Bottleneck phenomena in capillary turbulence on the surface of cryogenic liquids

Leonid V. Abdurakhimov, Max Yu. Brazhnikov,

Alexander A. Levchenko, Igor A. Remizov

Laboratory of quantum crystals,
Institute of Solid State Physics RAS
142432, Chernogolovka, Russia

Capillary Turbulence

The dispersion relation

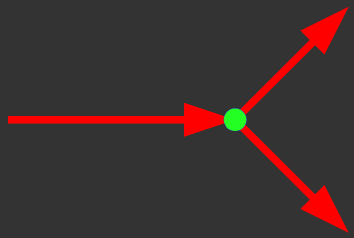
of capillary waves:

$$\omega^2 = \sigma/\rho \, k^3$$

Three-wave interaction:

$$\boldsymbol{k}_1 = \boldsymbol{k}_2 + \boldsymbol{k}_3$$

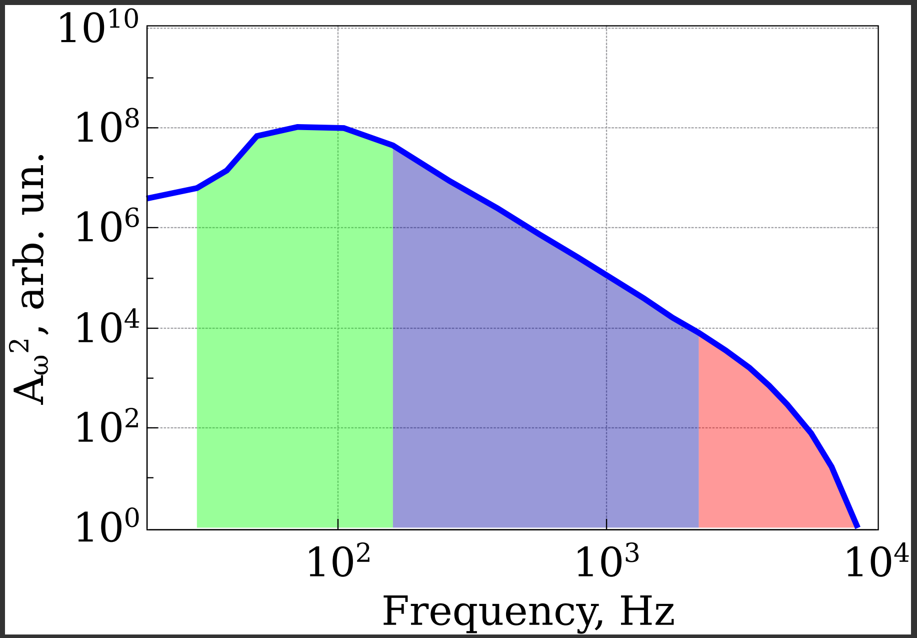
$$\omega_1 = \omega_2 + \omega_3$$



Kolmogorov-Zakharov spectrum

of capillary turbulence:

$$<|\eta_\omega|^2> = C\, P^{1/2}\, (\sigma/\rho)^{1/6}\, \omega^{-17/6}$$



- Pumping at low frequencies
- Inertial range: energy transfer via nonlinear wave interaction
- Dissipation at high frequencies

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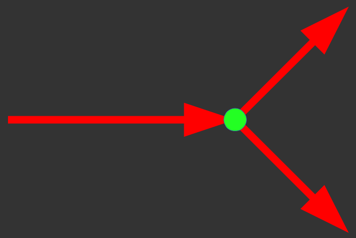

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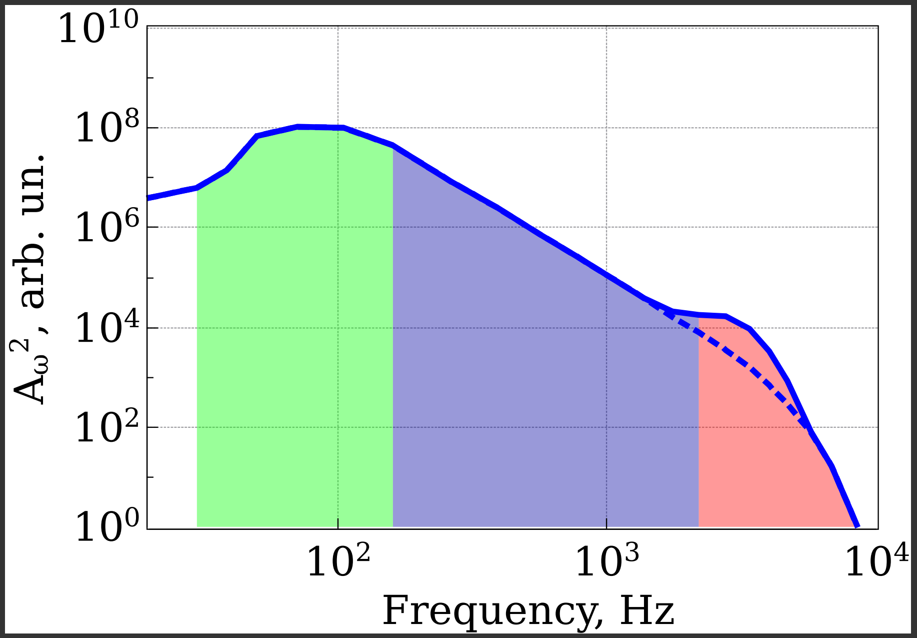
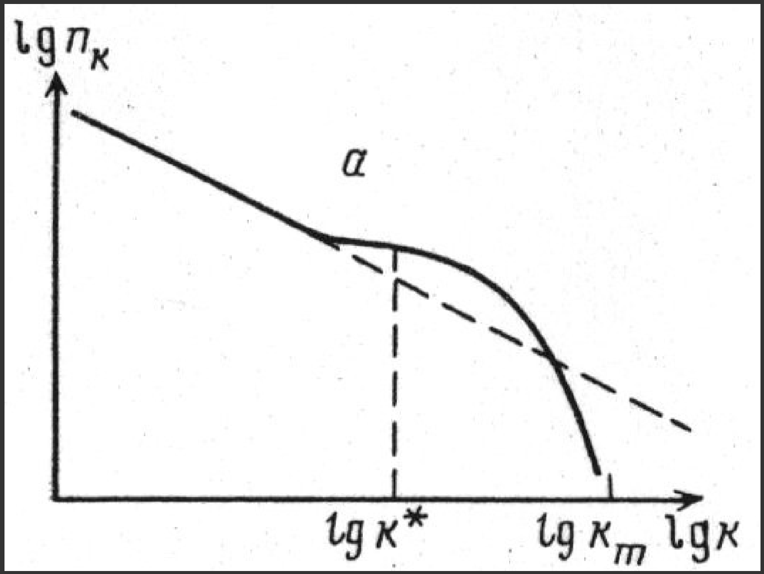


- Pumping at low frequencies
- Inertial range: energy transfer via nonlinear wave interaction
- Dissipation at high frequencies

Bottleneck due to dissipation

A reduction of energy flux due to low occupation numbers n_k in the dissipation domain leads to the energy accumulation near the high frequency edge of the inertial range.

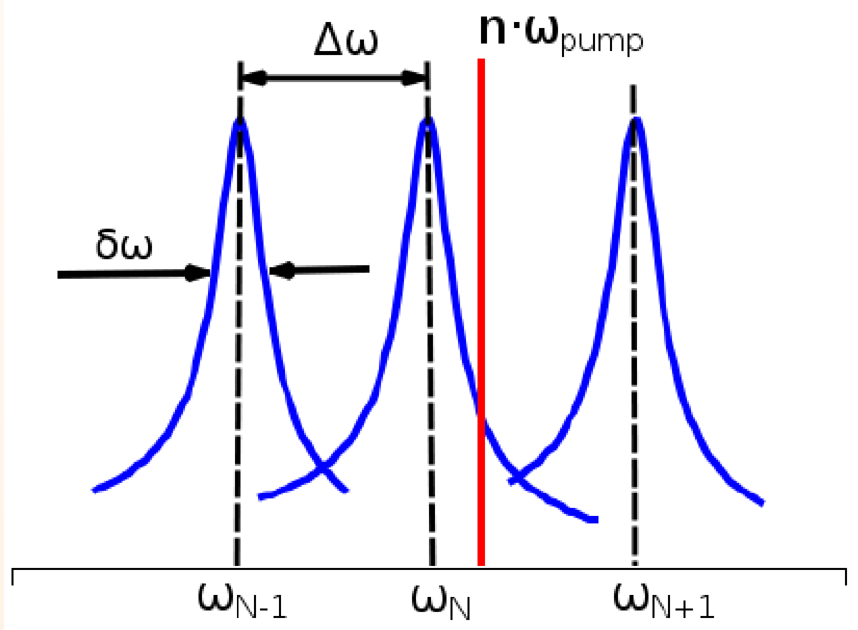


G. E. Falkovich, I. V. Ryzhenkova

JETP, **98**, 1931 (1990)

$$dn_k / dt = St_{nl}\{n_k\} - \gamma_k n_k$$

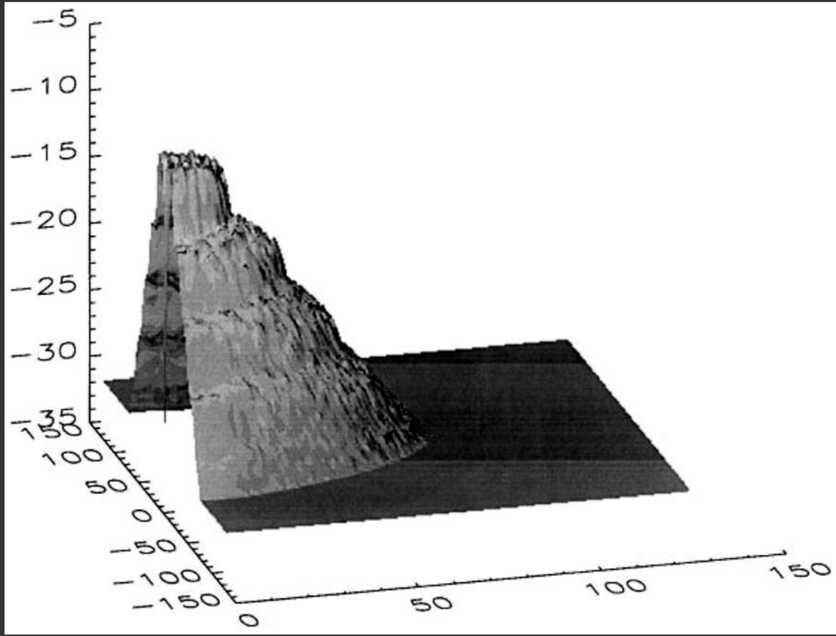
Bottleneck due to discreteness



The resonant processes

$$\omega_1 = \omega_2 + \omega_3$$

can be forbidden in discrete systems, if resonant modes are not broadened enough.



Spectrum of elevation in the case of frozen turbulence.

A. N. Pushkarev, V. E. Zakharov
Physica D 135 (2000) 98–116

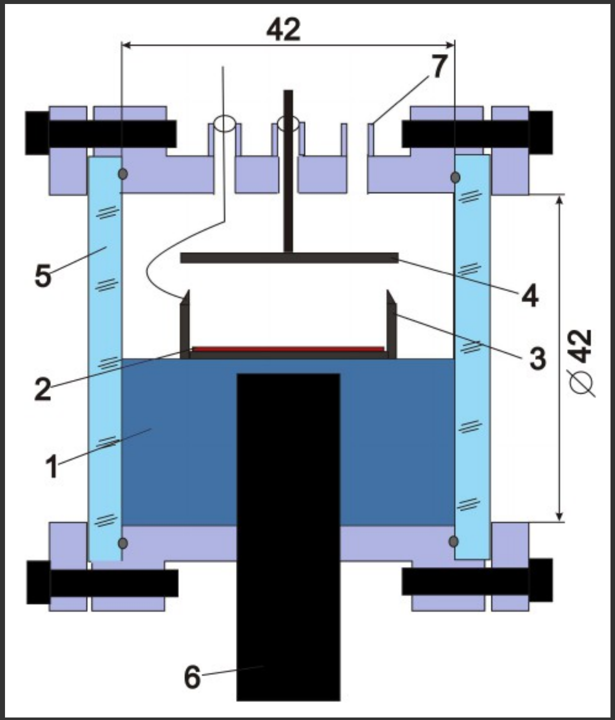
Properties of liquid hydrogen, helium and water

	H_2 T=15K	He T=4.2K	H_2O T=300K
Density, ρ, g/cm^3	0,076	0,145	1,0
Surface tension, σ, dyne/cm	2,7	0,12	77
Capillary length, λ cm	0,19	0,030	0,28
Capillary frequency, Hz	16	41	12
Nonlinear coefficient $V \sim (\sigma/\rho^3)^{1/4}$, $cm^{9/4}/g^{1/2}\ sec^{1/2}$	8,9	2,5	3,0
Viscosity, υ, cm^2/sec	0,0026	0,0002	0,01
Relative width of the inertial range, $\omega_{damping}/\omega_{drive}$	60	65	20

Experiments with cryogenic liquids

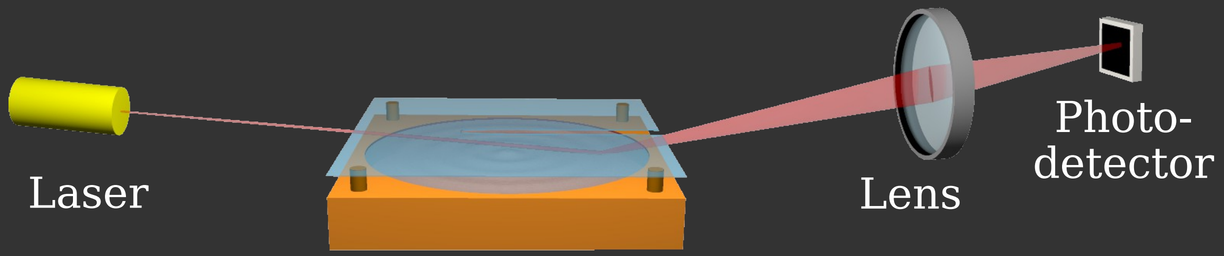
- Thanks to high nonlinearity and low viscosity the inertial range is wider than in common liquids (water).
- Possibility to create quasi-2D charged layer below the surface. Electric field can be used to oppose the gravity, thus extending capillary range to low-frequency domain.
- Small density allows excitation of the surface waves by weak oscillating electrical field. The driving force acts directly on the surface.
- The spectral characteristics of the driving force are well controllable.

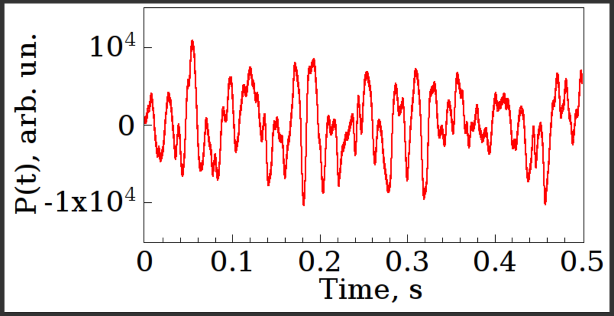
Experimental setup and techniques



1 – insulator (textolite), 2 – radioactive plate, 3 – copper cup, 4 – conductive plate, 5 – quartz windows, 6 – copper rod (connected to the helium bath at T = 1.7K), 7 – inlet capillary

Experimental setup and techniques





Recorded signal —
the power of reflected light $P(t)$

The correlation function of the surface elevation

$$<|\eta_\omega|^2> = <|\varphi_\omega/k|^2> \sim \omega^{4/3} <|\varphi_\omega|^2>,$$

$\varphi_\omega = k\ \eta_\omega$ — the wave steepness

The instrumental function $\Phi(\omega)$: $\varphi_\omega^{\ 2} \sim P_\omega^{\ 2}/\Phi(\omega)$

Narrow beam ($ka<<\pi$, a – size of laser spot):

$\Phi(\omega) \simeq 1 \quad \rightarrow \quad <|\eta_\omega|^2> \sim \omega^{4/3} P_\omega^{\ 2}$

Wide beam ($ka>>\pi$):

$\Phi(\omega) \sim \omega^{4/3} \quad \rightarrow \quad <|\eta_\omega|^2> \sim P_\omega^{\ 2}$

Experimental results: He-II

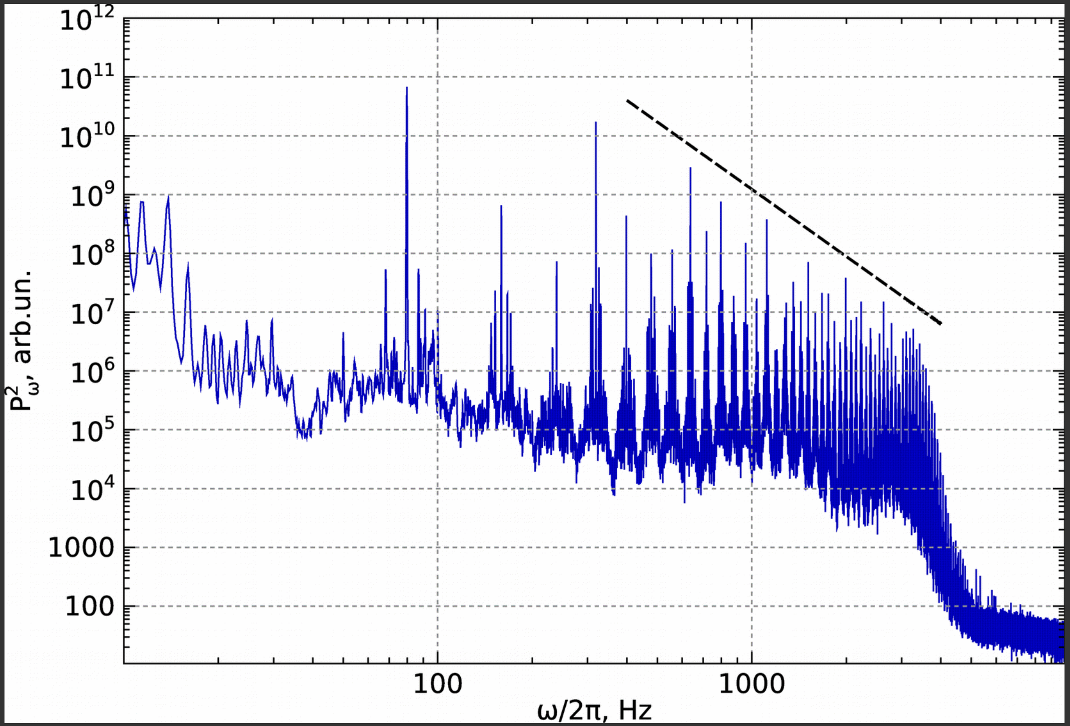
Experimental cell:
diameter – 30 mm
depth – 4 mm

Superfluid He-II,
T=1.7 K

Driving at the resonant frequency $f_p = 80$ Hz
(n ~ 30)

Dashed-line $P_\omega^{\,2} \sim \omega^{-3.7}$

High-frequency boundary $\omega_b \sim 4$ kHz

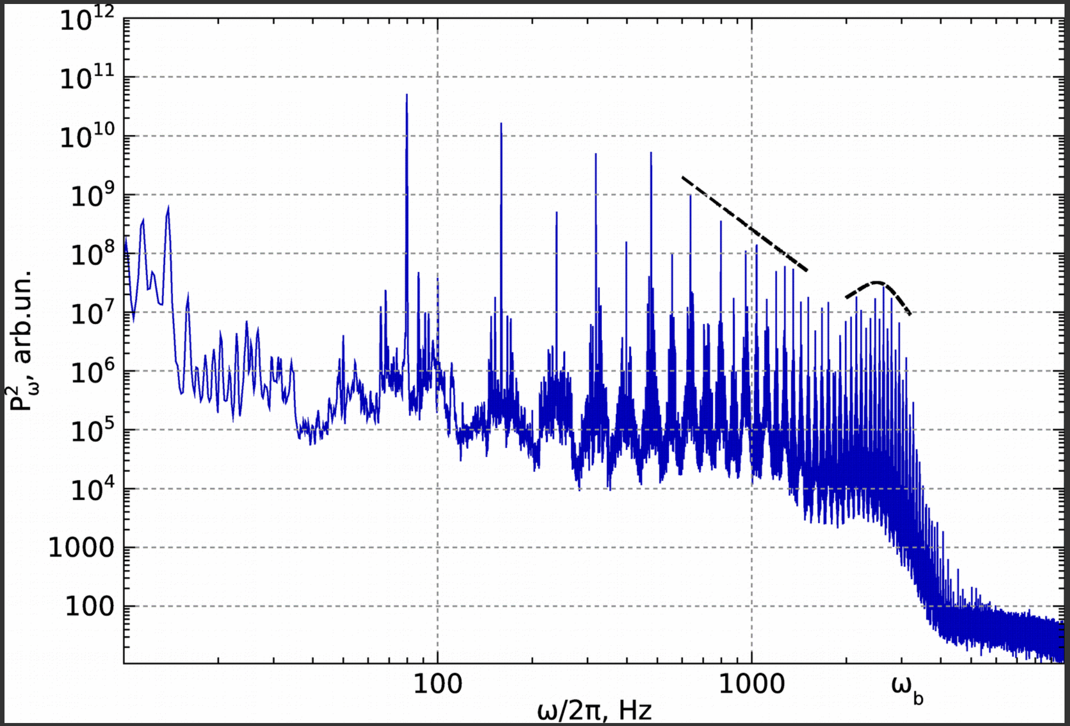


Experimental results: He-II

Turbulent spectrum after reducing the driving voltage to 10 V

Dashed-line $P_\omega^2 \sim \omega^{-3.7}$

A local maximum is observed at ~ *2.5 kHz*



Local maximum is observed in number of experiments.
Prerequisites:

- harmonic driving force
- moderate driving amplitudes

He-II: Influence of discreteness

Viscous broadening:

$$\delta\omega_\nu = 2\,\nu\, k^2 \sim (\rho/\sigma)^{2/3}\, \omega^{4/3}$$

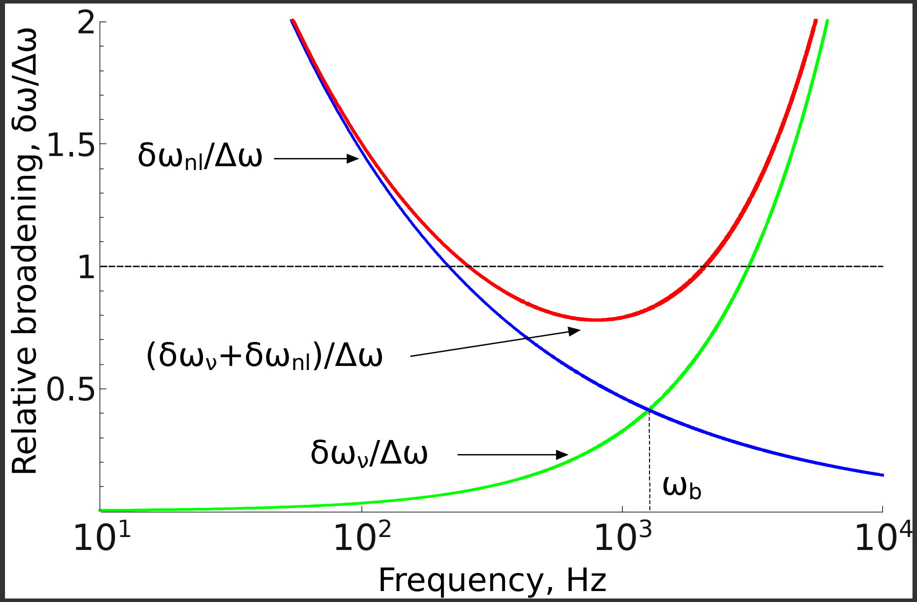
Nonlinear broadening:

$$\delta\omega_{nl} = \tau_{nl}^{\;-1} \sim f(n_k)\, \omega^{-1/6}$$

Distance between neighbouring resonances:

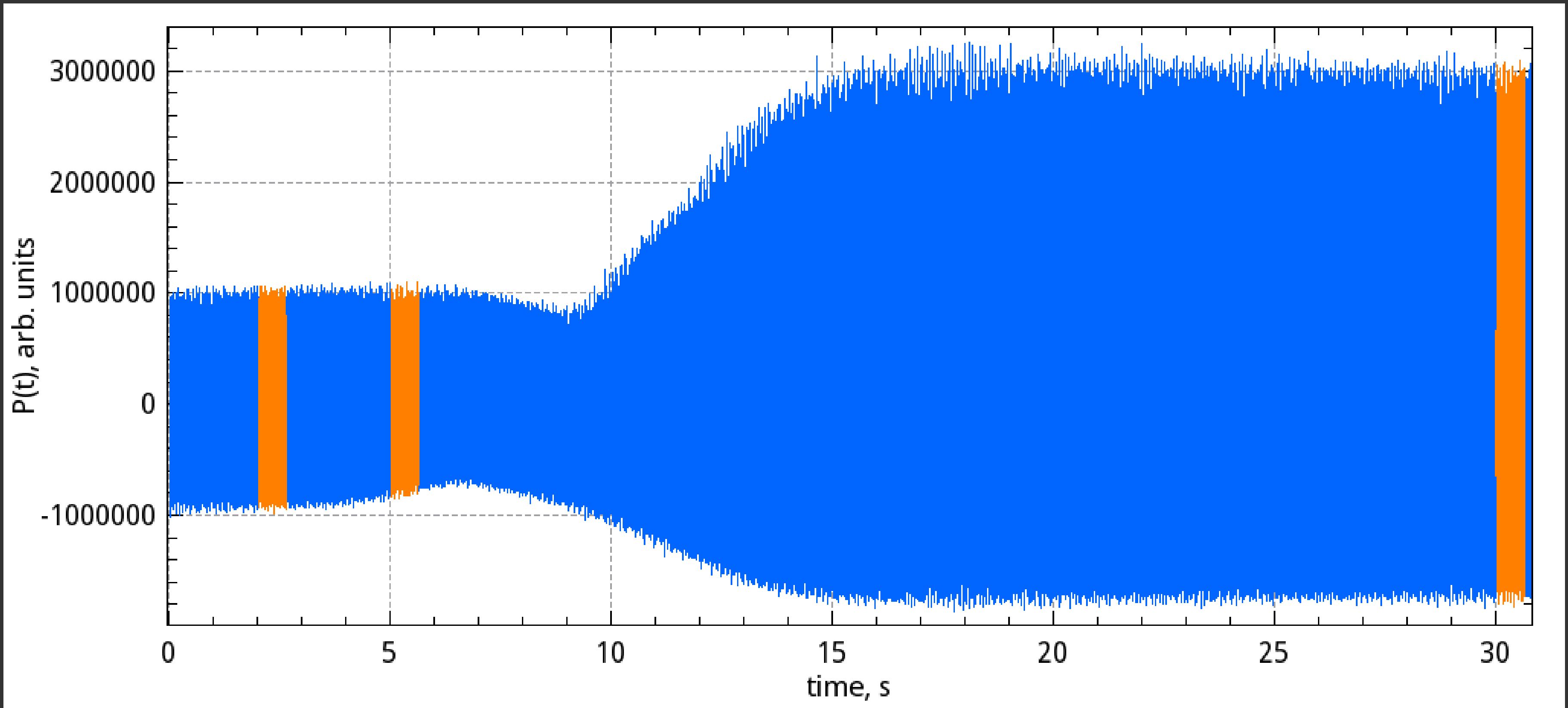
$$\Delta\omega \approx 3\pi/D\, (\sigma/\rho)^{1/3}\, \omega^{1/3}$$

$f(n_k)$ is estimated from the condition at the high frequency boundary: $\delta\omega_{nl} \sim \delta\omega_\nu$



The discrete regime can be realized when $\delta\omega / \Delta\omega < 1$, which is possible at moderate amplitudes of pumping.

Experimental results: H_2



Experimental cell: diameter – 60 mm, depth – 4 mm

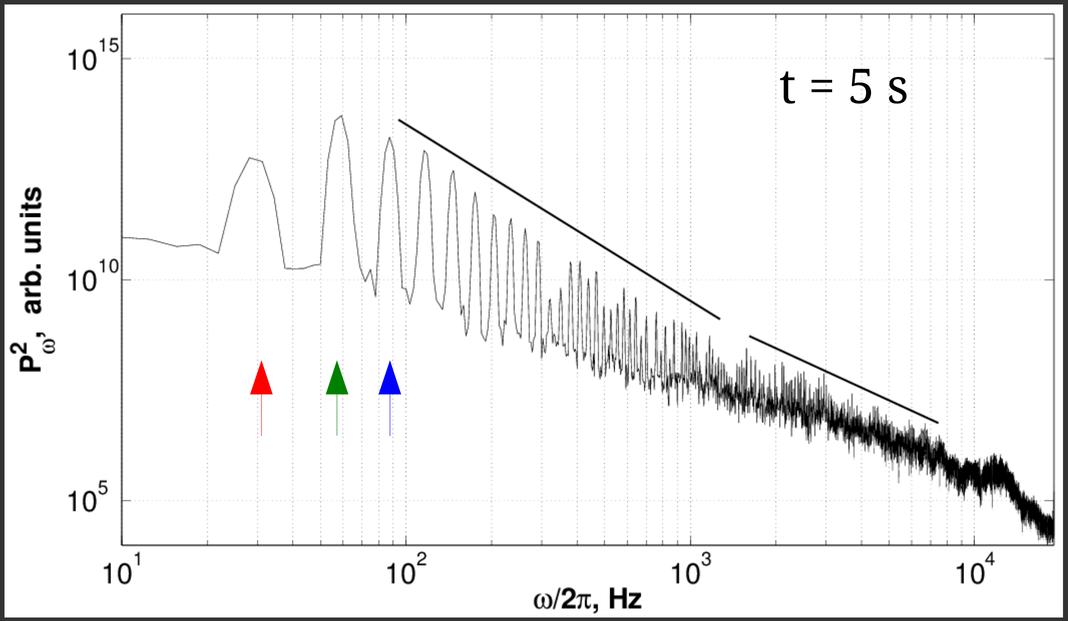
Driving at the resonant frequency $f_p = 58.4\ Hz$ (n=15)

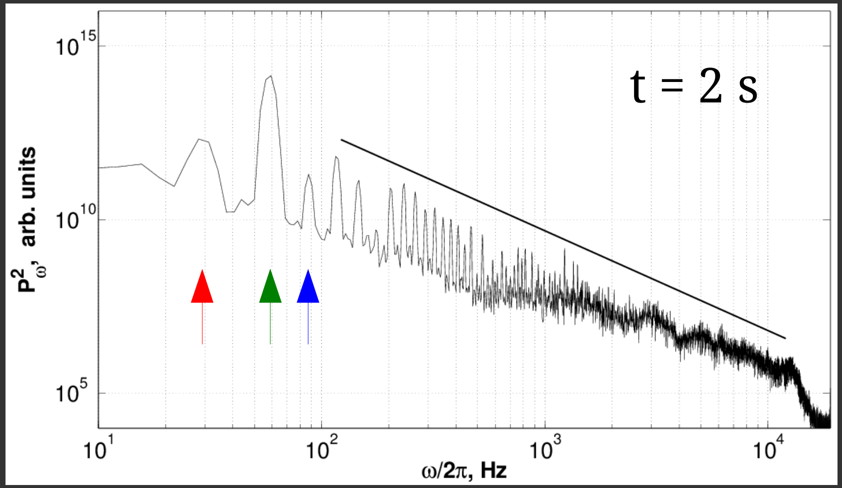
Experimental results: H_2

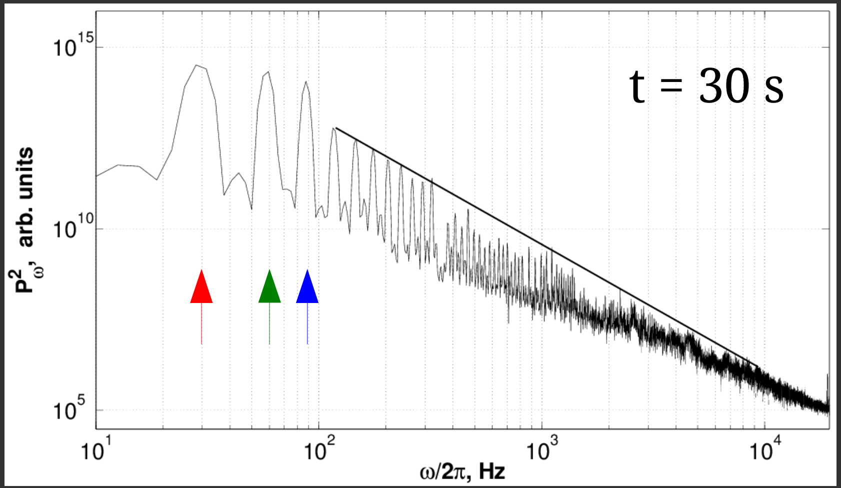
$f_p = 58.4\ Hz$

$f_1 = 29.2\ Hz$

$f_3 = 87.6\ Hz$







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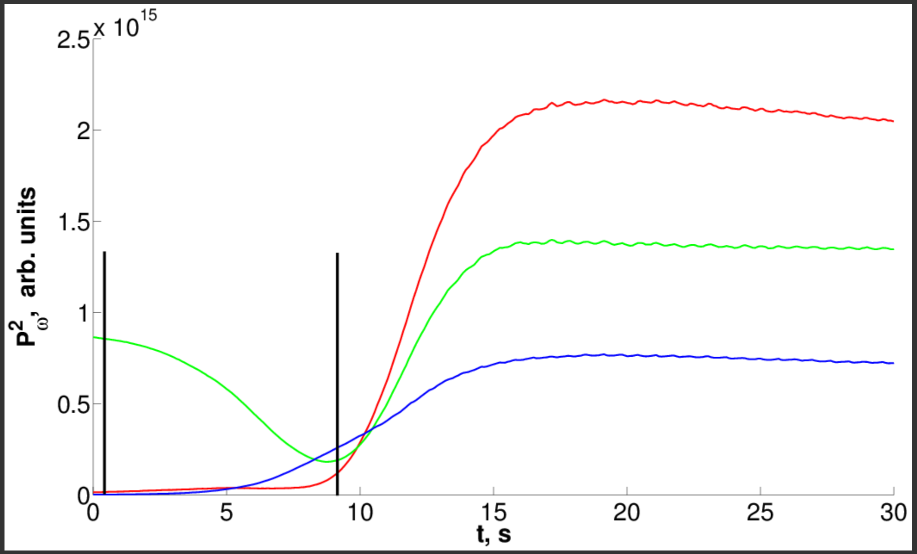

The generation of subharmonic at $f_p/2$ is accompanied by a significant wave energy loss at the main frequency (~90%). In the same time window the energy accumulation is seen in the high frequency domain. Unlike He-II the eigenmode spectrum of oscillations on the surface of H_2 can be considered as quasi-continuous above 1kHz.

Summary

The energy accumulation (bottleneck) has observed in system of capillary waves on the surface of superfluid helium and liquid hydrogen.

In the case of superfluid helium the bottleneck is observed on the steady state spectra of capillary turbulence generated by harmonic pumping. The discreteness of eigen modes is a primary cause for formation of the bottleneck.

In the case of liquid hydrogen the bottleneck is seen during reorganization of the turbulent cascade related to the generation of subharmonic at half of the driving frequency and combination harmonics. The increased energy flux towards high frequencies and finiteness of the dissipation scale result in temporary wave energy accumulation near the high frequency boundary of the inertial range.