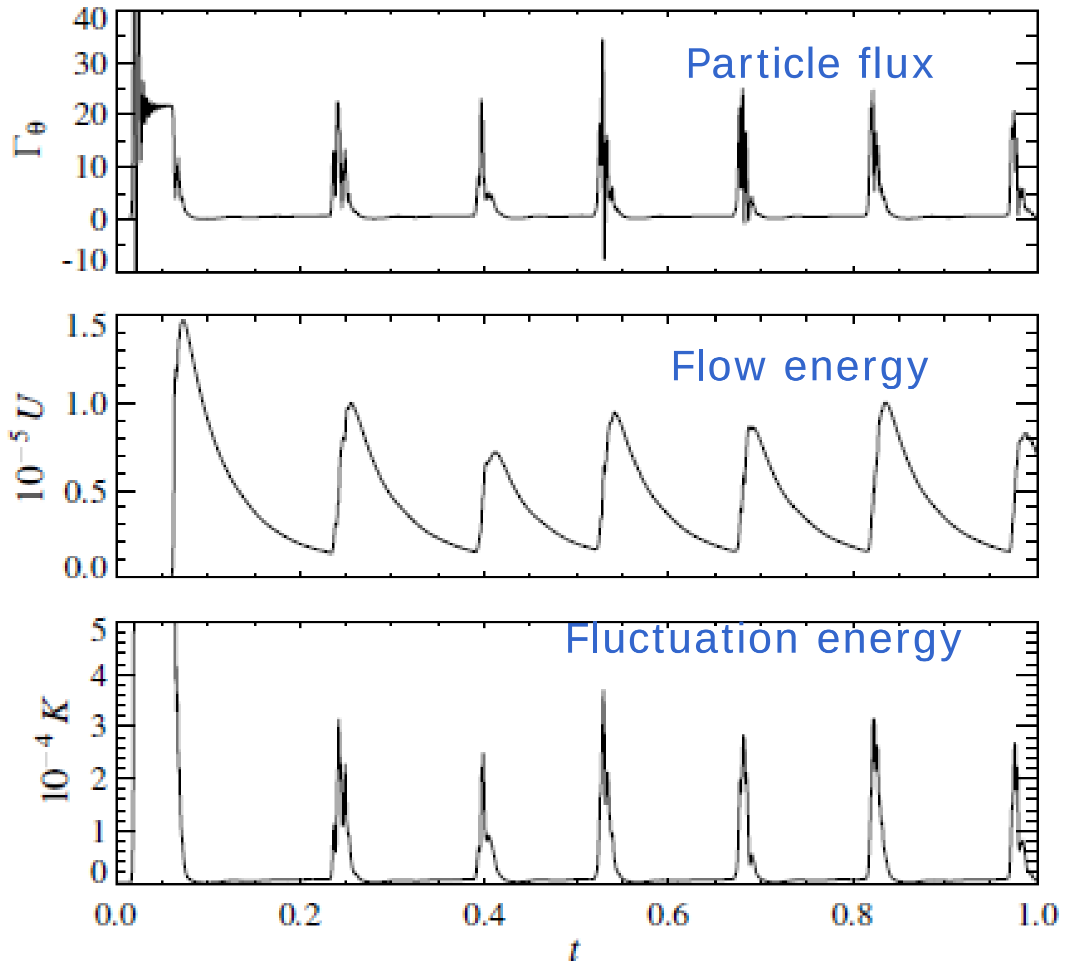

Γ_θ flux; U, K energy in the flow, fluctuations

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Simulation of convection model, plasma in an inhomogeneous magnetic field.

The turbulent intensity and the radial particle flux across the magnetic field is strongly modulated by the zonal flow generation.

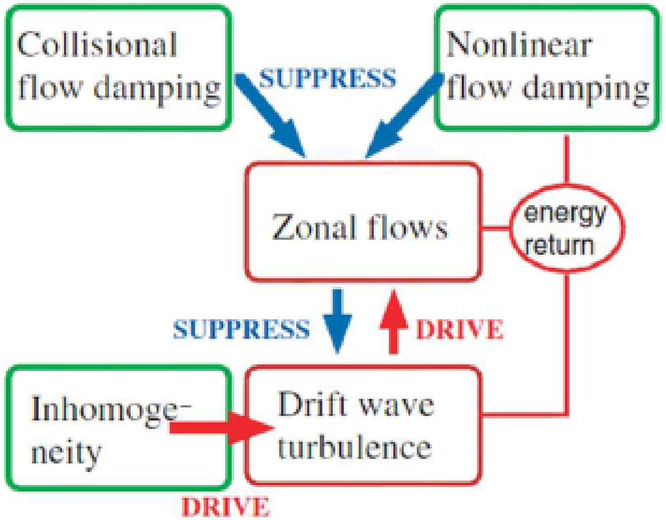
Typical behaviour.

Model :: Turbulence – flow interacting

- Closing the loop of shearing and Reynolds work

  (∗ Self-Regulating System)

- Spectral 'Predator-Prey' equations



Prey → Drift waves,

Diamond et al 2011

$$\partial_t N = \gamma N - \alpha V^2 N - \Delta\omega N^2,$$

Predator → Zonal flow,

$$\partial_t V^2 = \alpha N V^2 - \gamma_d V^2 - \alpha_2 V^4.$$

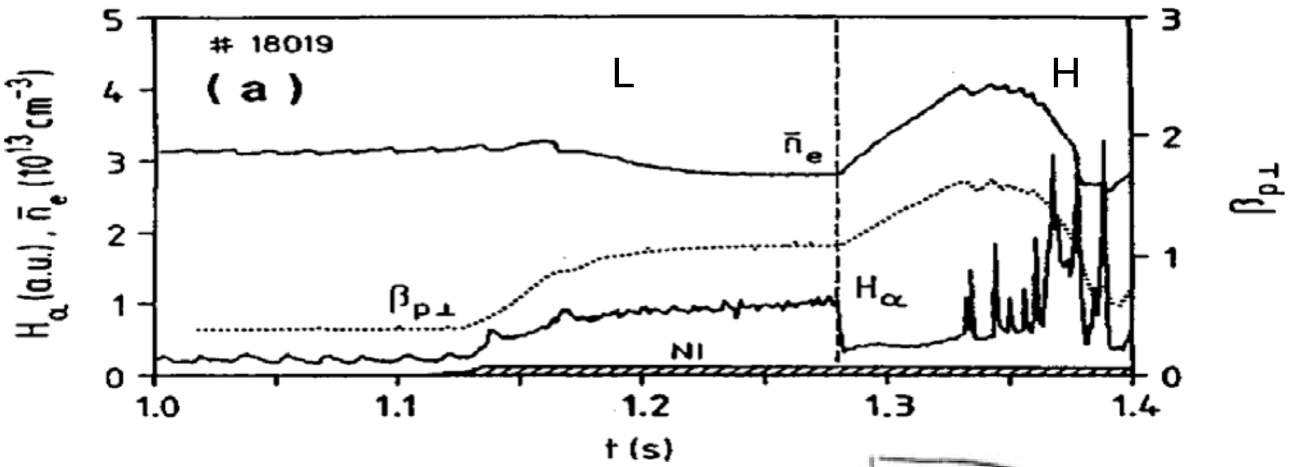
V – Flow velocity predator

N - turbulence energy prey

Various solutions incl. limit cycle solutions

High confinement mode in Tokamak discovered on ASDEX in 1982

Wagner, ASDEX, PRL 1982, 1989



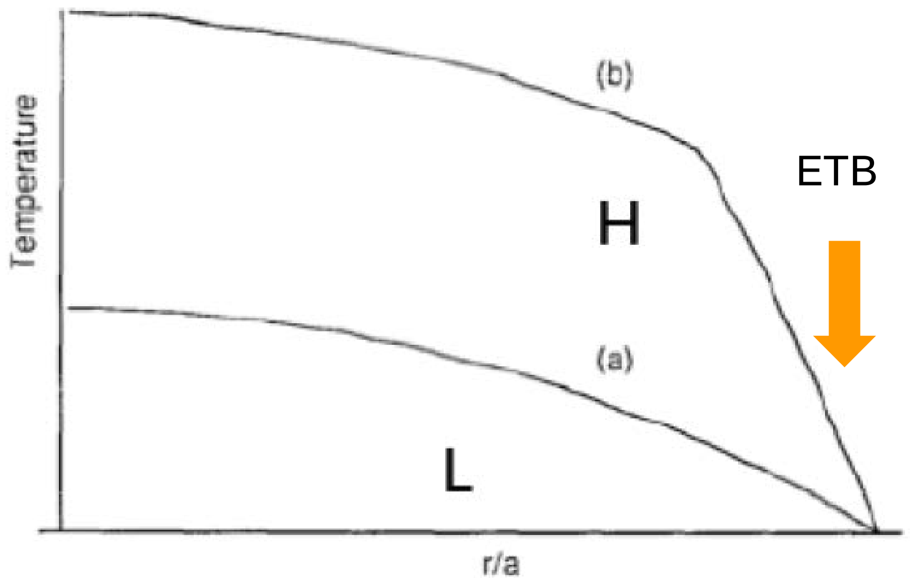
Transition – abrupt in response to a slowly changing parameter- energy input:

"Phase transition"

Temperature (pressure) profiles in L- and H-mode.

H-mode essential for a viable operation of a fusion reactor – and ITER

Transport barrier (ETB) is set up near the edge – mediated by sheared poloidal flows



L-H transition modeling: predator – prey models

Two predators:

$$\frac{d}{dt}I = I\left(G - a_1 I - a_2 E_{\mathrm{mf}} - a_3 E_{\mathrm{zf}}\right),$$ Turbulent intensity

$$\frac{d}{dt}E_{\mathrm{zf}} = 2E_{\mathrm{zf}}\left(\frac{b_1 I}{1 + b_2 E_{\mathrm{mf}}} - b_3\right),$$ Zonal flow shear

$$\frac{d}{dt}G = Q(t) - G\left(c_1 I + c_2\right)$$ Pressure gradient: $E_{mf} = c_3 G^4$

(E_{mf} mean flow shear)

3 coupled ODE – Kim & Diamond PRL 2003 --- 0 space dimension

Detailed analysis of the dynamical properties – finding conditions for L-H transition from the bifurcation properties.

M. Dam et al Phys. Plasma **20**, 102302 (2013);

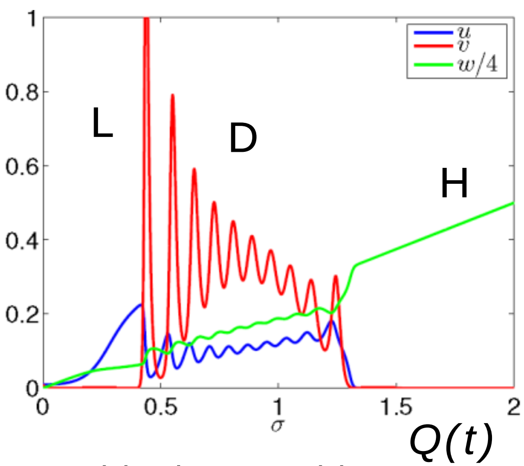
Reproduce quantitatively experimental observations – but no predictions!

Apply the results as guide-line for modelling based on first principle models.

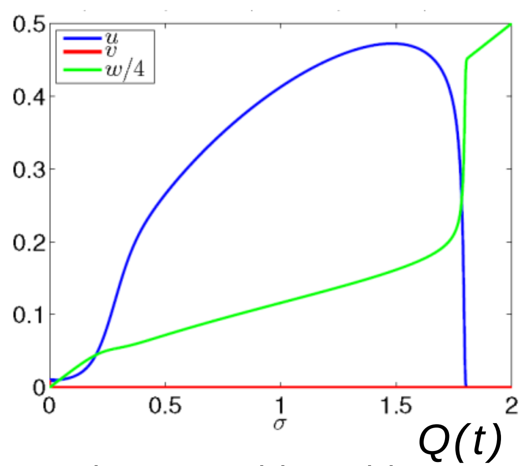
L-H Transition scenarios

Typical transition scenarios observed in Tokamaks

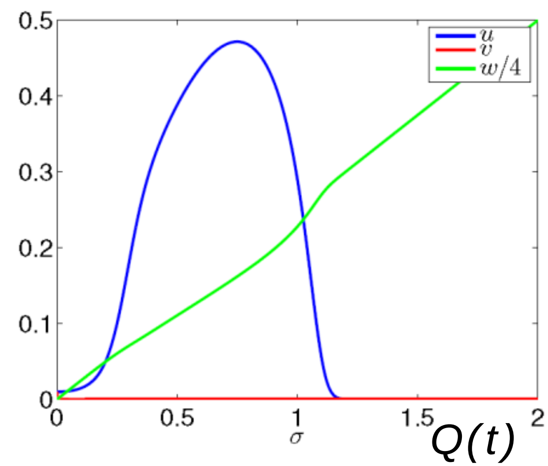
Control-parameter ion energy input – *Q(t)*



Dithering transition
Slow heating*



Sharp transition with
hysteresis – no ZF**



Smooth transition – no ZF

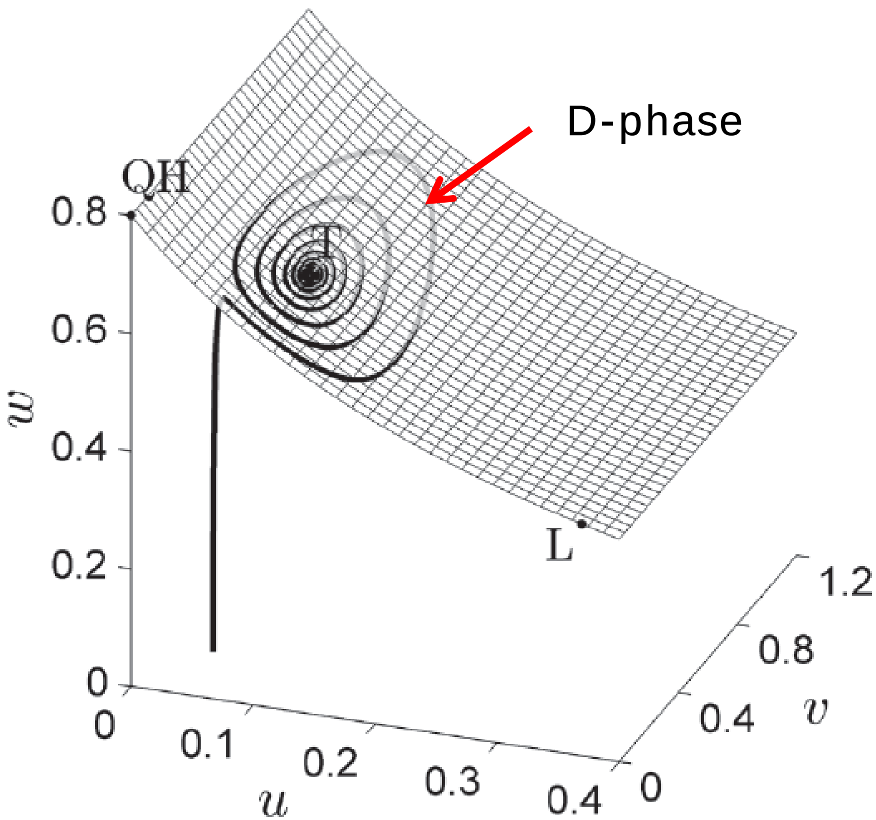
u– turb instensity, v – zonal flow shear, w-density gradient – propto mean flow

*Bifurcation analysis shows a stable fix point in the D-phase transforms into and unstable fixpoint - Hopf bifurcation → enter into H-mode – stable equilibrium

**No stable fixpoint in D-phase and direct transition from L-equilibrium to H-equilibrium.

Slow transition dynamics

The dynamics is essential ”2D”: *u* and *v* – w ”slaving variable” – a reduced 2 ODE model reproduce the 3 ODE results – for the slow transition.



$$\dot{u} = u\left(w - u - v - w^4\right)$$

$$\dot{v} = \mu_1 v\left(\frac{u}{1+\mu_4 w^4} - \mu_2\right)$$

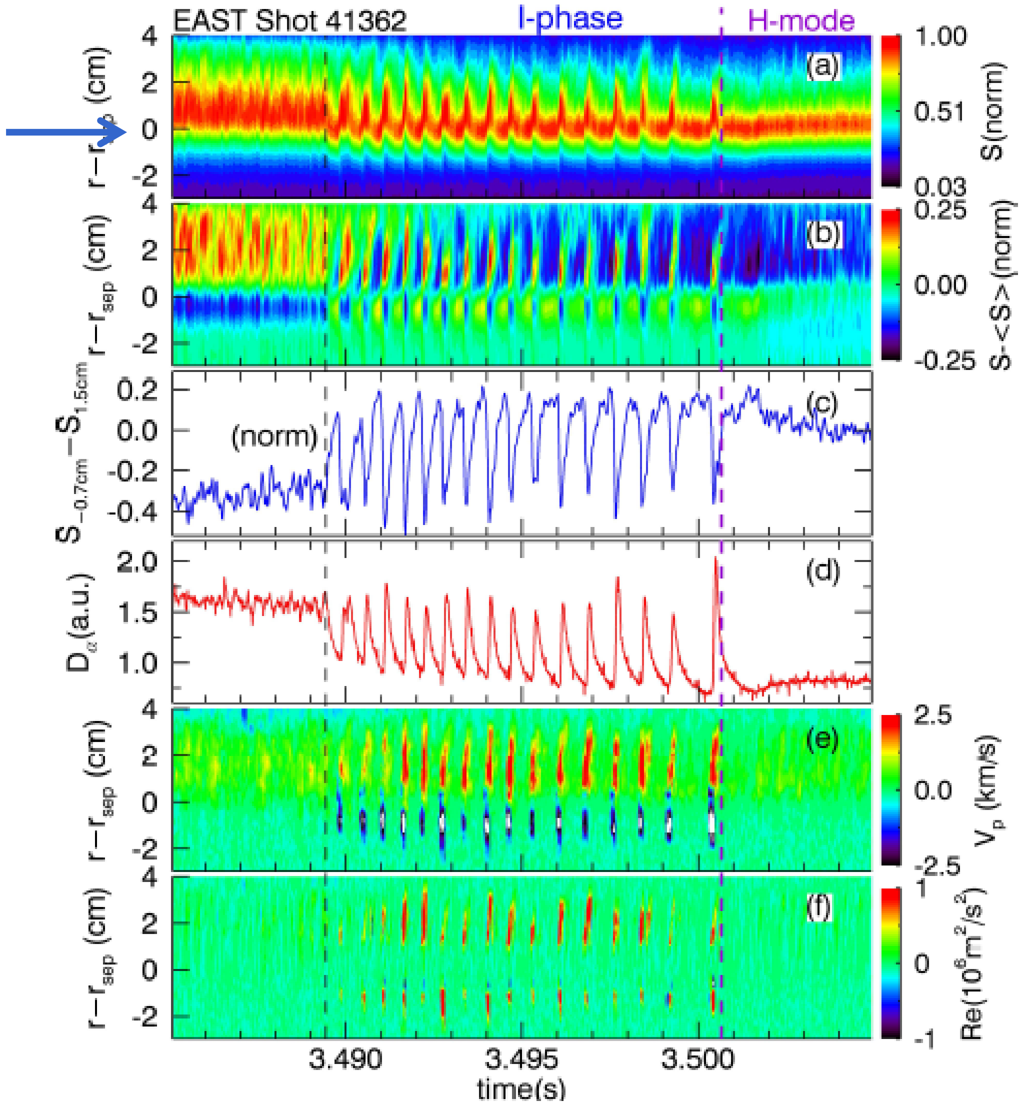
$$w = \frac{\sigma}{1+\mu_3 u}$$

Critical manifold

L-H transition with dithering phase

Gas puff imaging, GPI - Deuterium

EAST Tokamak, Hefei CN



Pressure evolution

Pressure fluctuations

Pressure difference – ”gradient”

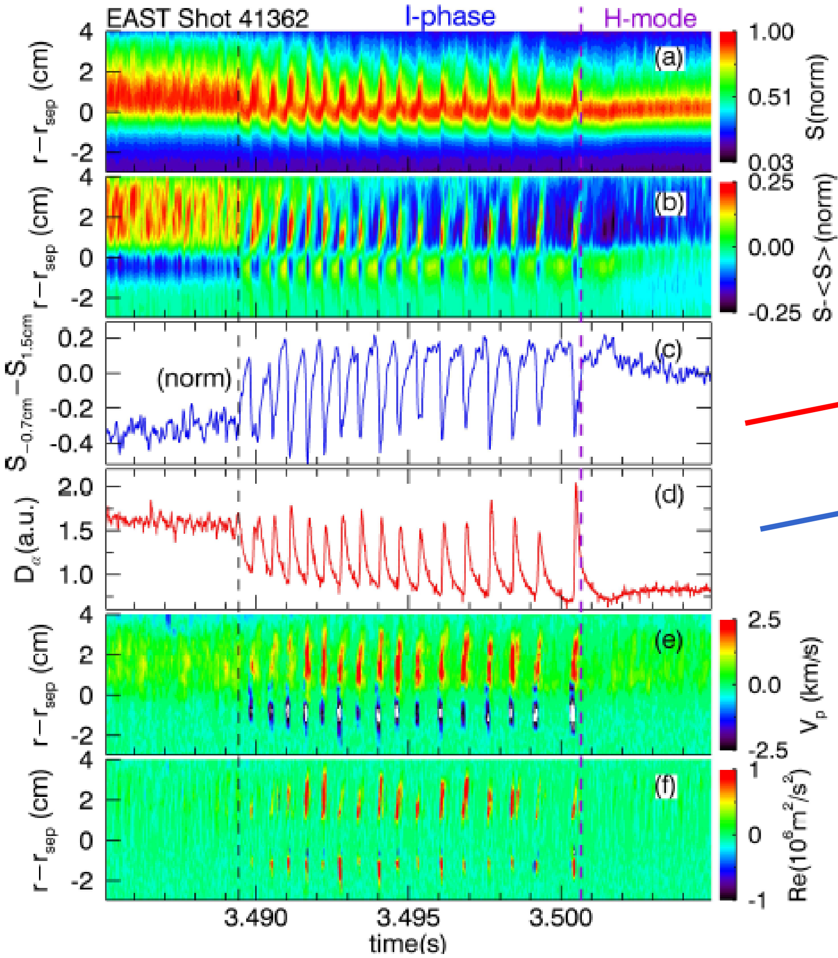
”Particle transport”

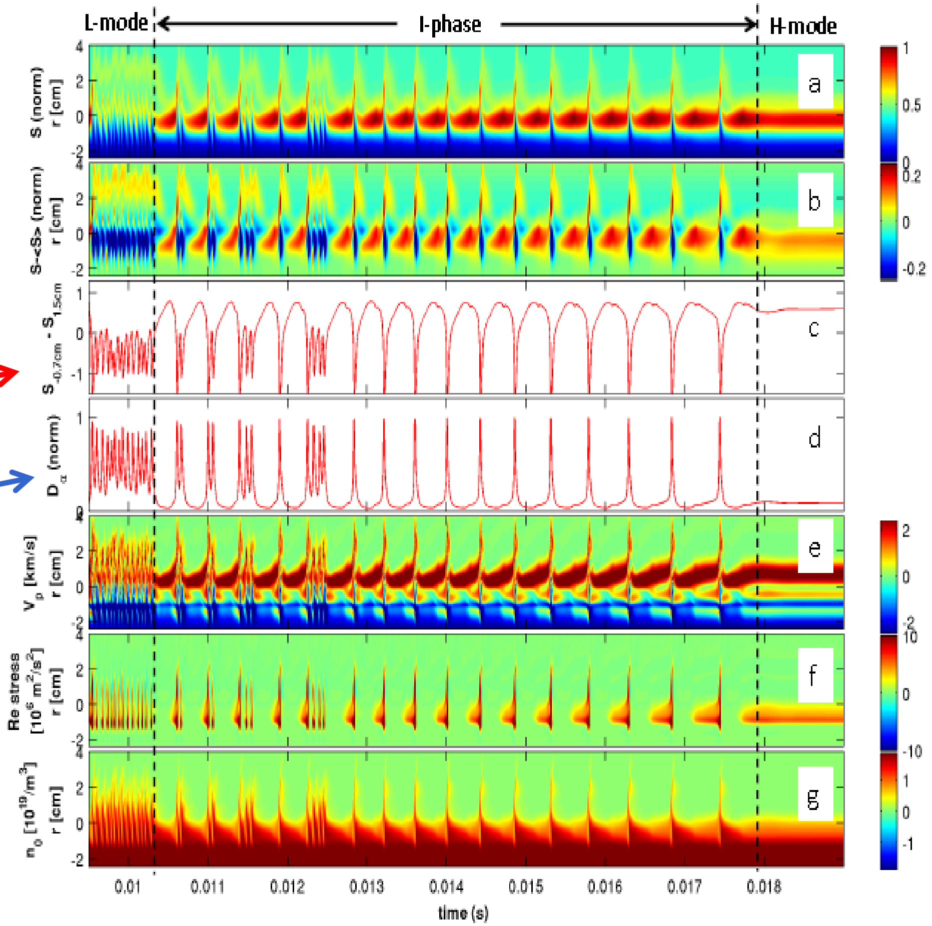
Poloidal velocity

Reynolds stress

G.S. Xu et al. *Nuclear Fusion* ***54***, *013007 (2014).*

L-H transistion: Simulation results





Experiments

EAST Tokamak, Hefei CN

Simulations – first principle model:

4-field fluid model - HESEL

Nielsen et al US – EU --TTF Workshop September 2014

Summary

- Zonal flows are generated by rectifying small scale turbulent fluctuations

- Zonal flows, sheared flows – transport barrier wrt. turbulent transport

- Flow generation by homogenization of potential vorticity PV in rotating fluids

- Role of in magnetized plasma - flow generation and transport barrier.

- Flows are essential ingredients in the L-H transition in magnetically confined plasma

- Modelling –Predator-Prey type models – towards first principle models