VII Int. Conference on “Solitons, Collapses and Turbulence”, Chernogolovka 2014

GENERATION OF SUBHARMONICS IN THE 2-D TURBULENCE SPECTRA ON THE SURFACE OF LIQUID HYDROGEN

I.A. Remizov, M.Yu. Brazhnikov, A.A. Levchenko and L.P. Mezhov-Deglin

ISSP RAS

… “ the theory of KZ spectra looks elegant and self-contained. Therefore this theory should not be left aside. But further developments and justification are needed to strengthen it…” V.E. Zakharov

ON THE SURFACE OF LIQUID HYDROGEN
IN THE RECTANGULAR CELL:
observations of the direct capillary KZ-cascades accompanied by generation of the low frequency subharmonics including capillary and gravity waves.

GODPARENTS OF OUR STUDIES OF THE WAVE TURBULENT PHENOMENA ON THE SURFACE AND IN BULK OF QUANTUM LIQUIDS ARE:

***GODFATHER* - V.E. ZAKHAROV,**

***COLLECTIVE GODMATHER* - LABORATORY OF QUATUM CRYSTALLS, ISSP RAS**

In collaboration with Landau Inst.Theor. Phys., Lancaster University (P.V.E. McClintock), and NY City College (G.V. Kolmakov).

- *As a result of this nonlinear interaction of the theory and experiment the twin brother – joint seminar "Nonlinear waves”* **-** *was established, headed by V. Lebedev (ITP) and A. Levchenko (ISSP)*

Short content of the report.

•***Experimental investigations*** of the wave turbulence on the charged surface of liquid hydrogen in the rectangular cell supplied with the external collector (capacitor approx.)

• ***Results of observations***:

*Formation of the direct KZ cascade of capillary waves on monochromatic excitation at the resonant frequencies f_p in the range of 8 - 30Hz;

**Generation of low frequency subharmonics (down to 0.8 Hz) and appearance of the combinational harmonics in the direct KZ cascade on detuning the drive frequency and/or increasing the amplitude of the driving voltage $U_{ac} = U_p \sin \omega t$;

***Reversible evolution with time of the KZ cascade and the amplitudes of subharmonics on switching by step the drive frequency f_p or the driving voltage U_p.

Discussion:

*Near linear dispersion law on the charged surface in the range $1<k<10$ cm^{-1} (dispersion relation of of the decay type) at voltages $U_{dc} \geq 800$ V ($E>2.3$ kV/cm);

** Formation of the direct KZ cascade of capillary waves ***due to the nonlinear 3-wave interaction:*** $f_p + f_p \rightarrow 2 f_p$, etc. , on driving at frequencies $f_p \geq 8$Hz;

generation of the low frequency subharmonics $f^s_{1,2} \geq 5$ Hz and generation of the combinational harmonics direct KZ cascade due to ***3-wave interaction: $f_p \rightarrow f^s_1 + f^s_2$; $f_p + f^s_{1,2} \rightarrow f_{comb}$ on small detuning of the drive frequency at high U_p;

****observations of the low frequency subharmonics generated *in the range of* $f^s_3 \leq 1$ *Hz* ***due to the 4-wave process***: $f^s_1 + f^s_1 \rightarrow f^s_2 + f^s_3$; (analog gravity waves);

*****evolution of the turbulent cascades with time on changing by step the drive frequency or amplitude;

Comparison with results of other recent studies.

Wave turbulence theory by V.E. Zakharov

Turbulence in a system of interacting waves is manifested in various physical systems. Wave turbulence theory based on a kinetic equation for a wave ensemble predicts a steady-state scale-invariant solution that describes a constant flux of energy towards smaller scales, which is referred to as a direct energy cascade. Such a power-law spectrum can be viewed as the wave analog of the Kolmogorov spectrum of hydrodynamic turbulence, and is referred to as the Kolmogorov-Zakharov (KZ) spectrum of wave turbulence [1,2].

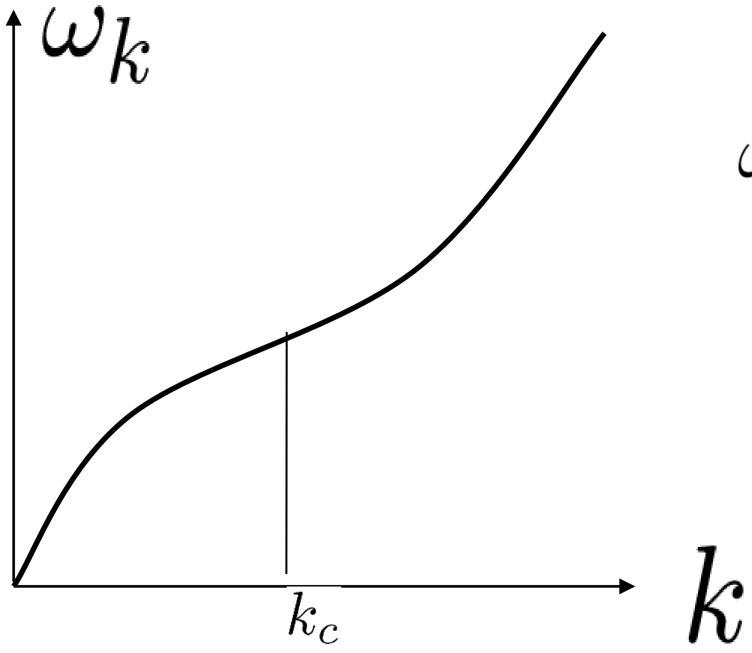
[1] V. E. Zakharov, V. S. L'vov, G. Falkovich, Kolmogorov Spectra of Turbulence I. (Springer, Berlin, 1992).

[2] Vladimir Zakharov, Frederic Diasc, and Andrei Pushkarev, One-dimensional wave turbulence. Physics Reports 398 (2004) 1 – 65

"…KZ spectra describe the transport of integrals of motion (energy, wave action, momentum) to the regions of small or large scales. In our opinion KZ spectra play a central role in wave turbulence. There is strong experimental evidence in support of this point of view. KZ spectra for capillary wave turbulence were observed independently in three laboratories (at the Physics Department of the University of California, Los Angeles [87], at the Niels Bohr Institute, Denmark [79,42] and at the Institute of Solid State Physics, Russia [14–17])… the theory of KZ spectra looks elegant and self-contained. Therefore this theory should not be left aside. But further developments and justification are needed to strengthen it…"

Waves on an unrestricted flat surface

Dispersion law for linear capillary-gravity waves (small amplitude)



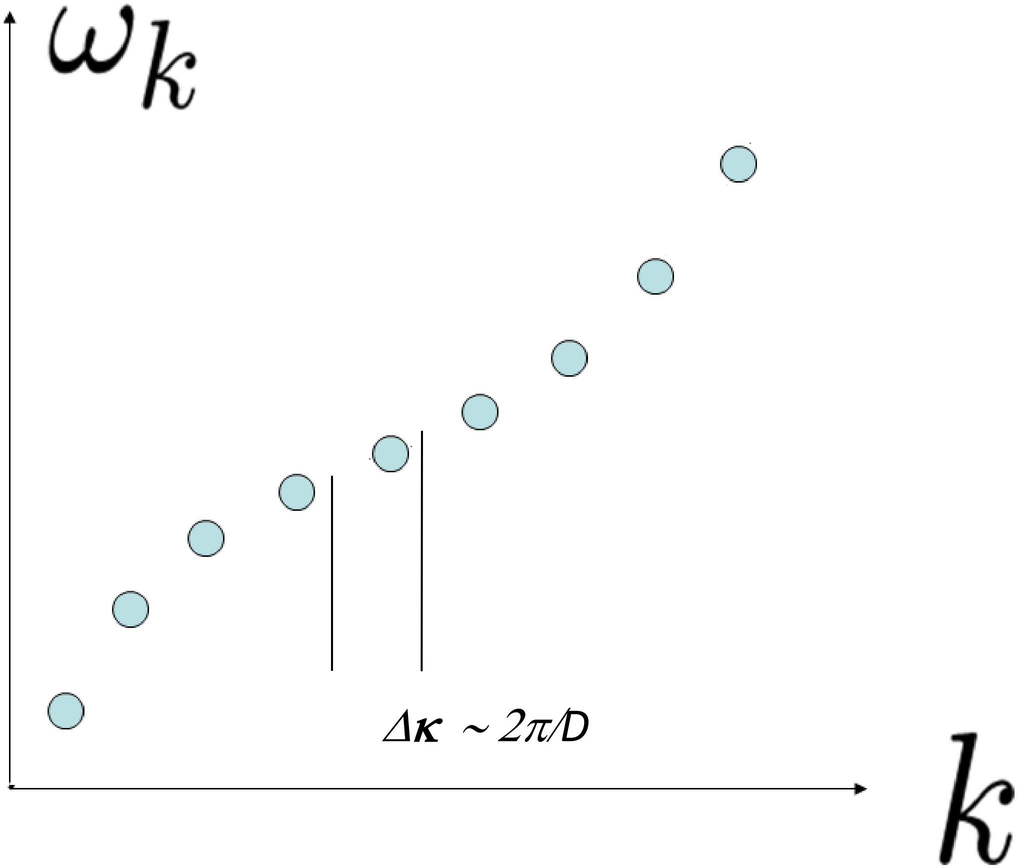
$$\omega_k^2 = gk + \frac{\alpha}{\rho}k^3$$

$k_c = (\alpha/\rho g)^{-1/2}$

Liquid hydrogen: $\lambda_c = 1.2$ cm; $k_c \approx 5\ \text{cm}^{-1}$, $\omega_c/2\pi \approx 10$ Hz

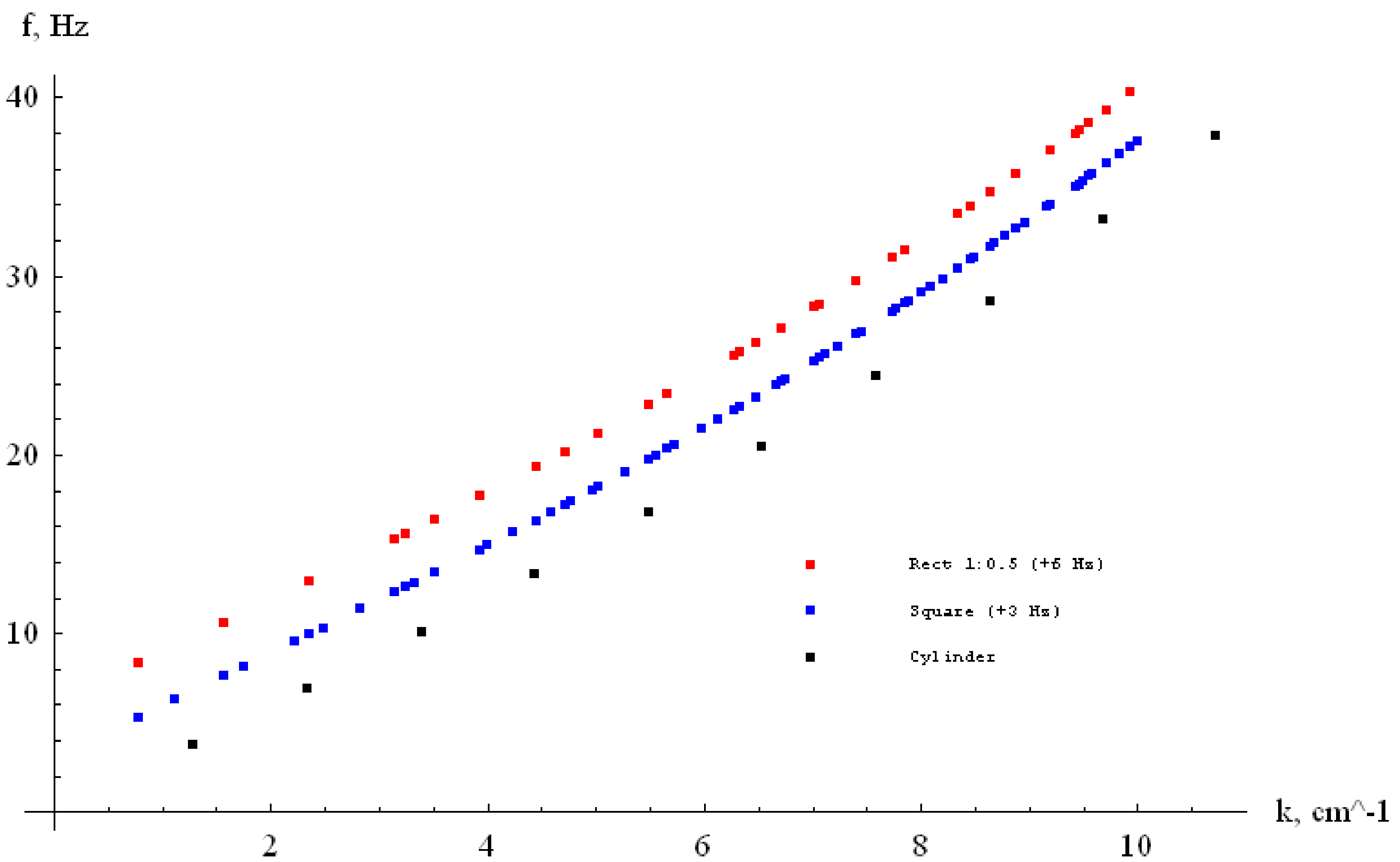
Superfluid He II : $\lambda_c = 0.17$ cm ; $\omega_c/2\pi = 25$ Hz

Discrete spectrum in a cylindrical cell (deep fluid, D – cell diameter)

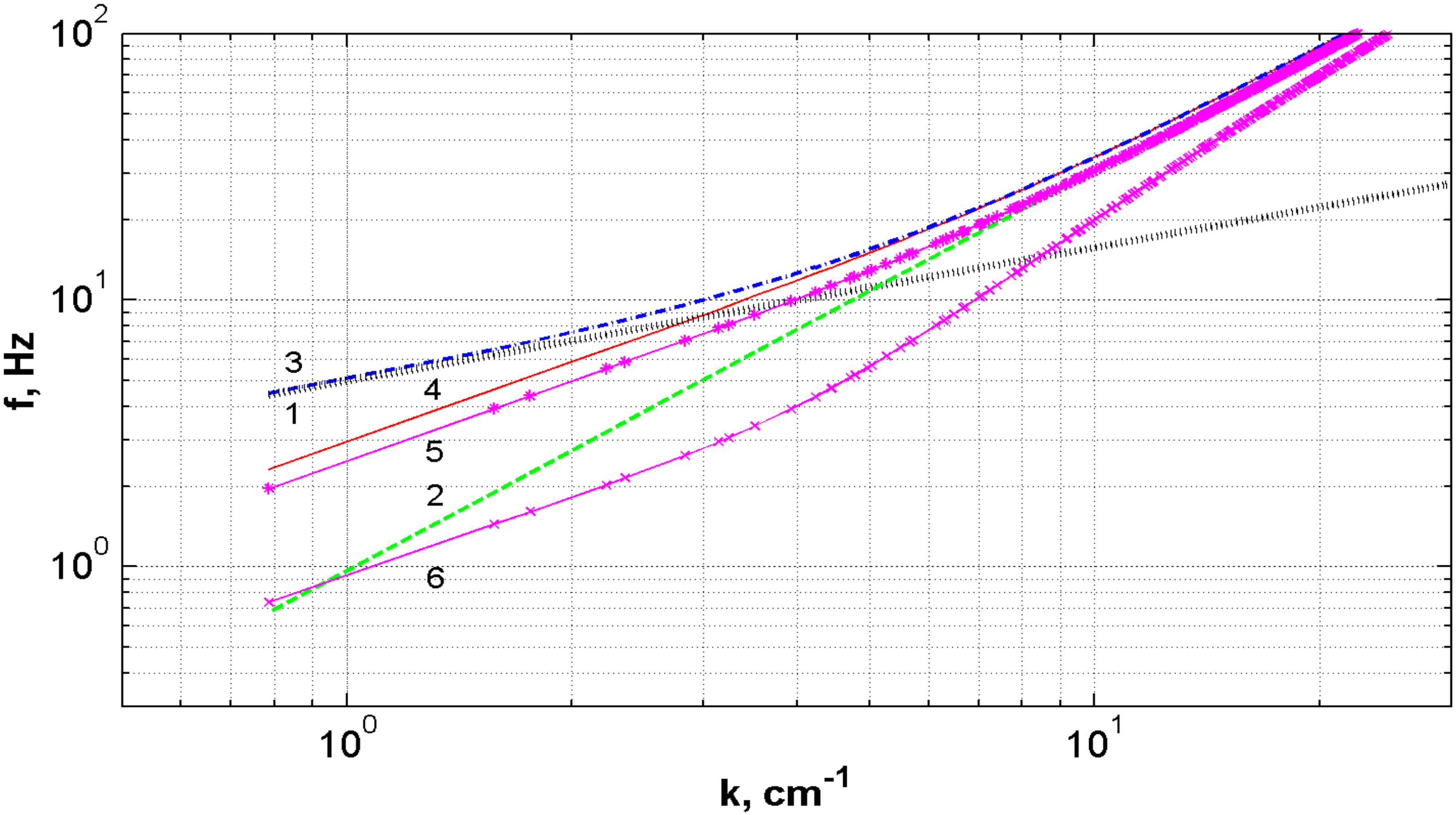


Distance between two resonances in *k space*

$$\Delta k \sim \pi/D$$



Surface oscillations. The resonant modes in a cell filled with liquid hydrogen:
cylinder 6 cm in diameter (dark points ▪ **); **a square 3x3 cm (**blue points: shifted to 3Hz**), and
a rectangular channel 4x2 cm in dimensions (red points, shifted to 6 Hz) .
Number of modes: cylinder – 9 (radial symmetry); square 66; rectangular cell – 50.



Waves on a surface of liquid hydrogen.

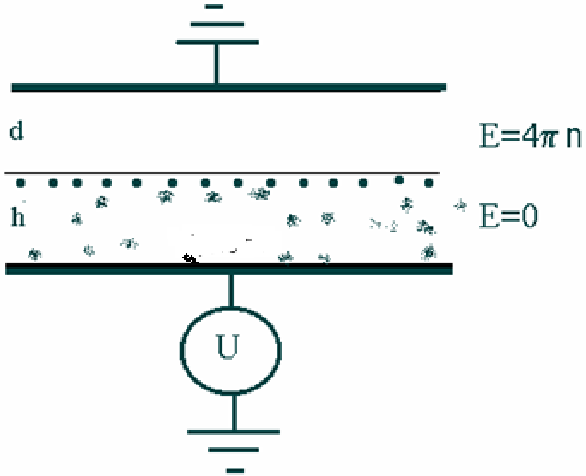
Dispersion law: $\mathbf{w^2 = th(kh)\,(gk + (\sigma/\rho)\,k^3 - (P/\rho)\,k^2\,cth(kd))}$

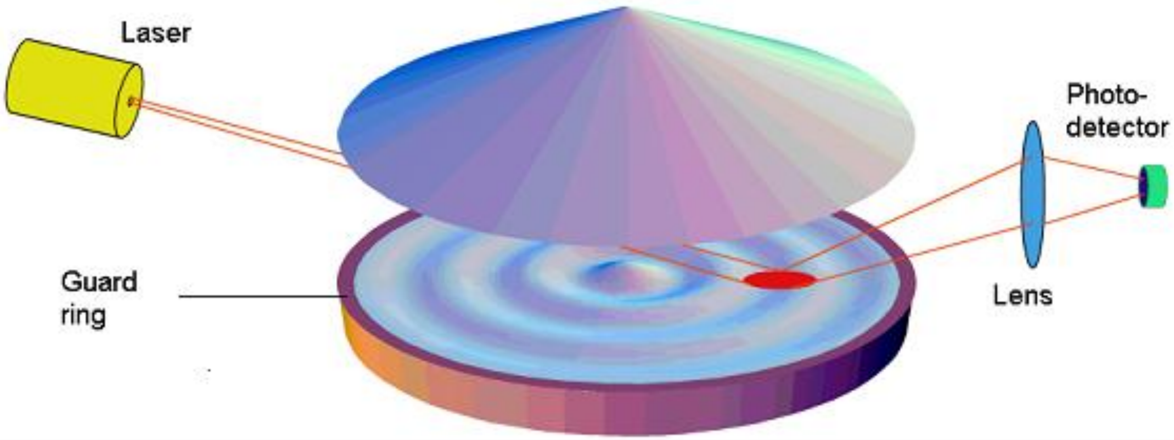
Curves **1**, 2, 3 –deep fluid approximation, **1** –**gravitational waves only** (σ ~0); **2 – the capillary waves in microgravity g~0**; **3-gravitational-capillary waves in the gravity field**.

Curves 4,5,6- liquid in the rectangular cell: cell depth h=0.35 cm; linear dimensions 2.0x4.0 cm; the gap *charged surface- collector* d=0.35 cm. Electric field pressure $\mathbf{P=(U_{dc}/d)^2/8\pi}$

4- U_{dc} =0;; 5 - U_{dc} =800 V; 6- U_{dc} =1800 V. :

Method of measurements





Oscillations of the fluid surface elevation **η (r, t)** were detected through variations of the power **P(t)** of the laser beam reflected from the surface. The wave power law spectrum **η_ω ~ P_ω** was calculated via Fourier time transformation of the signal **P(t)**.

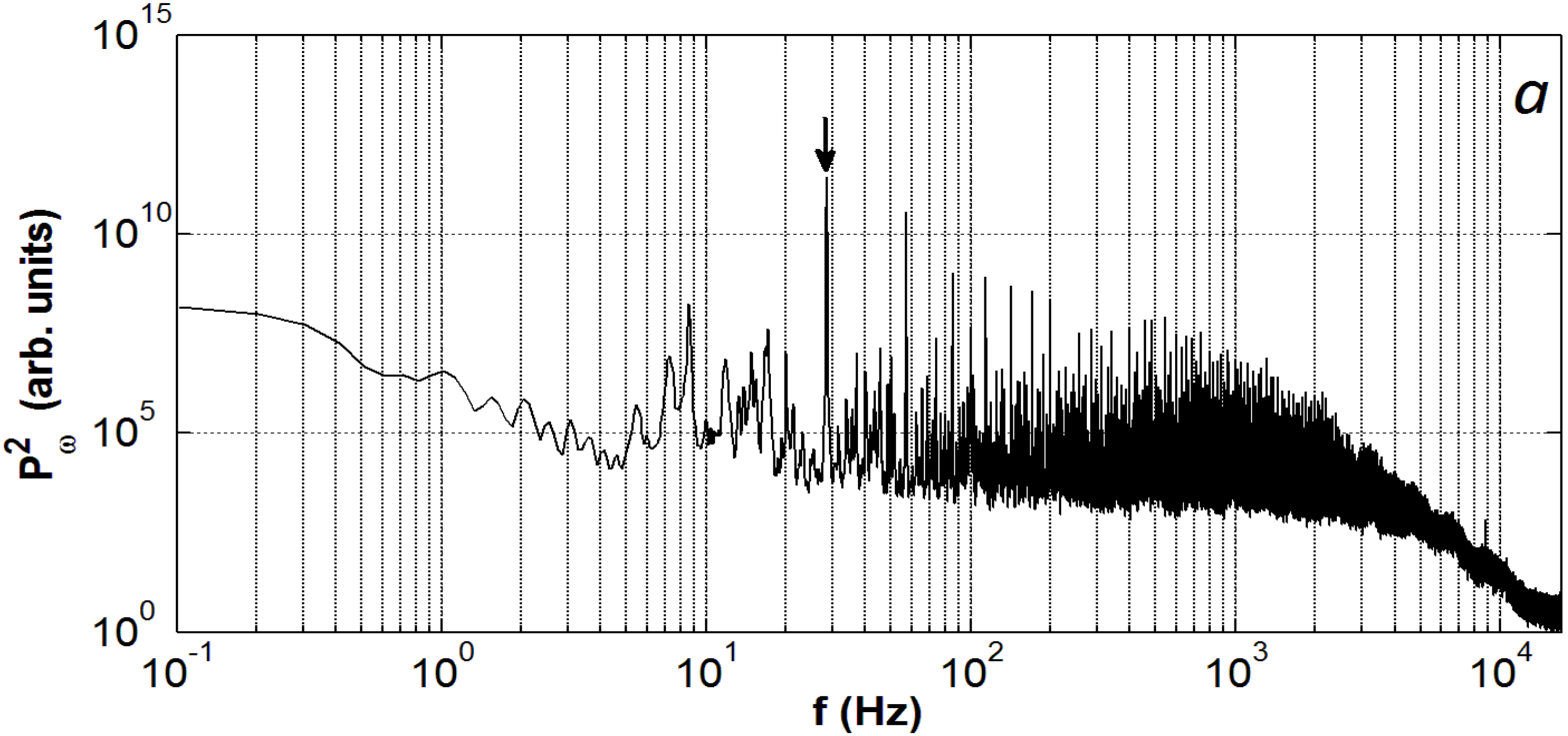
Direct KZ cascade , capillary waves, monochromatic driving: the correlation function

$I(\omega) = <|\eta_\omega|^2> = (\rho_\omega/\sigma k^2)\, n_\omega = CQ^{1/2} * (\sigma/\rho)^{1/6} * \omega^{-21/6}$

$<|\eta_\omega|^2> = <|\varphi_\omega/k|^2> = \{<|\varphi_\omega|^2>/k^2\} = \omega^{-4/3} * <|\varphi_\omega|^2>$

η_ω, k- the wave number , $\varphi_\omega = k\eta_\omega$ - the angle amplitude.

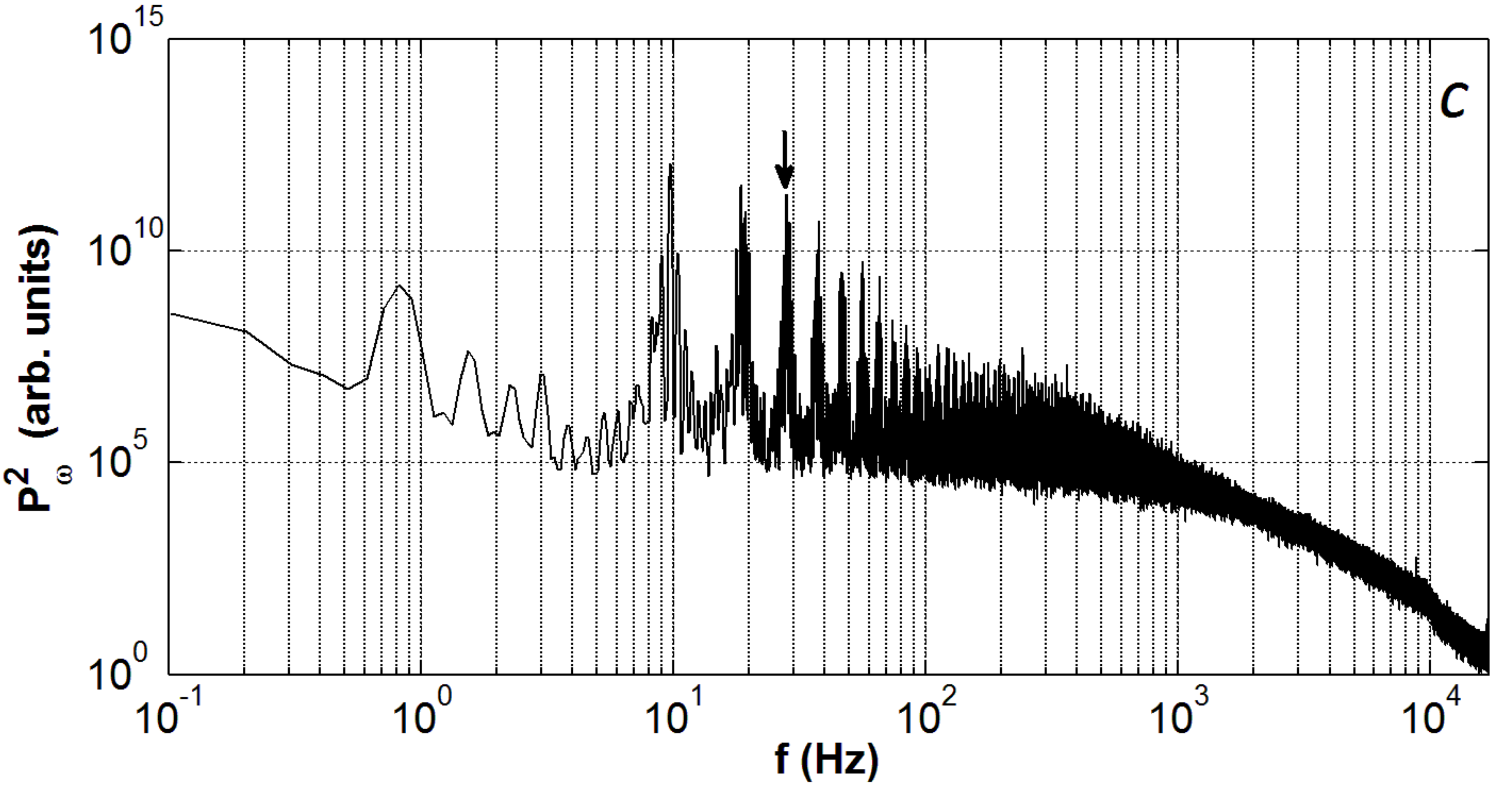
Weak low frequency oscillations, $I(\omega) \leq 10^{-11}$ cm^2 s . For the narrow beam ($ka << \pi$) the square of the Fourier component of the reflected power $<|\eta_\omega|^2> \sim P_\omega^2 * \omega^{-4/3}$; and for the wide beam ($ka >> \pi$, a – the linear dimension of the light spot) $<|\eta_\omega|^2> \sim P_\omega^2$.



Liquid hydrogen in the rectangular cell with linear dimensions 4.0x2.0 cm; the cell depth h =0.35 cm; the gap between the charged surface and the outer collector d=0.35 cm.

The power spectrum P_ω^2:

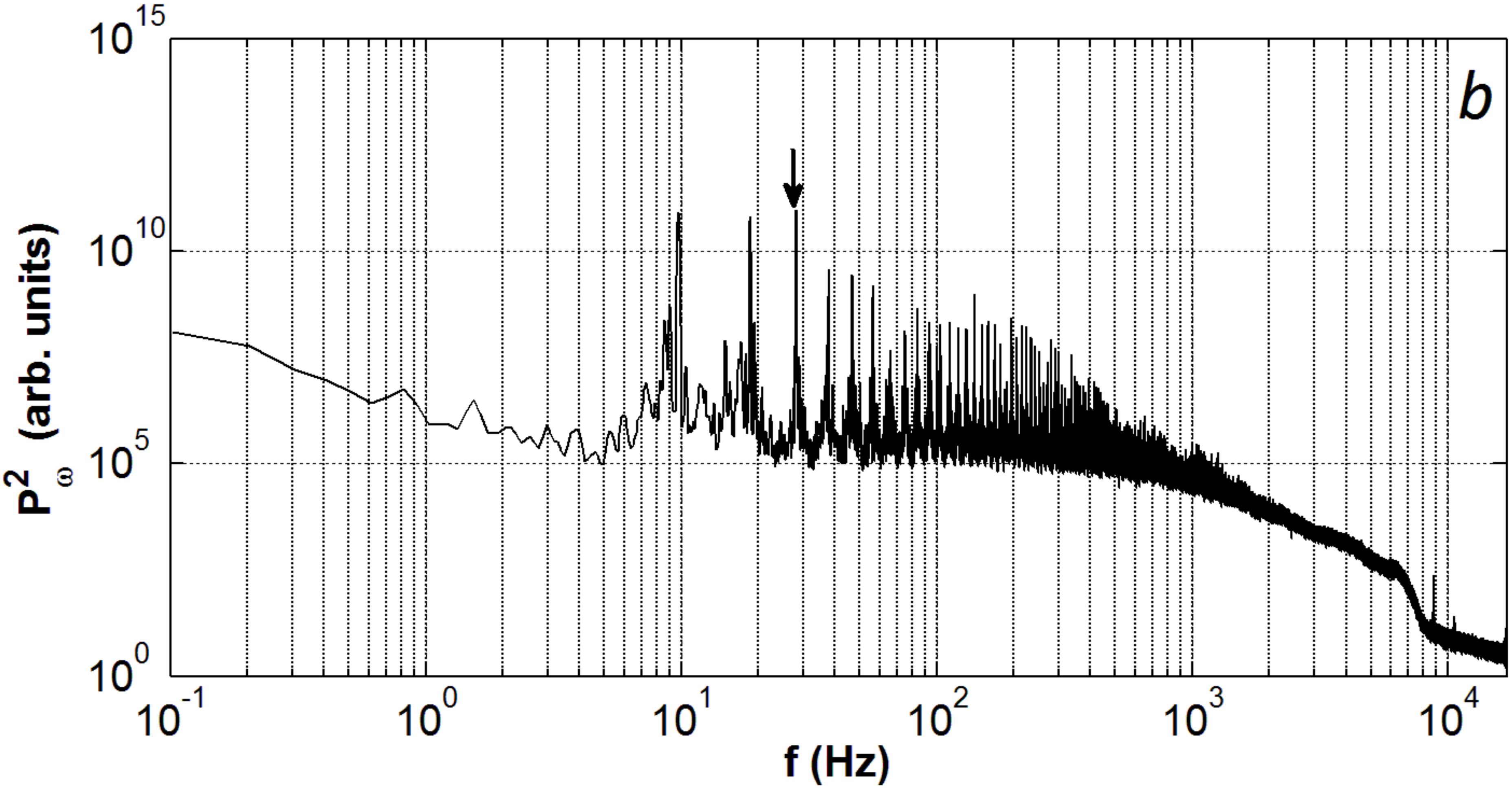
U_{dc} = 800 V, U_p=220 V, f_p= $\omega/2\pi$ =28.5 Гц (shown by the arrow).



Evolution of shape of the power spectrum $P\omega^2$ with lowering the drive frequency from 28.5 to 28.16 Hz at the same voltages:

U_{dc} = 800 V, U_p=220 V, f_p= 28.16 Гц (shown by the arrow).

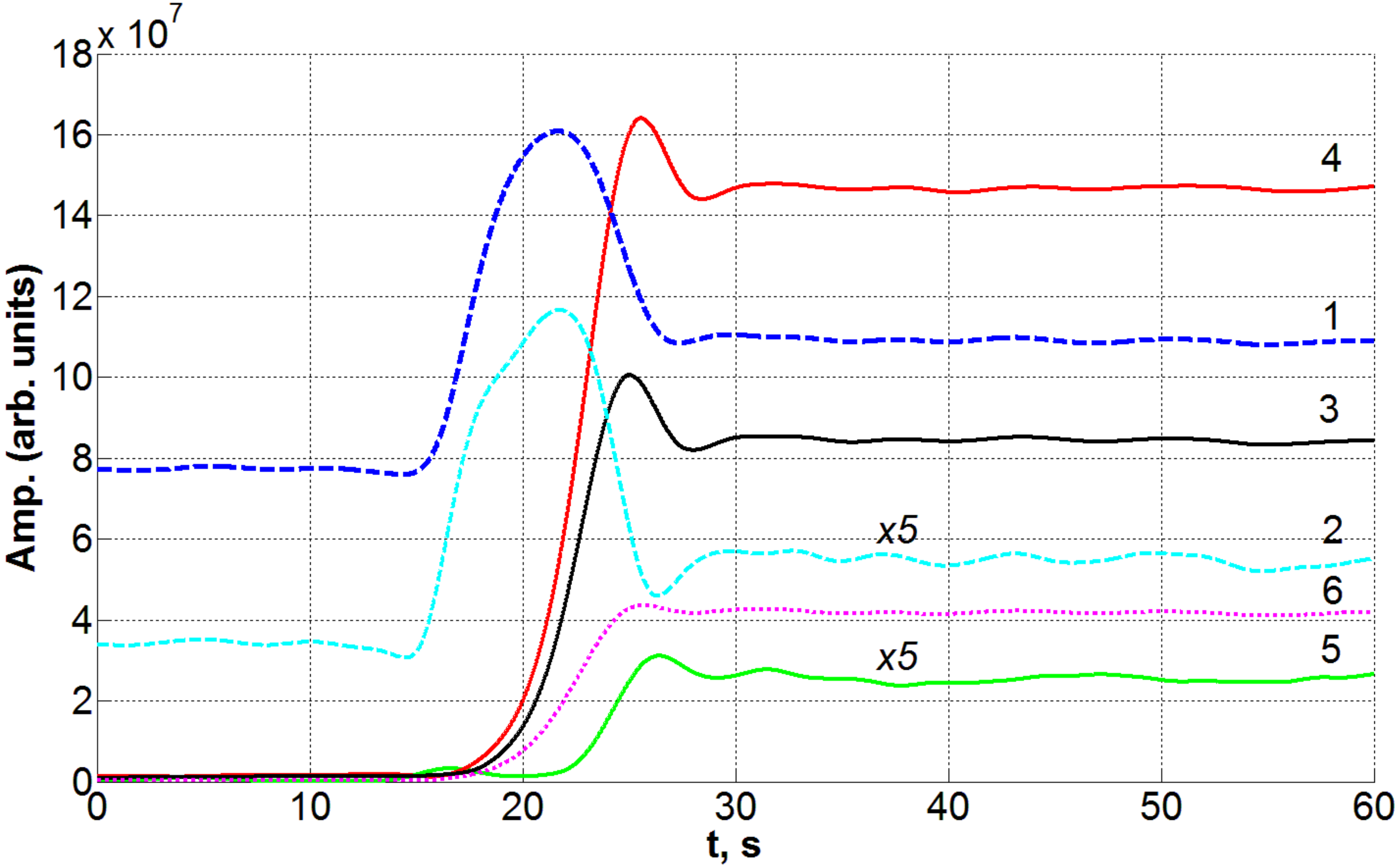
One can see three subharmonics f_1=18.64, f_2 =9.73 и f_3=0.82 Hz, and the proper combinational peaks on frequencies $f_p + f_2$ and $f_p + f_1$ in the direct KZ cascade



The next evolution of shape of the power spectrum Pω² with lowering the drive amplitude U_p from 220 to 125 V. at the same drive frequency f_p = 28.16 :

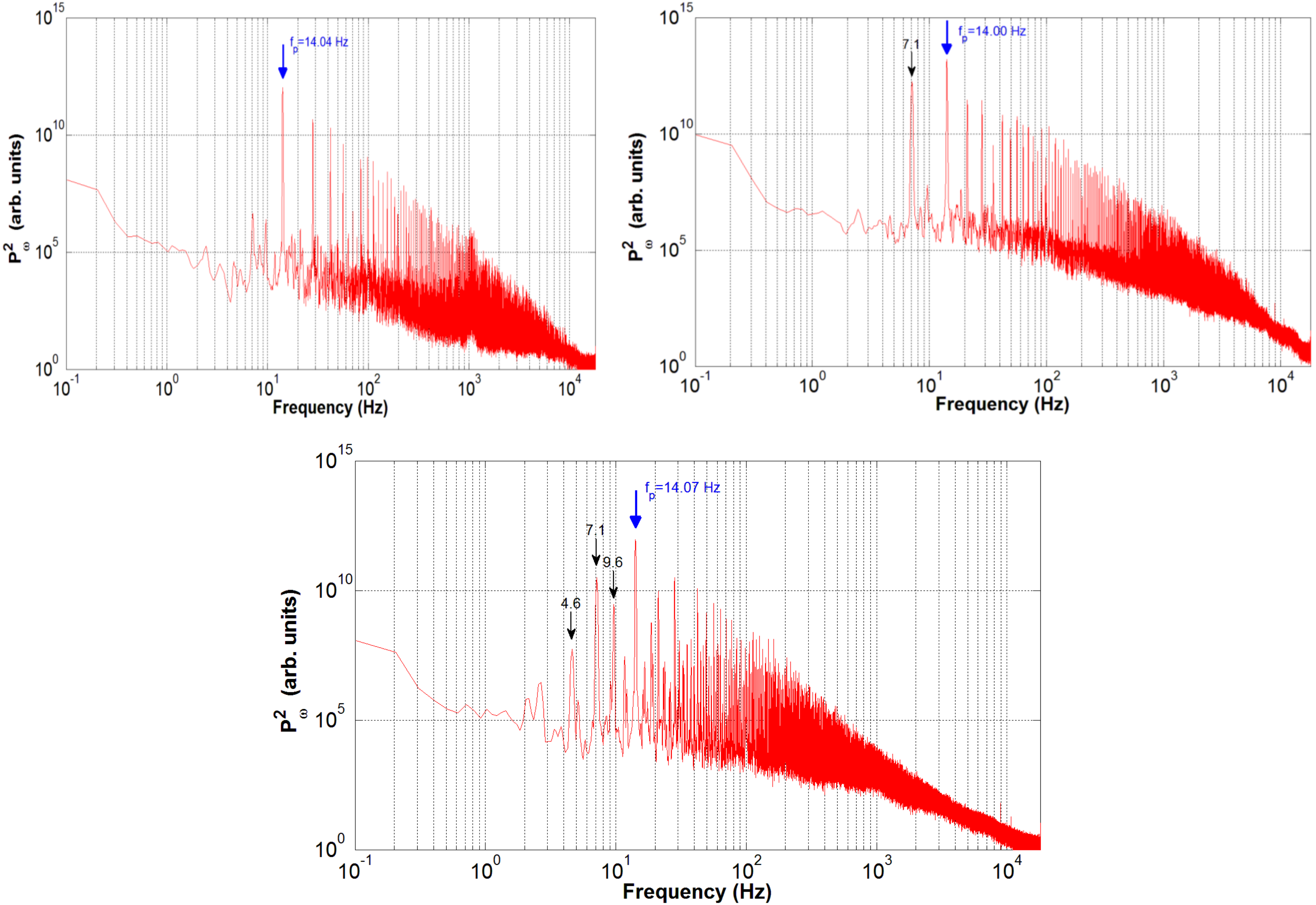
U_{dc} = 800 V, U_p=125 V, f_p= 28.16 Гц (shown by the arrow).

There exist only TWO subharmonics f_1=18.64, f_2 =9.73 Hz, and the proper combinational peaks on frequencies $f_p + f_2$ and $f_p + f_1$ the direct KZ cascade

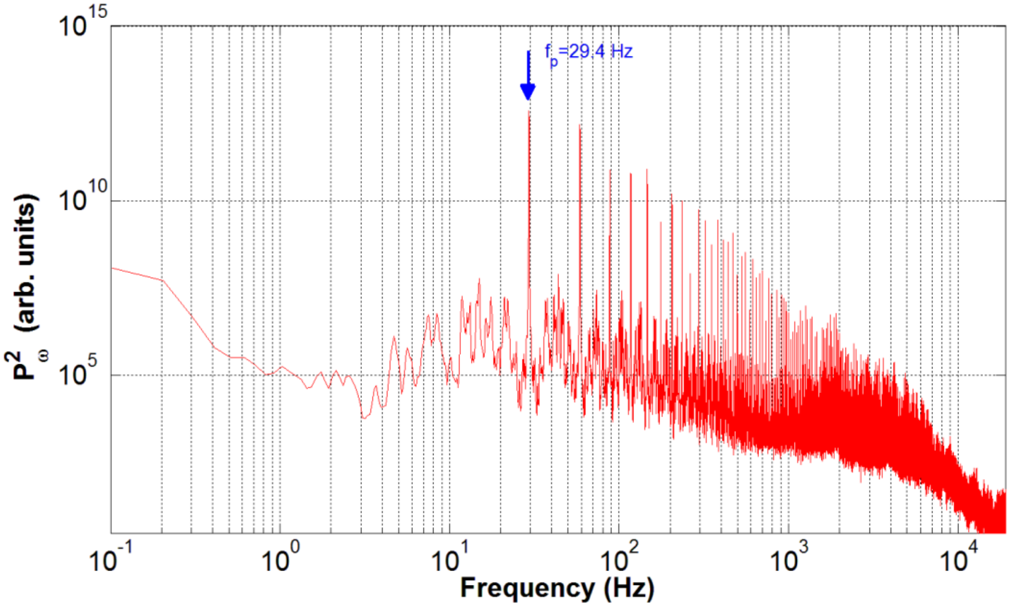


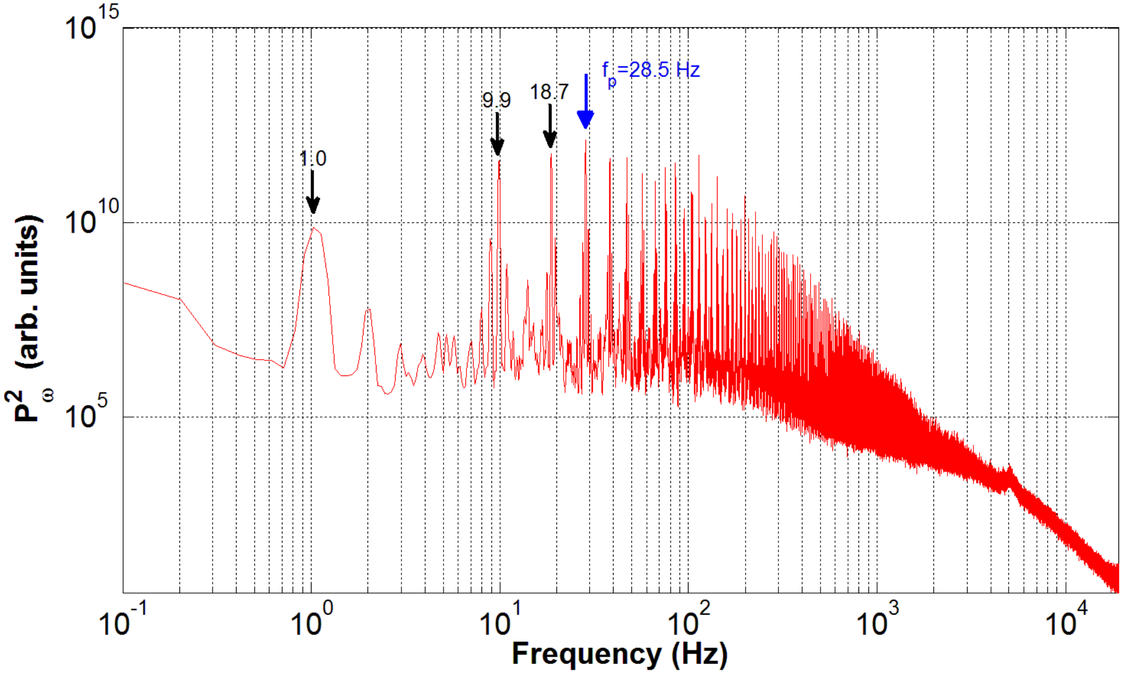
The time evolution of the waves amplitudes on increasing by step of the drive amplitude U_p from 125 до 220 V at the same drive frequency f_p=28.16 Hz at the moment t=15 sec.

Harmonics 1- f_p=28.16 ; 2 -- $2f_p$ =56.3; 6-combinational harmonics at $f_p + f_2$=37.9 Hz; subharmonics 3 - f_1=18.6; 4- f_2=9.7 and 5 - f_3=0.82 Hz

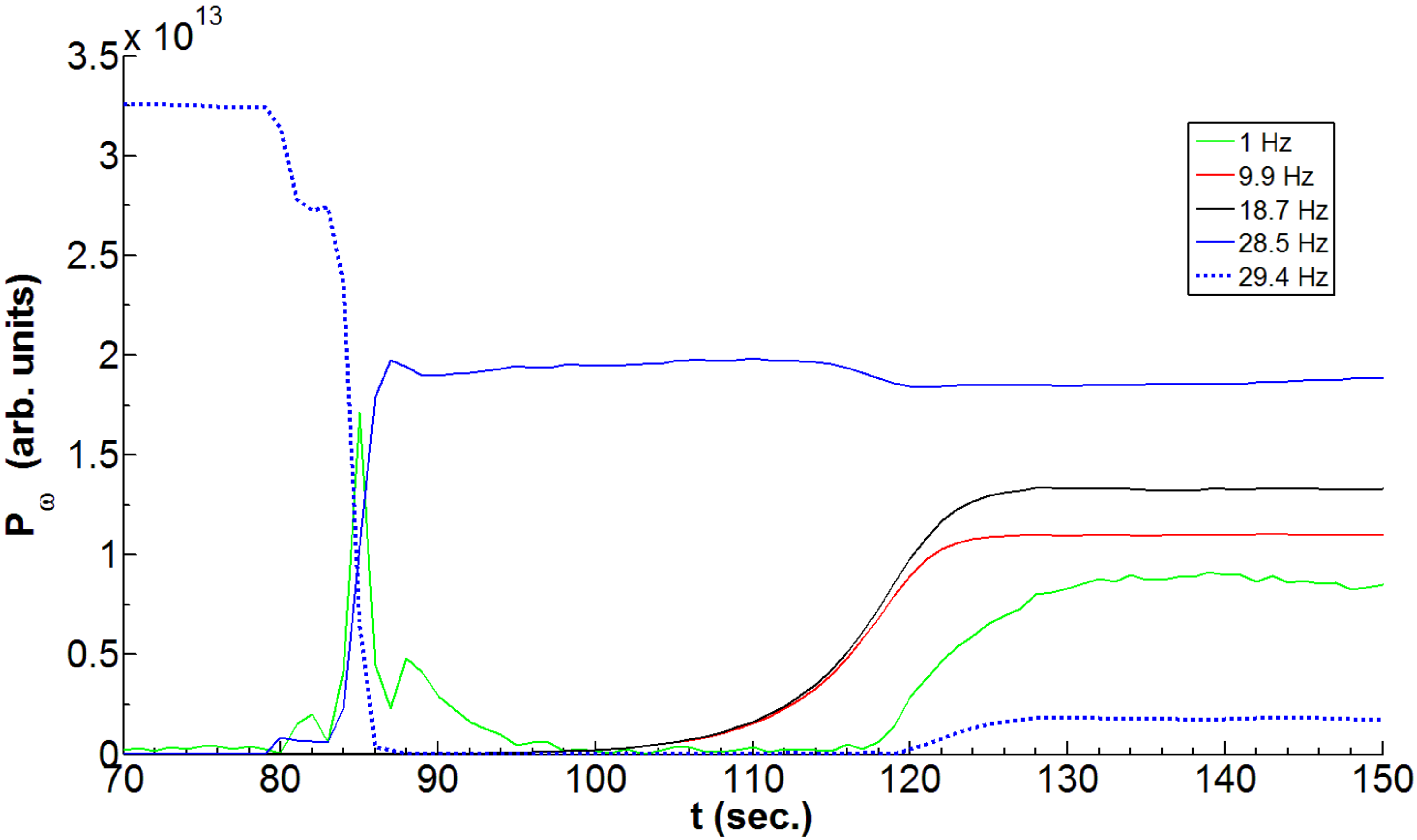


P²(ω); fp= 14.04; 14.00 and 14.07 Hz; Up= 244 V

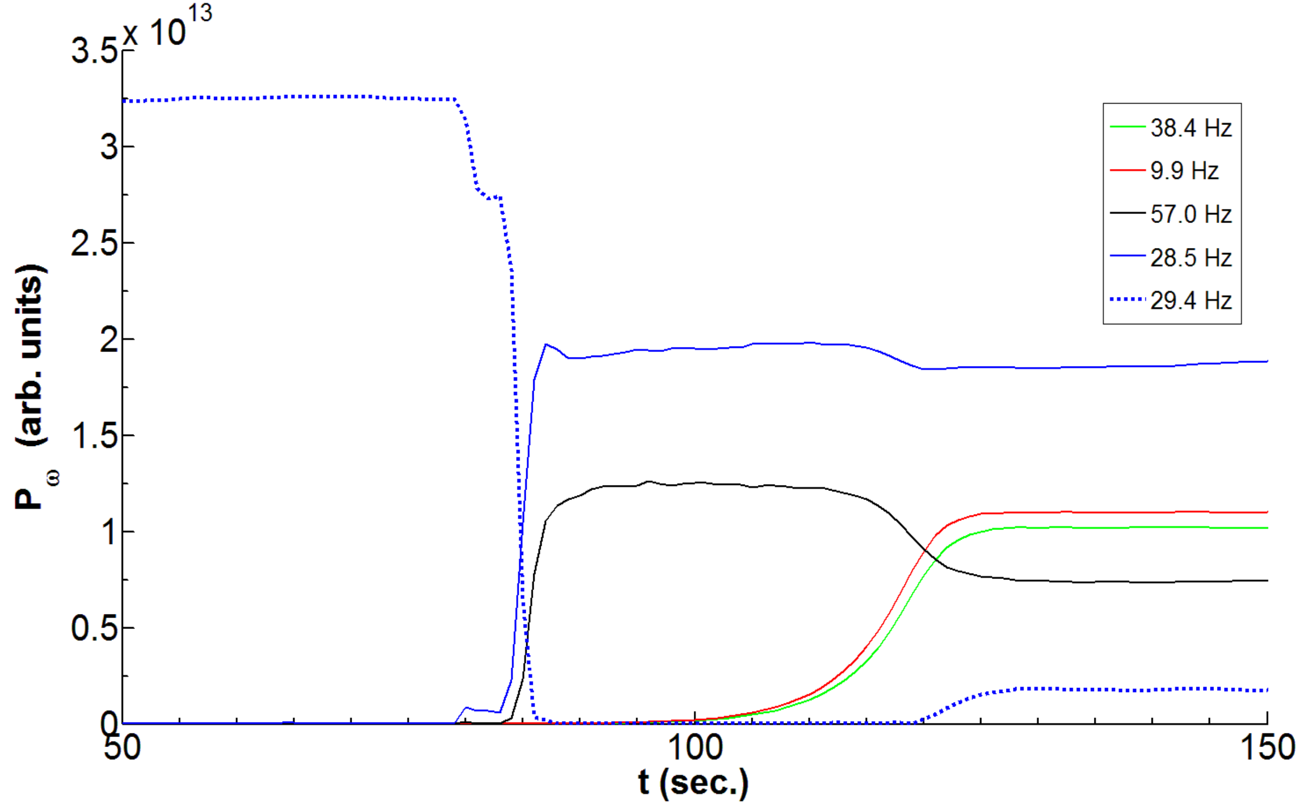




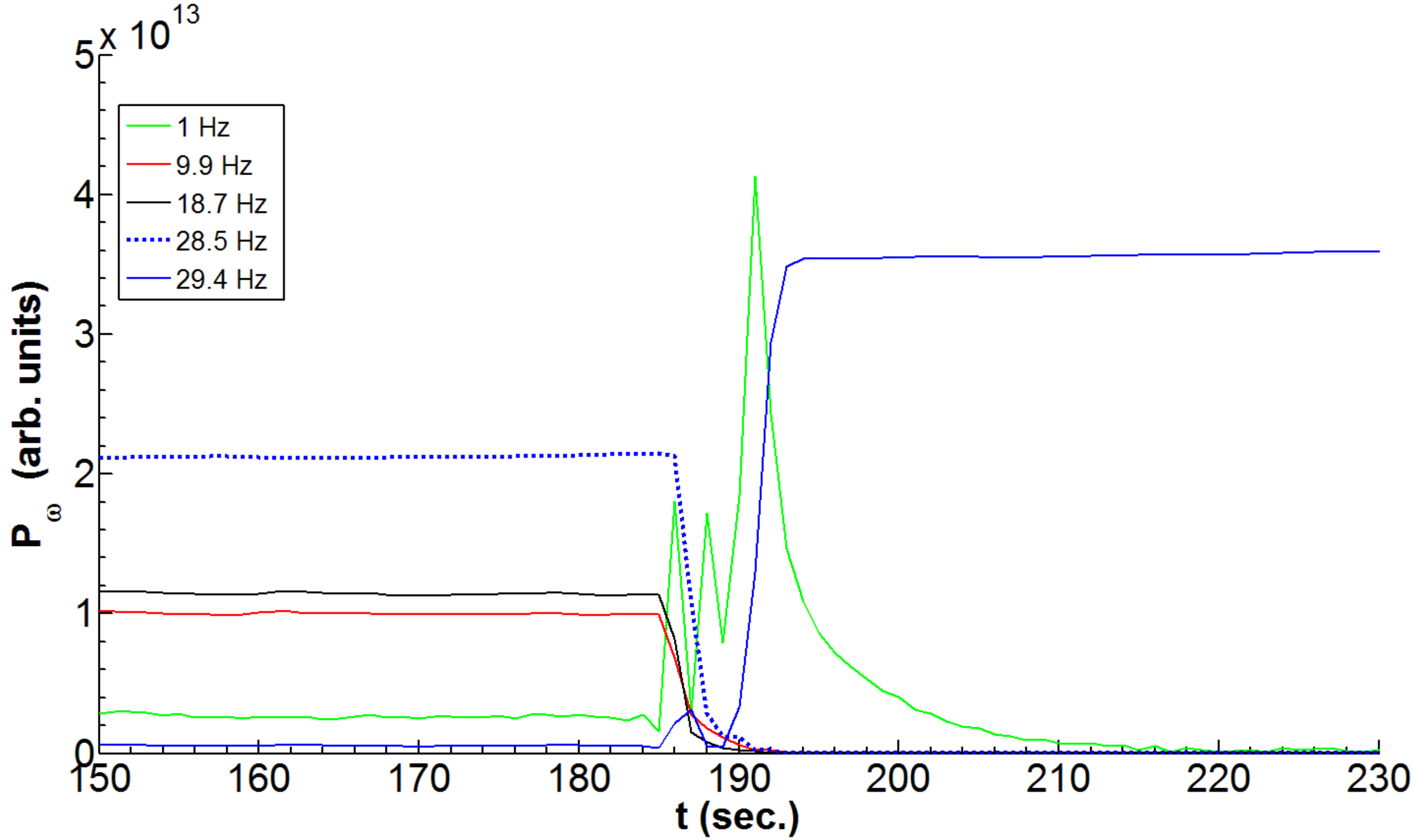
$P^2(\omega)$; **fp= 29.4 and 28.5 Hz; Up= 175 V**



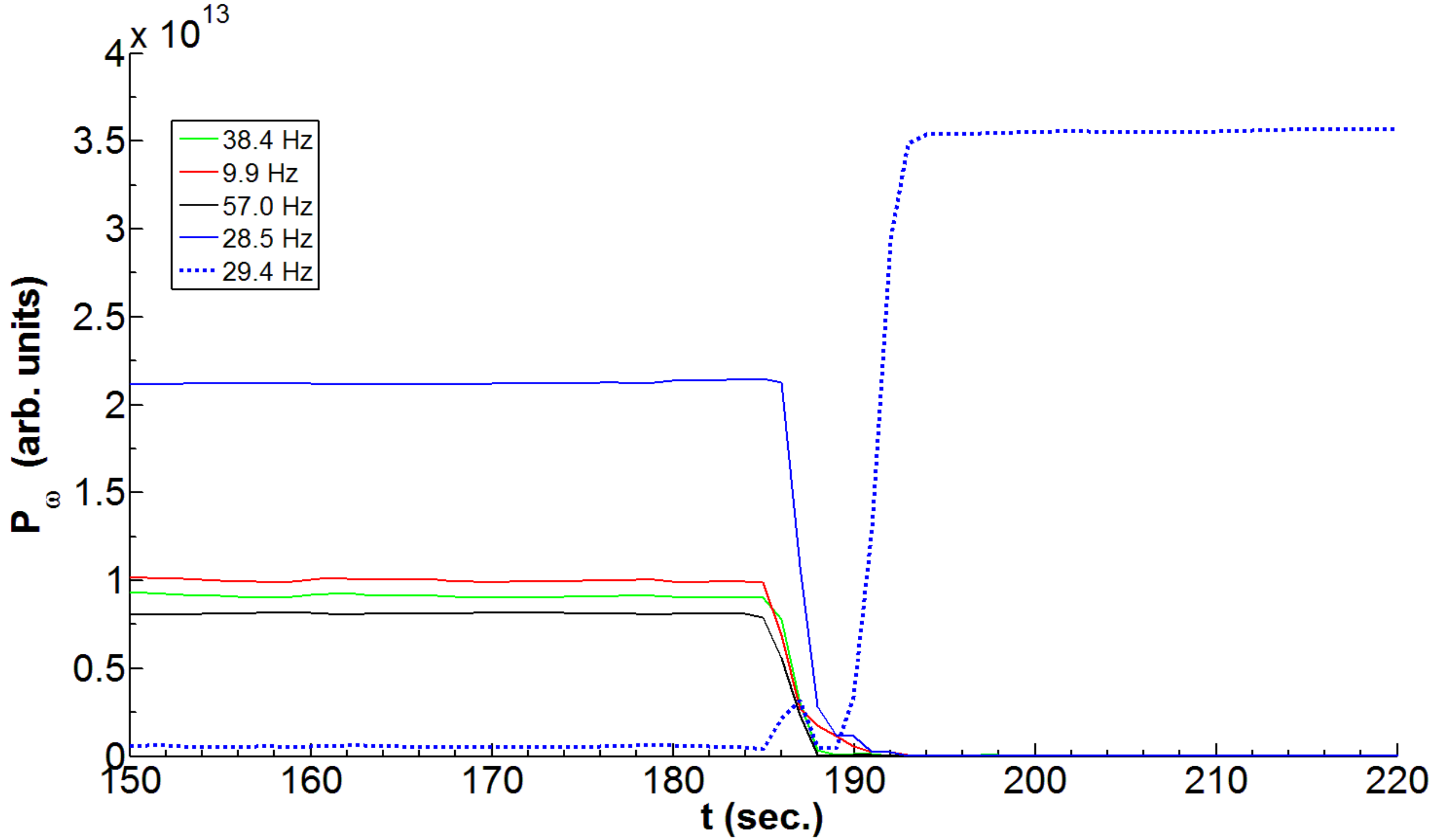
Evolution of the peak's amplitudes on lowering the drive frequency from 29.4 to 28.5 Hz



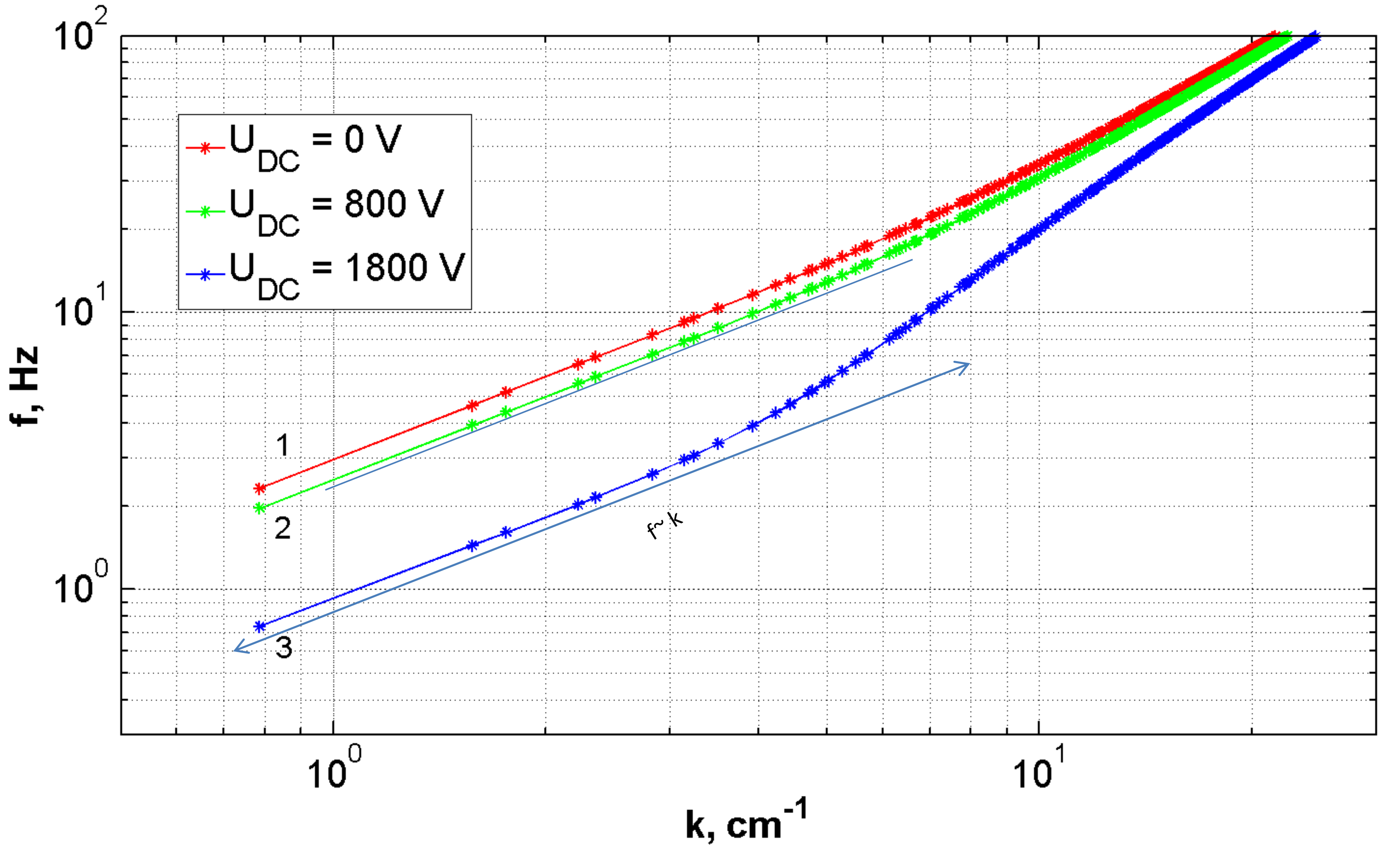
Evolution of the peak's amplitudes on lowering the drive frequency from **29.4 (•••)** to f_p **=28.5Hz;** f_{2p}**=57.0 ;** f_1**=9.9;** f_c**=9.9 + 28.5 =38.4 Hz.**



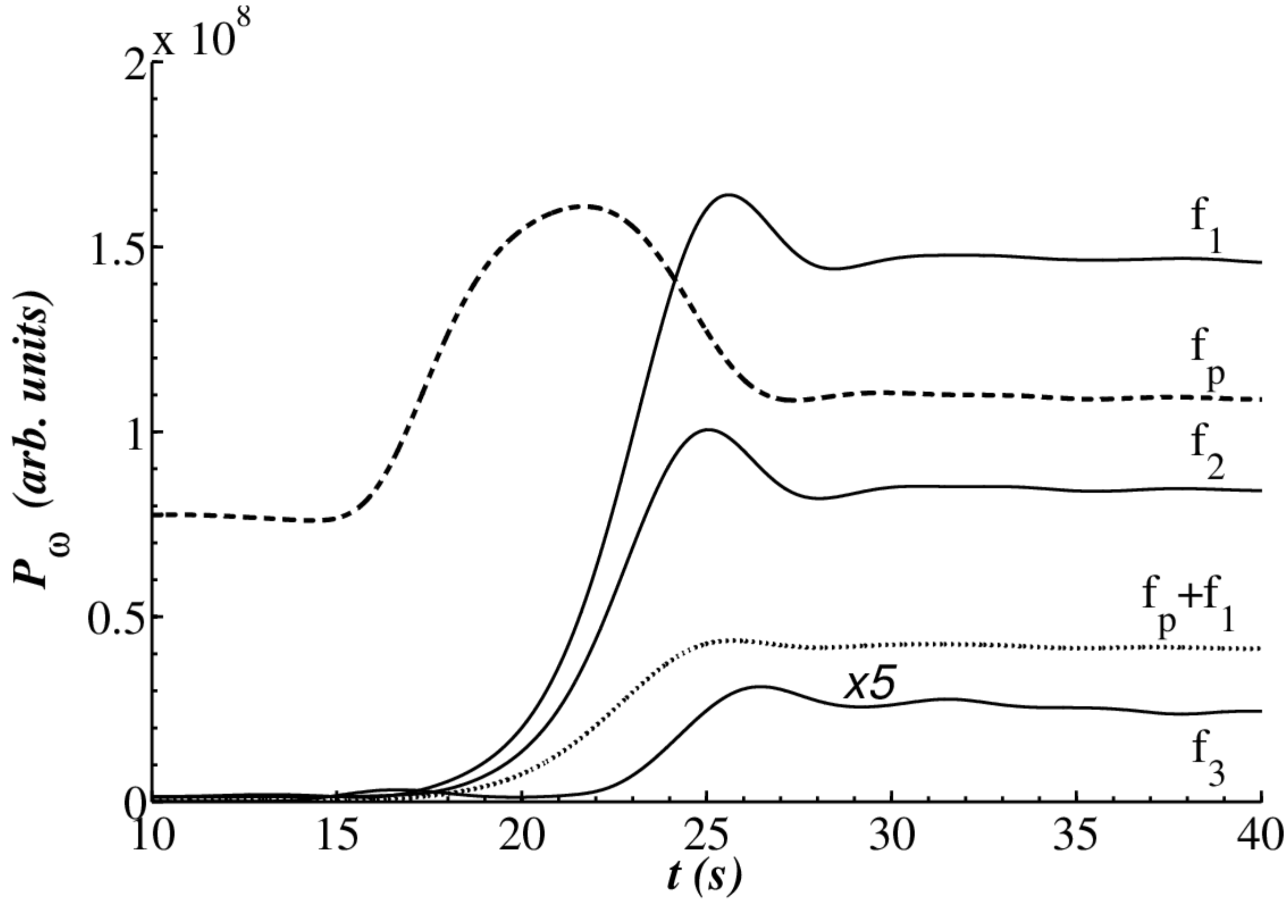
Evolution of the peak's amplitudes on detuning the drive frequency fp from 28.5 to 29.4 Hz. Subharmonics f1= 18.7; f2= 9.9; f3= 1 Hz



Attenuation of amplitudes of waves at **f_1= 9.9 Hz** (subharmonics), the drive frequency **fp = 28.5**, twin frequency **f2p = 57.0** and combinational frequency **fc = 38.4** in the direct KZ cascade on increasing the pump frequency **to 29.4 (….) Hz** at fixed drive voltage Up



Dispersion relations in the capacitor approximation



Dependence Pω (t) on increasing U_p from 125 to 220V:

harmonics (direct cascade) f_p=28.16 and 2f_p =56.3 (not shown);

f_{comb} (direct cascade)= f_p + f_2=37.9 Hz;

subharmonics f_1=18.6, f_2=9.7 ($f_1 + f_2 = f_p$); f_3 = 0.82 Hz ($f_3 = 2 f_2 - f_1$)

Conclusions

We found that in this spatially-restricted 2-D system (the surface waves in liquid hydrogen in the rectangular cell) the energy (or the wave action) may flow in the both directions opposite to that in unrestricted media. Specifically, for surface capillary waves in the open geometry energy flows towards small scales, while in the restricted geometry in the system of the capillary-gravitational waves interacting with the bottom and walls of the cell the energy may also flow towards large scales. That the direction of the energy flux can be reversed in bounded systems is truly remarkable and this can significantly affect the energy balance in the system. In particular, it leads to formation of large-scale waves of high amplitudes similar to “rogue" waves that sometimes observed in the ocean.

Bi-directional Energy Cascade in Surface Capillary Waves

L.V. Abdurakhimov, G.V.Kolmakov, A.A.Levchenko, Yu.V. Lvov , and I.A. Remizov

(PRL, under consideration)

Wave turbulence revisited: Where does the energy flow?

L. V. Abdurakhimov, I. A. Remizov, A. A. Levchenko, G. V. Kolmakov, Yu. V. Lvov

http://arxiv.org/1404.1111v1 [physics. flu-dyn] 3 Apr. 2014

- **Based on results of our experiment on superfluid He-II and subsequent simulations, the mechanism responsible for formation of the bi-directional energy cascade can be understood as follows. A system of nonlinear waves with no damping at low frequencies establishes a thermodynamic equilibrium spectrum in the low-frequency domain, which carries no energy flux and which temperature is proportional to the pumping rate [1]. However, in experiments with liquids of final depth, low-frequency wave damping occurs due to the viscous drag at container's walls [2,3]. The latter results in the decrease of the low-frequency wave amplitudes. The system tends to restore the thermodynamic equilibrium spectrum and in effect, a steady energy flux towards the low-frequency domain is formed.**

- **[1] E.Balkovsky, G.Falkovich, V.Lebedev and I.Y.Shapiro, Phys. Rev. E 52, 4537 (1995).**
- **[2] M. Brazhnikov, A. Levchenko, and L. Mezhov-Deglin, Instrum. Exp. Tech. 45, 758 (2002).**
- **[3] M.Yu.Brazhnikov, G.V.Kolmakov, A.A.Levchenko, and L.P.Mezhov-Deglin, JETP Lett. 82, 565 (2005).**

Wave turbulence revisited: Where does the energy flow?

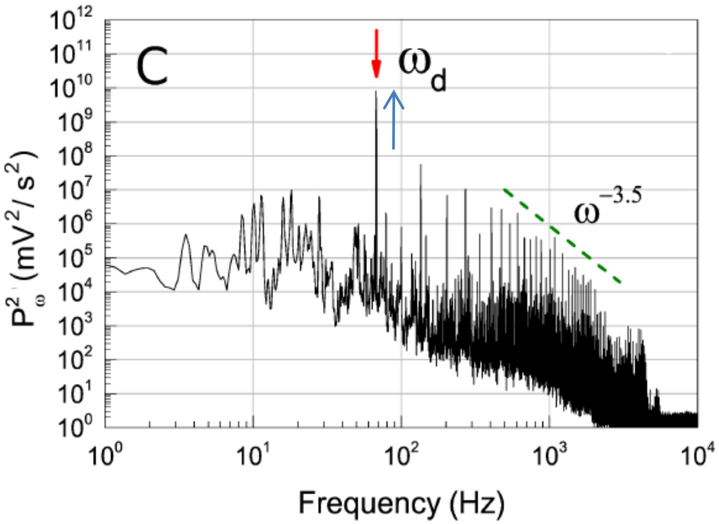
L. V. Abdurakhimov, I. A. Remizov, A. A. Levchenko, G. V. Kolmakov, Yu. V. Lvov

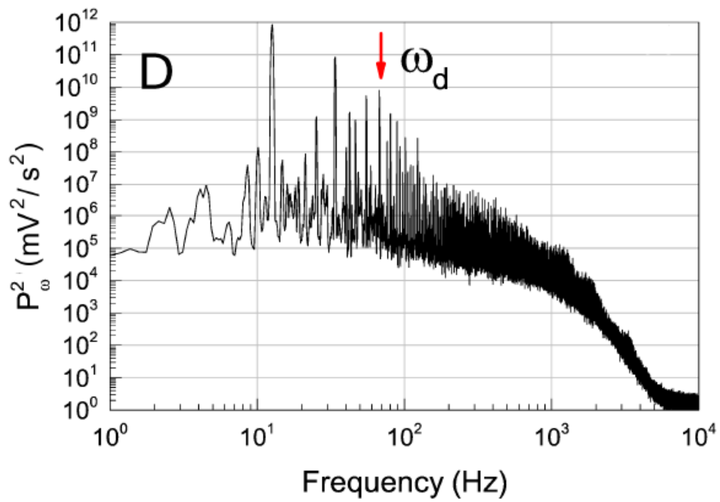
submitted to PRL; http://arxiv.org/pdf/1404.1111v1.pdf

In the paper submitted to PRL the authors have demonstrated that in the restricted geometry (a cell of set dimensions) energy flux from the driving scale towards the damping region can be formed for capillary waves even if the damping occurs at frequencies lower that the driving frequency. This bi-directional energy flux provides a continuous energy source for sustained low-frequency wave oscillations in the presence of finite damping. Furthermore, bi-directional energy flux provides an effective global coupling mechanism between the scales.

We studied nonlinear capillary waves on the surface of superfluid He-II in the cylindrical cell. Based on our experiment and computer simulations, the mechanism responsible for formation of the bi-directional energy cascade can be understood as follows. A system of nonlinear waves with no damping at low frequencies establishes a thermodynamic equilibrium spectrum in the low-frequency domain, which carries no energy flux and which temperature is proportional to the pumping rate (*E.Balkovsky, G.Falkovich, V.Lebedev and Yu. Shapiro, Phys. Rev. E 52, 4537 (1995).* However, in experiments with liquids of final depth, low-frequency wave damping occurs due to the viscous drag at container's walls [20]. The latter results in the decrease of the low-frequency wave amplitudes. The system tends to restore the thermodynamic equilibrium spectrum and in effect, a steady energy flux towards the low-frequency domain is formed.

Turbulent cascade on the surface of liquid helium at T=1.7 K.
Driving forces are 4V (C) and 14 V (D) at w_d = 68 Hz

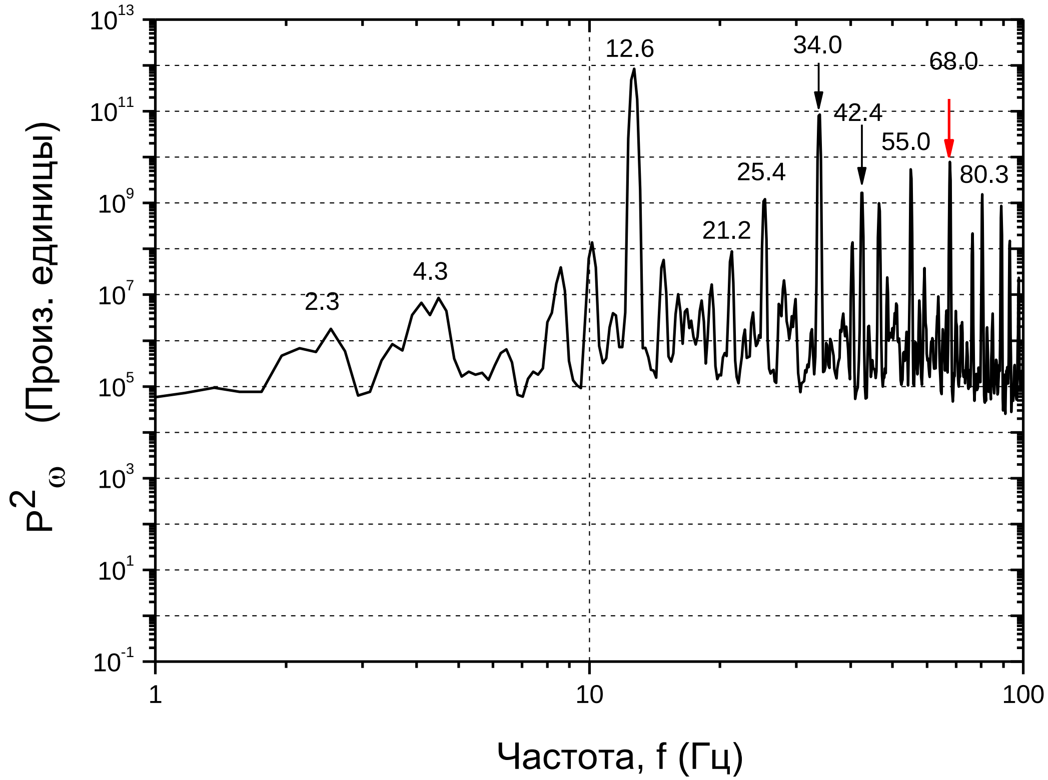




Nonlinear waves at the surface of superfluid He-II in the cylindrical cell of the inner diameter D = 60 mm
w_d =32Hz ; T = 1.7 K

B

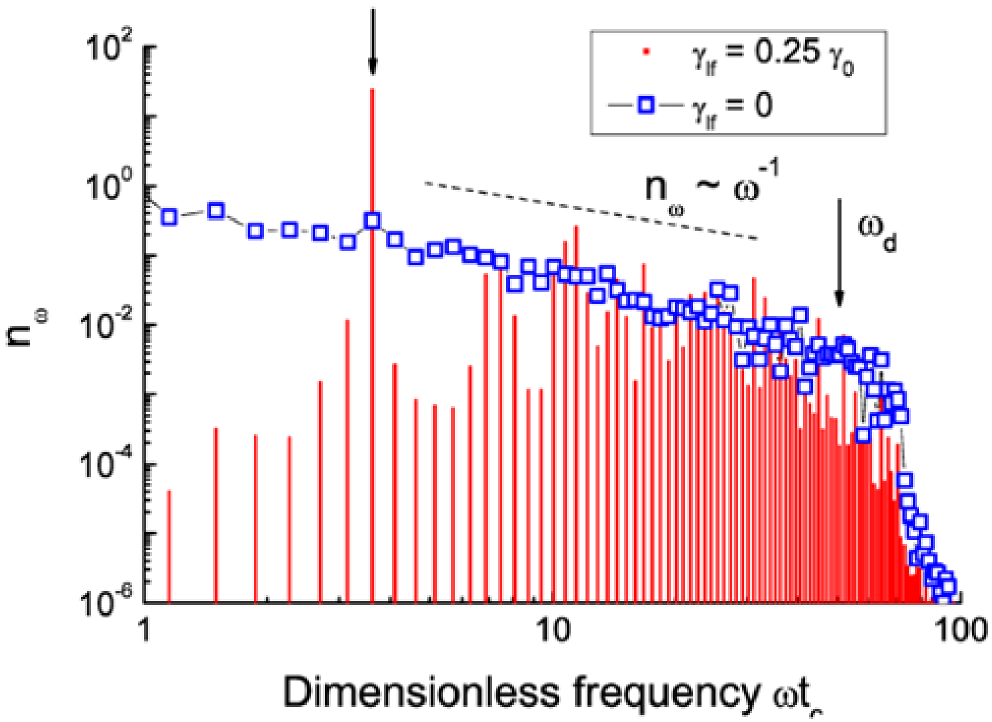
10 mm



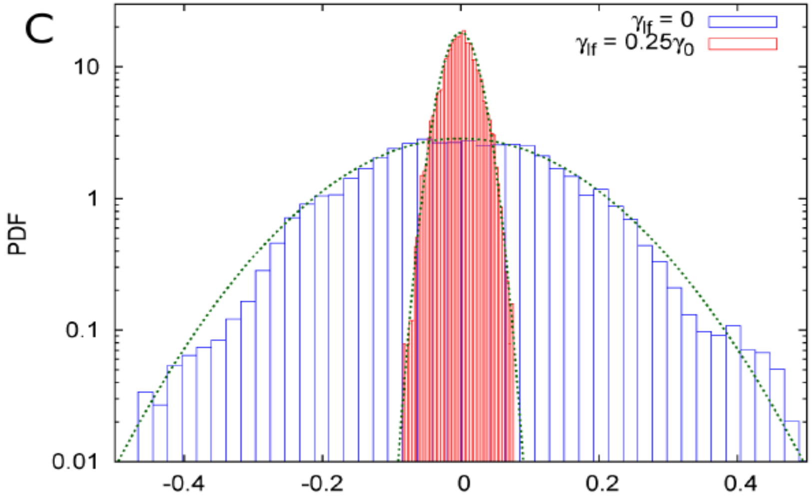
10^{13}
10^{11}
10^{9}
10^{7}
10^{5}
10^{3}
10^{1}
10^{-1}
P^2_ω (Произ. единицы)
12.6
34.0
68.0
42.4
55.0
25.4
80.3
21.2
4.3
2.3
1
10
100
Частота, f (Гц)

$$\frac{da_k(t)}{dt} = -i\sum_{k_1,k_2} V_{k,k_1,k_2} D_{k,k_1,k_2} a_{k_1}(t) a_{k_2}(t) e^{i(\omega(k)-\omega(k_1)-\omega(k_2))t} - 2i\sum_{k_1,k_2} V^*_{k_1,k,k_2} D_{k_1,k,k_2} a_{k_1}(t) a^*_{k_2}(t) e^{i(\omega(k)+\omega(k_2)-\omega(k_1))t} - \gamma(\omega(k)) a_{\boldsymbol{k}}(t). \quad (4)$$

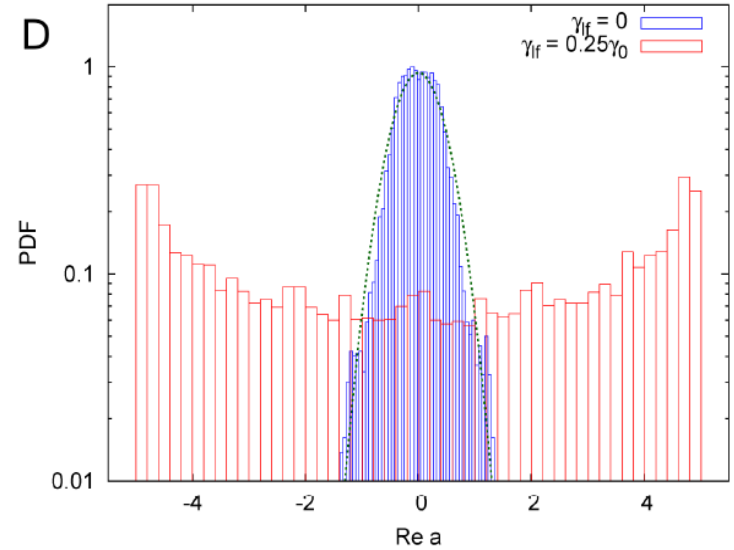
$$N(k) = \langle |a_k(t)|^2 \rangle$$



Numerical steady-state spectrum of sustained surface oscillations.



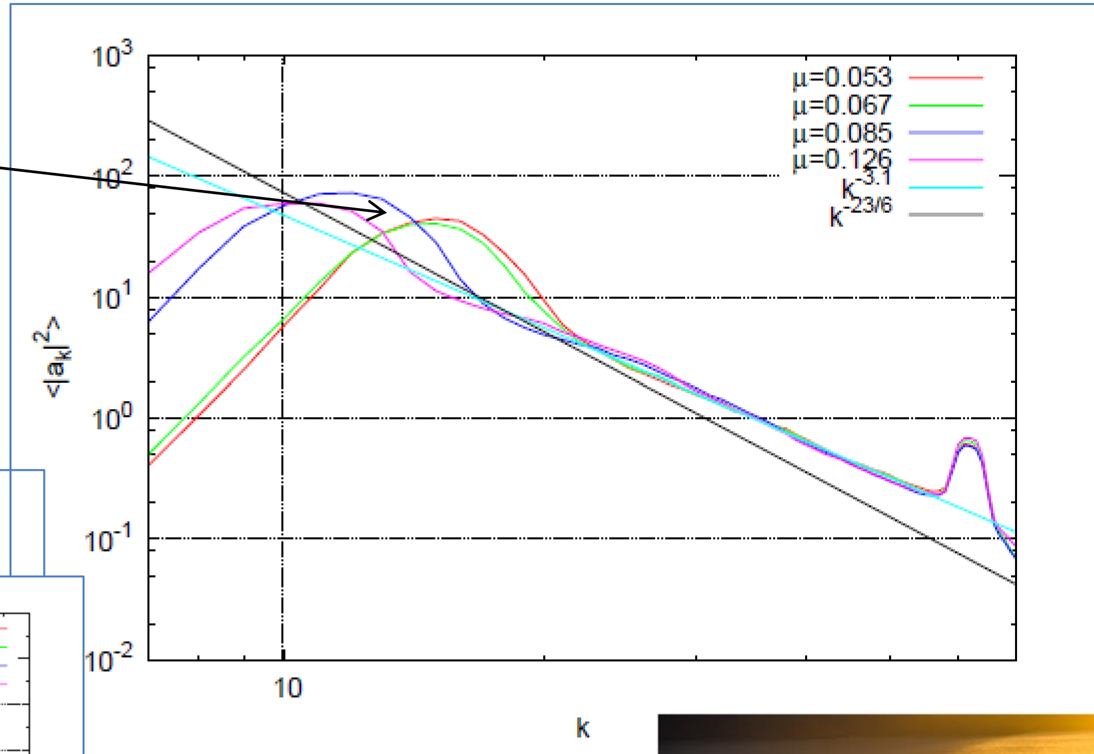
PDF for 20th mode in the absence of low-frequency damping and at damping.

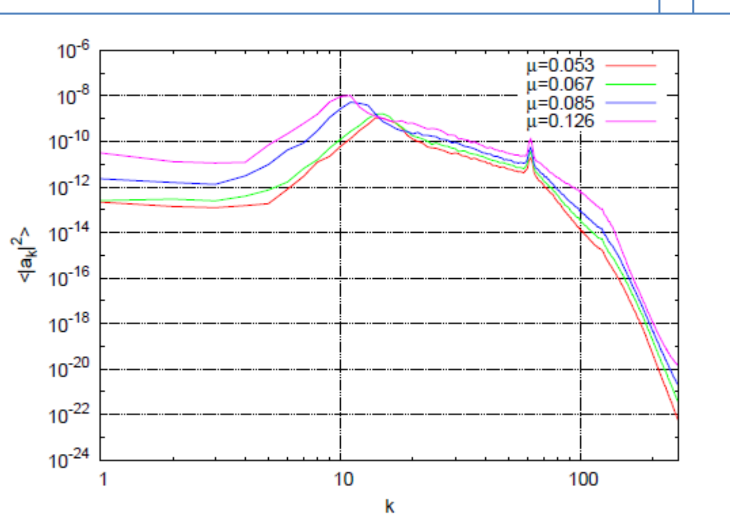


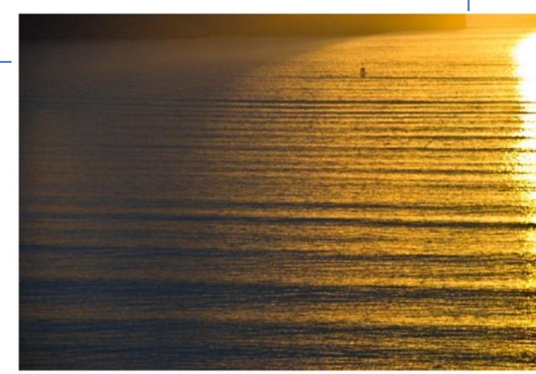
PDF for 10th mode in the absence of low-frequency damping and at damping.

Formation of inverse cascade

Condensate







A. O. Korotkevich, *Influence of the condensate and inverse cascade on the direct cascade in wave turbulence*, Math. Comput. Simul., doi:10.1016/j.matcom.2010.07.009 (2010); arXiv: 0911.0741.